# Life prediction of ZPW-2000A track circuit equipment based on SVDD and gray prediction

WANG Rui-feng, JIA Nan
(*School of Automation & Electrical Engineering, Lanzhou Jiaotong University, Lanzhou 730070, China*)

**Abstract**: Evaluation of the health state and prediction of the remaining life of the track circuit are important for the safe operation of the equipment of railway signal system. Based on support vector data description (SVDD) and gray prediction, this paper illustrates a method of life prediction for ZPW-2000A track circuit, which combines entropy weight method, SVDD, Mahalanobis distance and negative conversion function to set up a health state assessment model. The model transforms multiple factors affecting the health state into a health index named $H$ to reflect the health state of the equipment. According to $H$, the life prediction model of ZPW-2000A track circuit equipment is established by means of gray prediction so as to predict the trend of health state of the equipment. The certification of the example shows that the method can visually reflect the health state and effectively predict the remaining life of the equipment. It also provides a theoretical basis to further improve the maintenance and management for ZPW-2000A track circuit.

**Key words**: track circuit; health state assessment; life prediction; support vector data description (SVDD); gray prediction

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## 0 Introduction

With its wide application in railway signal system, ZPW-2000A jointless track circuit has played a direct and critical role in the efficiency and safety of railway transportation. Nowadays, artificial regular maintenance and troubleshooting are often used in the prediction and health management of track circuit. But due to lack of intelligent assessment of the equipment, it might cause under-maintenance, over-maintenance and low equipment utilization etc. Therefore, it is of great theoretical and practical meanings to improve the management level by establishing a reasonable assessment of the health state of ZPW-2000A and by predicting its life.

To solve the problem, Zhao, et al.[1] proposed a track circuit diagnosis method based on genetic algorithm, which could judge several compensation capacitor failure and channel resistance fluctuations. Wang, et al.[2] analyzed the reliability of ZPW-2000A track circuit by means of failure mode influence analysis and fault tree analysis. To improve efficiency and accuracy of track circuit fault diagnosis, Wang, et al.[3] presented a fault diagnosis model based on least squares support vector machine. Wu, et al.[4] summarized the common failures of ZPW-2000A track circuit and established a fault diagnosis system based on decision tree C4.5 algorithm and expert system. Zhang, et al.[5] proposed a method to assess the health state of ZPW-2000A track circuit equipment based on fuzzy comprehensive evaluation and realized the life prediction. In the present, however, research on the state assessment of the track circuit is mostly confined to reliability analysis and fault diagnosis.

In this paper, a new health assessment method was applied to the maintenance management of ZPW-2000A track circuit with the establishment of a health state assessment model and a life prediction model.

## 1 Health state assessment model

Many factors jointly cause the degradation or failure of ZPW-2000A track circuit, so its degradation mechanism becomes complicated[6]. Because of the complex relationship between factors and variables, one single variable cannot describe the health state. Therefore, this paper presents a new health state assessment model based on support

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**Corresponding author**: WANG Rui-feng (carol_nn@126.com)

vector data description (SVDD), as shown in Fig. 1.

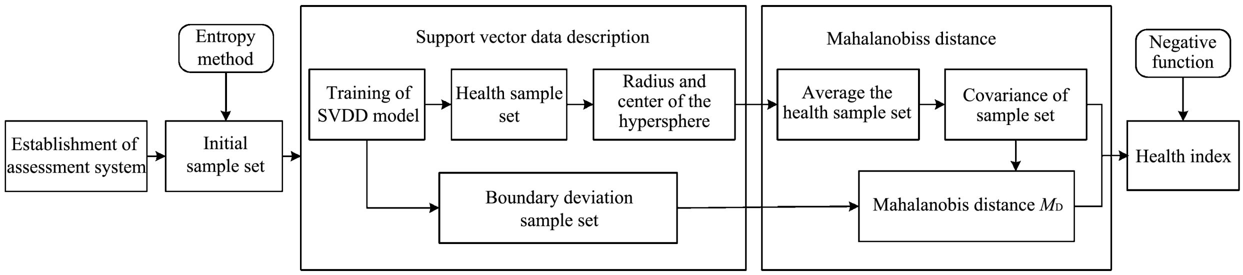


**Fig. 1 Health state assessment model**

The steps of the establishment of health state assessment model are as follows.

