Balancing control of unicycle robot by using sliding control

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Abstract: This paper discusses about balancing control of unicycle robot. Unicycle robot consists of pitch which acts like inverted pendulum and roll which acts like reaction wheel pendulum. The robot which does not have actuator located in yaw axis is made to derive the simple dynamics. Lagrange equation is applied to deriving dynamic equations. Obtained dynamic equations are used to design the sliding mode control. State variables of the designed control are pitch angle and roll angle. Sliding mode control has chattering problem, which is eliminated by using the sigmoid function as switching function. Finally the control performance and eliminated chattering problem is verified by simulation.

Key words: balancing control; sliding control; dynamic equation; lagrange; unicycle robot

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Balancing control of unicycle robot is to be discussed in this paper. The unicycle robot consists of pitch and roll. The former acts like inverted pendulum and the latter acts like reaction wheel pendulum. Two actuators are located in the wheel and disc of the unicycle robot, which can be used to control balance. Angles of robot that are called pitch angle and roll angle are obtained from the inertial sensor that is located in the central axis of the robot.

Sliding mode control is used to control the robot. The sliding mode control makes a system state reach the sliding surface in order to trammel the system state near the sliding surface. This means that an error of the system state is decaying and small. The control input of sliding control consists of equivalence control input and robust control input.

As Fig. 1 shows, the form of the disc is given to generate greater rotational inertia, and the system is designed whose central axis to be center of gravity.

Here, the equivalence control input maintains the system state near sliding surface. And the robust control input pushes the system state to sliding surface, when the system state stays away from sliding surface. The robust control input contains switch function to keep robust property. Because of such switch function, the chattering phenomenon occurred, which causes damage of actuators and produces some disturbance to the system state. In this paper, the chattering phenomenon is eliminated by using sigmoid function such as tanh function to produce the robust control input.

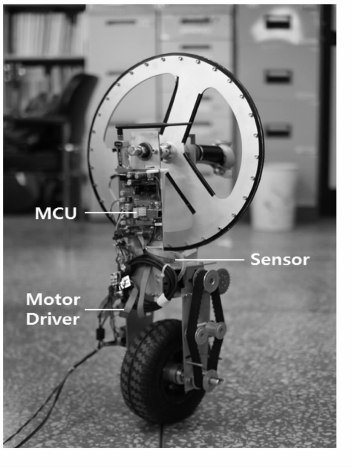

Fig. 1 Unicycle robot

Controlled variable signal which generated by controller is entered into the DC motor drive to generate appropriate torque which is needed to control the attitude. Research on the single-wheel (unicycle) robot has been ongoing since the 1980s in the U.S. and Japan. Schoonwinkel A of Stanford University proposed a linear dynamic model of the robot and presented its optimal motion control in his

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Ph. D. thesis in 1987. Prof. Yamafujii of Tokyo University also proposed a dynamic model of the robot as an upper turntable and lower rotating wheel. In the last few years, several unicycle robots have been implemented in the Intelligent Robot Laboratory at Pusan National University. The early unicycle model had actuators that were placed in the center of the robot to control the yaw direction. For this reason the dynamic equation was too complex to design the controller and it took a great deal of time to generate proper controlled variable. Therefore, the unicycle robot without yaw actuators is made and the dynamic equations of this robot are derived. The dynamic equations are applied to designing sliding mode control. The designed controller makes this robot keep the balance[1].

1 System structure

1.1 Sensor

The inertial measurement unit (IMU) sensor is placed on the center of the body to simultaneously get the degree (pitch, roll) of the robot. The center means that the center is the middle of roll axis and pitch axis. The sensor data is transmitted to main micro controller unit (MCU) by using RS-232 protocol and the interval of sensor data is 10 ms.

1.2 Main control

In roll and pitch control, we use the DC motor to rotate the disc of the top and the wheel of the bottom. The DC motor is driven by motor-drive module which consists of H-bridge. The motor-drive's input is pulse width modulation (PWM). The motor's torque and speed of rotation are changed by input PWM's width.

At interval of 10 ms, the MCU transmits control signal to the motor-drive to keep the balance according to the designed controller. The control signal of the MCU is PWM and the motor-drive generates power for DC motor to rotate according to the input signal.

Fig. 2 describes the overall system structure of the unicycle robot.

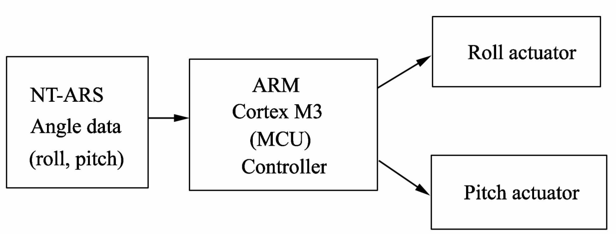

Fig. 2 System structure of unicycle robot

2 Dynamics of the robot

Set the axis as shown in Fig. 3. L denotes the length from the center of the wheel to the center of the body, R_D denotes the radius of the disc, R_W denotes the radius of the wheel, θ_R denotes the angle of the roll axis, θ denotes the angle of the wheel, ψ denotes the angle of the pitch axis, θ_D denotes the angle of the disc, M_1 denotes the wheel mass, M_2 denotes the body mass and M_3 denotes the disc mass.