1) Establishment of assessment system

The assessment system block diagram is shown in Fig. 2.

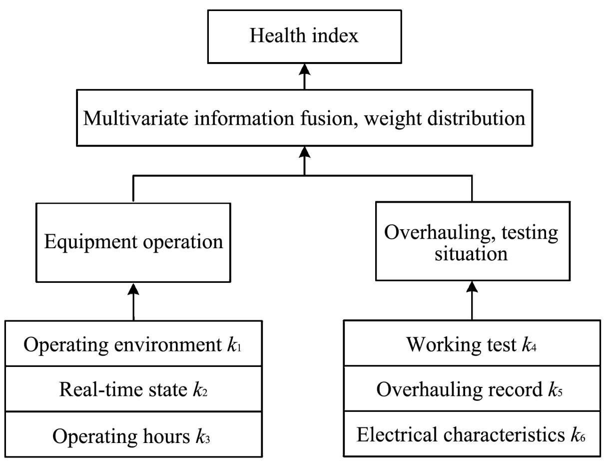


**Fig. 2 Block diagram of assessment system**

2) Determination of weight

For its strong objectivity, entropy method is used to give weight to each factor index after establishing the assessment system. Moreover, it also reflects the true level accurately[7].

3) Establishment of initial sample

As shown in Eq. (1), let the product of $x_{k_i}(j)$ and $d_i$ be the initial normal sample set, where $x_{k_i}(j)$ is the evaluation result of sample $j$ in factor index $k_i$, and $d_i$ is the corresponding weight. Taking into account the degradation curve, only the equipment less than 50% service time is selected as the reference object.

$$\boldsymbol{X}(j)=[x_{k_1}(j)\times d_1, x_{k_2}(j)\times d_2, \cdots, x_{k_6}(j)\times d_6]^{\mathrm{T}}. \tag{1}$$

4) Training of SVDD model

The SVDD model is trained by using the initial sample set. It can define a minimal hypersphere that contains high-dimensional mapped space sample sets and determines the radius and center of the hypersphere[8-9]. In order to reduce the impact of the deviation points, relaxation parameter $\xi_i$ and adjustment parameter $C_s$ are introduced, and both of them are given the default values, but the adjustment of $\xi_i$ and $C_s$ will not be discussed here.

5) Indentification of boundary deviation points

The trained hypersphere can separate the health sample set $\boldsymbol{X}_h$ (inside the hypersphere) and the boundary deviation sample set $\boldsymbol{X}_u$ (on the surface of and outside the hypersphere) from the initial sample set $\boldsymbol{X}(j)$. Supposing there are $p$ samples in $\boldsymbol{X}_h$ and $q$ in $\boldsymbol{X}_u$.

6) Calculation of Mahalanobis distance

Mahalanobis distance is used to calculate the distance between different samples, which can reflect the relationship of data better and avoid the influence of data dimension[10].

First, averaging the health sample set $\boldsymbol{X}_h$ as

$$\bar{\boldsymbol{X}}_h=[\bar{X}_{k_1}, \bar{X}_{k_2}, \cdots, \bar{X}_{k_6}]^{\mathrm{T}}, \tag{2}$$

$$\bar{X}_{k_n}=\frac{1}{p}\sum_{i=1}^{p}x_{k_n}(i),\ n=1,2,\cdots,6 \tag{3}$$

Then, calculating the covariance of the samples with different factors $k_a$ and $k_b$ by

$$Cov(X_{k_a}, X_{k_b})=\frac{\sum_{i=1}^{p}[(x_{k_a}(i)-\bar{X}_{k_a})(x_{k_b}(i)-\bar{X}_{k_b})]}{p-1},$$
$$a,b\in[1,2,\cdots,6]. \tag{4}$$

The covariance matrix $\boldsymbol{C}$ is

$$\boldsymbol{C}=\begin{bmatrix} Cov(X_{k_1},X_{k_1}) & Cov(X_{k_1},X_{k_2}) & \cdots & Cov(X_{k_1},X_{k_6}) \\ Cov(X_{k_2},X_{k_1}) & Cov(X_{k_2},X_{k_2}) & \cdots & Cov(X_{k_2},X_{k_6}) \\ \vdots & \vdots & & \vdots \\ Cov(X_{k_6},X_{k_1}) & Cov(X_{k_6},X_{k_2}) & \cdots & Cov(X_{k_6},X_{k_6}) \end{bmatrix}. \tag{5}$$

The Mahalanobis distance $M_D$ between the sample $\boldsymbol{X}(z)$ and the health sample set is obtained by

$$M_D = \sqrt{(\boldsymbol{X}(z)-\boldsymbol{X}_h)^T \boldsymbol{C}^{-1}(\boldsymbol{X}(z)-\boldsymbol{X}_h)}. \quad (6)$$

7) Calculation of health index

Taking into account the relationship between the degree of deviation and the degree of health, the negative function is chosen as the conversion model[11], and then its health index is

$$H = 100 \times e^{-bM_D}. \quad (7)$$

Let the average $M_D$ of $\boldsymbol{X}_u$ obtained in step 5 from the health sample is $M_{Dr}$, and its health index is 60, that is, $H_{\boldsymbol{X}_u} = 100 \times e^{-bM_{Dr}} = 60$, then

$$b = -\frac{\ln 0.6}{M_{Dr}}. \quad (8)$$

Finally, substituting Eq. (8) into Eq. (7), the health index $H$ can be calculated by

$$H = 100 \times e^{\frac{\ln 0.6}{M_{Dr}} M_D}. \quad (9)$$

## 2 ZPW-2000A track circuit equipment life prediction

Gray prediction theory can predict the trend of the system by analyzing the correlation between various factors in the system and finding the internal rules in the known cluttered data sequence[12]. It is widely used in the process of prediction due to various advantages such as strong adaptability and instantaneity, simple structure, fewer samples needed, etc.

The operation environment of ZPW-2000A track circuit is complex and the factors influencing health state are fuzzy and random. The health degree of ZPW-2000A track circuit can be characterized by small sample and partial information unknown, and it also has gray characteristics[13]. The life prediction of ZPW-2000A track circuit can be carried out based on gray prediction theory.

### 2.1 Health prediction based on gray prediction model

Let original sequence $\boldsymbol{x}^{(0)}$ be a set composed of $n$ ordered values as $\boldsymbol{x}^{(0)} = (x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n))$, where $x^{((0)}(k) = H_{ac}(k)$ is the actual health index, and $n$ is the number of sampling points.

Step 1: Calculating a new health index sequence $\boldsymbol{x}^{(1)}$ by exerting the accumulating generation operation (AGO) on the original health index sequence, there is

$$\boldsymbol{x}^{(1)} = (x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n)),$$

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i),\ k = 1,2,\cdots,n. \quad (10)$$

Step 2: Establishing gray differential equations, its $GM(1,1)$ is

$$\frac{d\boldsymbol{x}^{(1)}}{dt} + ax^{(1)}(k) = b. \quad (11)$$

Parameters $a$ and $b$ can be calculated by the least squares method, and then two estimated values $\hat{a}$ and $\hat{b}$ can be found. Their relationship is expressed as

$$U = \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = (\boldsymbol{B}^T \boldsymbol{B})^{-1} \boldsymbol{B}^T Y,$$

$$\boldsymbol{B} = \begin{bmatrix} -\frac{1}{2}(x^{(1)}(1)+x^{(1)}(2)) & 1 \\ -\frac{1}{2}(x^{(1)}(1)+x^{(1)}(2)) & 1 \\ \vdots & 1 \\ -\frac{1}{2}(x^{(1)}(n-1)+x^{(1)}(n)) & 1 \end{bmatrix},$$

$$\boldsymbol{Y} = (x^{(0)}(2), x^{(0)}(3), \cdots, x^{(0)}(n))^T. \quad (12)$$

Step 3: Substituting parameters $\hat{a}$ and $\hat{b}$ into Eq. (11), then is

$$\hat{x}^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak} + \frac{b}{a},$$

$$k = 0,1,\cdots,n-1. \quad (13)$$

Exerting the inverse accumulating generation operation (IAGO) on $\hat{\boldsymbol{x}}^{(1)}$, the predicted original sequence $\boldsymbol{x}^{(0)}$ is obtained as

$$\hat{x}^{(0)}(k+1) = (1-e^a)\left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak},$$

$$k = 0,1,\cdots,n-1. \quad (14)$$

Therefore, the predicted health index is

$$H_{pr}(k) = (1-e^a)\left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak},$$

$$k = 1,2,\cdots,n. \quad (15)$$

### 2.2 Process of solving remaining life

Up till now, multivariate variables have been transformed into a single health index which can reflect the health state of the equipment. According to health index, the remaining life can be calculated based on gray prediction theory. The calculation process is shown in Fig. 3.

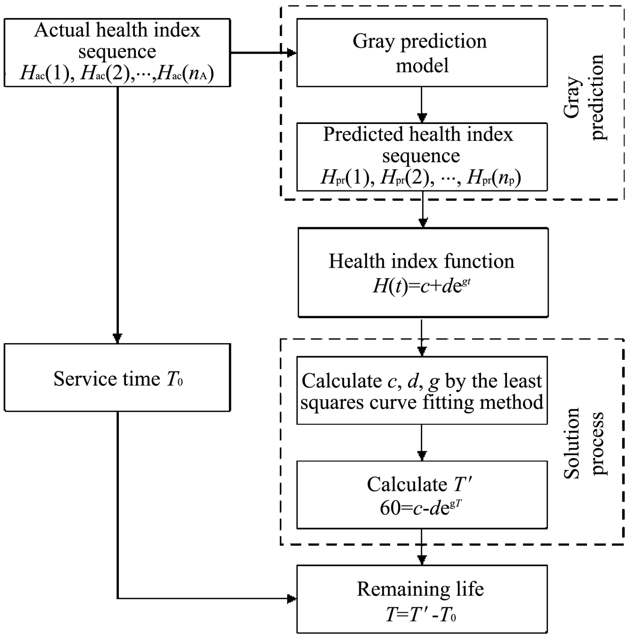


**Fig. 3 Calculation process of remaining life prediction**

Let the actual health index sequence be $H_{ac}(1), H_{ac}(2), \cdots, H_{ac}(n_A)$, where $n_A$ is the number of sampling points. The process of solving remaining life is as follows.

Analyzing the original sequence based on $GM(1,1)$ model and then inputting the actual health index sequence into the gray prediction model, the predicted health index sequence with the number of health index prediction points $n_p$ is obtained as

$$H_{pr}(1), H_{pr}(2), \cdots, H_{pr}(n_p). \tag{16}$$

Assuming that the equipment will not be repaired during service, the ideal health index curve generally shows three downward trends: monotone concave descent, monotone convex descent or monotonous fluctuation descent[14]. It can be seen that the trend of track circuit equipment is monotone convex downward, which is consistent with the exponential function. Substituting the predicted points into Eq. (15) and calculating $c$, $d$ and $g$ by the least squares curve fitting method, there is

$$H(t) = c + d\mathrm{e}^{gt},$$

$$c, g > 0;\ d < 0. \tag{17}$$

Let $H(T')=60$, the final predicted life $T'$ of the equipment can be calculated.

Assuming that the equipment has run for $T_0$, the remaining life $T$ is

$$T = T' - T_0. \tag{18}$$

# 3 Verification and analysis

## 3.1 Remaining life prediction of transmitter

Now, an example was chosen to verify the reasonableness of the model. For one transmitter that has retired from the track circuit, we took 36 groups of representative maintenance data from its first ten years of service as an initial sample to calculate its health index using the above methods and to predict its remaining life.

The appropriate field data were selected to calculate $H$ and $T$.

1) Calculation of weights

The entropy method was used to calculate the weights of the factor indexes, and the results are shown in Table 1.

**Table 1 Weights of factor indexes**

| Factor index | Weight | Factor index | Weight |
|---|---|---|---|
| $k_1$ | 0.177 7 | $k_4$ | 0.056 6 |
| $k_2$ | 0.327 6 | $k_5$ | 0.297 6 |
| $k_3$ | 0.054 7 | $k_6$ | 0.085 8 |

2) Establishment of initial sample set

The initial sample set was established by qualitatively analyzing each factor index according to the actual situation of the equipment.

Generally, the life cycle of the transmitter is 30 years. Considering the threshold value of the final life expectancy is 95% − 99.9% of the maximum value under normal working conditions[15], we chose 28.5 years as the maximum life span. As the factor index score values use percentile, the factor index score of the running time can be obtained by

$$x_{ij} = 60 + 40 \times \frac{28.5 - 10}{28.5} = 85.96.$$

Other subjective and qualitative factor index score values shown in Table 2 are graded by experts and site staffs. Combining the data from Tables 1 and 2, according to Eq. (1), the initial sample set can be calculated.

3) Training of SVDD model

The SVDD model was trained by using the initial sample set to determine the hypersphere parameters and to identify boundary deviations points. The trained health sample set and the deviated sample set are

$$\boldsymbol{X} = [X(1), X(2), \cdots, X(36)], \tag{19}$$

$$\boldsymbol{X}_{\mathrm{u}} = [X(4), X(6), X(8), X(10), X(11), X(23), X(24), X(26), X(31)], \tag{20}$$

$$\boldsymbol{X}_{\mathrm{h}}(j) = \boldsymbol{X} - \boldsymbol{X}_{\mathrm{u}}. \tag{21}$$

**Table 2 Factor index scores**

| $j$ | Scores | | | | | |
|---|---|---|---|---|---|---|
| | $k_1$ | $k_2$ | $k_3$ | $k_4$ | $k_5$ | $k_6$ |
| 1 | 85.96 | 86.32 | 91.85 | 93.05 | 91.96 | 91.85 |
| 2 | 94.77 | 92.59 | 97.56 | 96.82 | 98.48 | 90.94 |
| 3 | 93.24 | 81.38 | 90.70 | 90.49 | 95.20 | 98.73 |
| 4 | 98.74 | 95.27 | 96.50 | 99.65 | 97.53 | 90.86 |
| 5 | 96.56 | 82.83 | 98.03 | 95.61 | 96.32 | 93.67 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| 35 | 90.29 | 95.41 | 97.48 | 96.18 | 95.20 | 99.02 |
| 36 | 97.71 | 90.78 | 93.83 | 92.34 | 91.65 | 97.21 |

4) Calculation of health index

After calculating the average Mahalanobis distance from the deviated sample set to the health sample set ($M_{\mathrm{Dr}}=11.6757$) as well the coefficient ($b=0.0437$) according to Eq. (8), the final health index calculation equation is

$$H = 100 \times \mathrm{e}^{-0.0437 \times M_{\mathrm{D}}}. \tag{22}$$

5) Prediction of health index

The original health index sequence was obtained by calculating the health index of maintenance and troubleshooting records with $V_{\mathrm{t}}=1$ (Unit: a) and by building $GM(1,1)$ model, as listed in Table 3.

**Table 3 Actual health index of transmitter**

| $T$ (a) | $H_{\mathrm{ac}}$ | $T$ (a) | $H_{\mathrm{ac}}$ |
|---|---|---|---|
| 1 | 98.95 | 6 | 97.38 |
| 2 | 98.46 | 7 | 97.01 |
| 3 | 98.25 | 8 | 96.59 |
| 4 | 97.99 | 9 | 96.11 |
| 5 | 97.71 | 10 | 95.55 |

The data in Table 3 were input into the gray prediction model, the possible trend of the next 10 points are predicted, as shown in Table 4. The curves of the actual data and predicted health index are shown in Fig. 4. It can be seen that the health index of the transmitter decreases exponentially with the running time.

**Table 4 Predicted health index of transmitter**

| $T$(a) | $H_{\mathrm{pr}}$ | $T$(a) | $H_{\mathrm{pr}}$ |
|---|---|---|---|
| 11 | 94.92 | 16 | 90.13 |
| 12 | 94.20 | 17 | 88.72 |
| 13 | 93.38 | 18 | 87.12 |
| 14 | 92.43 | 19 | 85.30 |
| 15 | 91.35 | 20 | 83.21 |

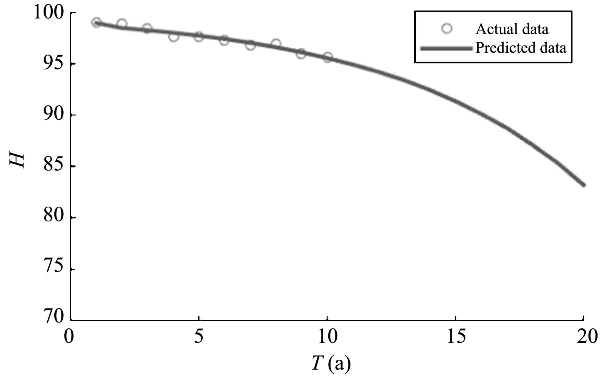


**Fig. 4 Predicted curves for tendency of health index**

6) Solution of function

The least squares curve fitting method is used for fitting the predicted data $H_{\mathrm{pr}}$ and solving the curve coefficient, and then the final health index function is

$$H(t) = 100.5546 - 1.4419\mathrm{e}^{0.1243t}. \tag{23}$$

7) Calculation of remaining life

Let $H(T')=100.5546-1.4419\mathrm{e}^{0.1243T'}=60$, then $T'=26.84$ a. As known that the service time $T_0=10$, the remaining life $T$ is

$$T = T' - T_0 = 26.84 - 10 = 16.84 \text{ a}. \tag{24}$$

Calculating the remaining life of the sample at each time point by using Eqs. (10)-(18), the results of the remaining life prediction of the transmitter by this method is shown in Fig. 5, and the actual remaining life is given as a reference standard.

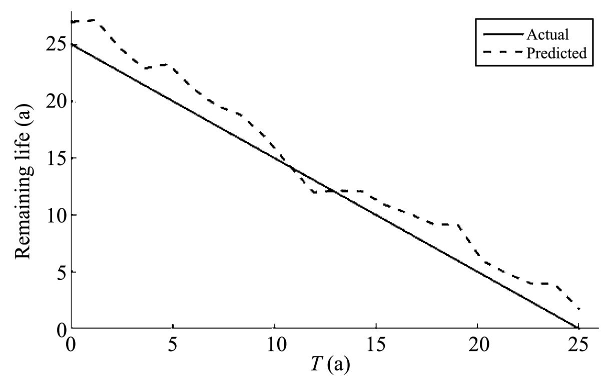


**Fig. 5 Prediction results of remaining life**

## 3.2 Model verification

In order to verify the effectiveness of this method, defining $M_{\mathrm{PE}}(k)$, which can measure the prediction method's error at a single monitoring point, namely

$$M_{\mathrm{PE}}(k) = \frac{1}{r}\sum_{h=1}^{r} | P_h(k) - A_h(k) |, \tag{25}$$

where $P_h(k)$ and $A_h(k)$ is the predicted remaining life and the actual remaining life of the sample $h$ at the time $k$; $r$ is the number of samples. According to Eq. (25), the smaller $M_{\mathrm{PE}}(k)$, the smaller the prediction error and the higher the prediction accuracy.

Calculating the remaining life of the retired sample using Zhang's method[5] and this method and then calculating their $M_{\mathrm{PE}}$ by Eq. (25), the comparison results of different methods are shown in Fig. 6.

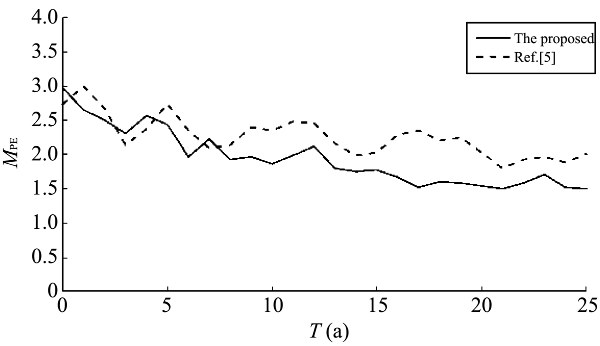


**Fig. 6 $M_{\mathrm{PE}}$ comparison of different methods**

As shown in Fig. 6, the average $M_{\mathrm{PE}}$ by the proposed method in this paper is less than that of Zhang's method. Moreover, its long-term prediction error is small and its curve is smooth. It is proved that this method is reasonable and advantageous, that is, this method can obtain more accurate remaining life prediction.

## 4 Conclusion

In this paper, a health assessment model based on SVDD has been constructed to detect health state in real time and to quantify the health state of equipment under multi-state factors. At the same time, a life prediction model based on gray prediction has been established. In this model, the health index is regarded as the input to predict the remaining life under the equipment health threshold.

1) This paper presents a new method for equipment health assessment. It can reflect the real-time health state of the track circuit equipment and make up for the shortage of the current applications, which means that it is necessary to study on the health of the track circuit equipment.

2) The predicted life curve of the track circuit equipment in this paper can accurately characterize the degradation process of health state, which state that the method is reasonable and feasible.

3) This method is convenient for maintenance personnel to grasp the equipment state more intuitively and accurately and for railway departments to provide scientific basis and guidance for maintenance decisions. It also provides a new idea and theoretical support to realize the intelligent management of railway signal equipment.

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# 基于SVDD和灰色预测的ZPW-2000A轨道电路设备寿命预测研究

王瑞峰，贾　楠

（兰州交通大学 自动化与电气工程学院，甘肃 兰州 730070）

**摘　要：** 轨道电路作为铁路信号的重要基础设备，对其进行健康状态评估和剩余寿命预测可以有效保障设备安全运行。本文描述了基于支持向量数据描述法(support vector domain description，SVDD)和灰色预测的ZPW-2000A轨道电路寿命预测方法。采用熵权法、SVDD、马氏距离和负向转换函数等建立健康状态评估模型，将影响健康状态的多因素转化为反映轨道电路设备健康状态的指标，即健康度 $H$；根据健康度，建立基于灰色预测的ZPW-2000A轨道电路设备寿命预测模型，预测健康状态未来的发展趋势。实例验证表明，该方法能直观地看出轨道电路设备健康状态水平，并有效地预测设备的剩余寿命，为进一步提高维修管理水平提供了理论依据。

**关键词：** 轨道电路；健康状态评估；寿命预测；支持向量数据描述；灰色预测

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