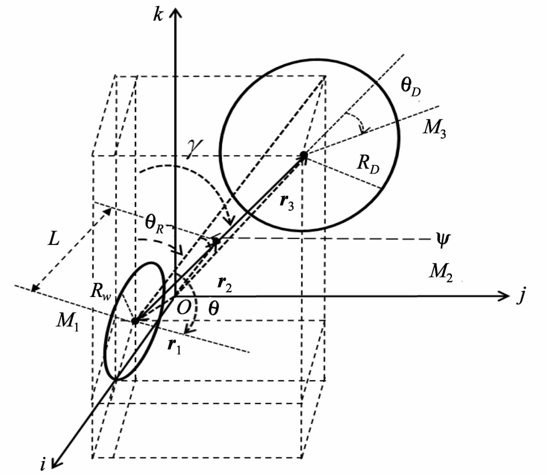

Fig. 3 Model of unicycle robot

The parameters of the unicycle robot are given in Table 1.

Table 1 Parameters of the unicycle robot

Symbol	Value
M_1	1.225 kg
m	1.300 kg
M_2	4.889 kg
M_3	4.964 kg
R_W	0.110 m
R_D	0.200 m
L	0.330 m
J_d	0.529 5 kg·m^2
J_ψ	0.720 8 kg·m^2
J_w	0.007 9 kg·m^2
J_{mo}	0.000 1 kg·m^2

In this section, dynamic equation of the unicycle robot is derived. There are other methods which can get dynamic equations of some systems. Here, the Lagrange equation is used to derive dynamic equation[2-3].

Generally, Lagrange equation needs the kinetic and potential energy to obtain the dynamic equation.

First, position vectors are considered to get kinetic energy, which are

$$\boldsymbol{r}_1 = R\theta\hat{i} + R\sin\theta_R\hat{j} + R\cos\theta_R\hat{k},$$

$$\boldsymbol{r}_2 = (R\theta + L\sin\psi_R)\hat{i} + (R\sin\theta_R + L\cos\psi\sin\theta_R)\hat{j} + (R\cos\theta_R + L\cos\psi\cos\theta_R)\hat{k},$$

$$\boldsymbol{r}_3 = (R\theta + 2L\sin\psi_R)\hat{i} + (R\sin\theta_R + 2L\cos\psi\sin\theta_R)\hat{j} + (R\cos\theta_R + 2L\cos\psi\cos\theta_R)\hat{k}, \quad (1)$$

where $\boldsymbol{r}_1$ denotes the position vector of the wheel; $\boldsymbol{r}_2$ denotes the position vector of the body and $\boldsymbol{r}_3$ denotes the position vector.

And the velocity vectors of the body are obtained from $\boldsymbol{v}_i = \frac{\mathrm{d}\boldsymbol{r}_i}{\mathrm{d}t}$, $i = 1,2,3$. The velocity vectors are

$$\boldsymbol{v}_1 = R\dot{\theta}\hat{i} + R\dot{\theta}_R\cos\theta_R\hat{j} - R\dot{\theta}_R\sin\theta_R\hat{k}$$

$$\begin{aligned}\boldsymbol{v}_2 = {} & (L\dot{\psi}\cos\psi + R\dot{\theta})\hat{i} + (-L\dot{\psi}\sin\psi\sin\theta_R + R\dot{\theta}_R\cos\theta_R + L\dot{\theta}_R\cos\psi\cos\theta_R)\hat{j} + \\ & (-L\dot{\psi}\cos\theta_R\sin\psi - R\dot{\theta}_R\sin\theta_R - L\dot{\theta}_R\cos\psi\sin\theta_R)\hat{k},\end{aligned}$$

$$\begin{aligned}\boldsymbol{v}_3 = {} & (2L\dot{\psi}\cos\psi + R\dot{\theta})\hat{i} + (-2L\dot{\psi}\sin\psi\sin\theta_R + R\dot{\theta}_R\cos\theta_R + 2L\dot{\theta}_R\cos\psi\cos\theta_R)\hat{j} + \\ & (-2L\dot{\psi}\cos\theta_R\sin\psi - R\dot{\theta}_R\sin\theta_R - 2L\dot{\theta}_R\cos\psi\sin\theta_R)\hat{k}. \quad (2)\end{aligned}$$

According to Eq. (2), the transition kinetic energy T_1 is

$$T_1 = \frac{1}{2}M_1(\boldsymbol{v}_1 \cdot \boldsymbol{v}_1) + \frac{1}{2}M_2(\boldsymbol{v}_2 \cdot \boldsymbol{v}_2) + \frac{1}{2}M_3(\boldsymbol{v}_3 \cdot \boldsymbol{v}_3). \quad (3)$$

According to Eq. (3), the kinetic energy is

$$\begin{aligned}T_1 = {} & \frac{1}{4}(2L^2(M_2 + 4M_3)\dot{\psi}^2 + 4L(M_2 + 2M_3)R\dot{\psi}\dot{\theta}\cos\psi + 2(M_1 + M_2 + M_3)R^2\dot{\theta}^2 + \\ & (L^2M_2 + 4L^2M_3 + 2M_1R^2 + 2M_2R^2 + 2M_3R^2 + 4L(M_2 + M_3)R\cos\psi + \\ & L^2(M_2 + 4M_3)\cos2\psi)\theta_R^2). \quad (4)\end{aligned}$$

And kinetic energy also contains the rotational energy. The rotational energy is

$$T_2 = \frac{1}{2}J_w\dot{\theta}^2 + \frac{1}{2}J_{mo}n^2(\dot{\theta} - \dot{\psi})^2 + \frac{1}{2}J_\psi\dot{\psi}^2 + \frac{1}{2}J_d(\dot{\theta}_R + \dot{\theta}_D)^2, \quad (5)$$

where J_w is rotational inertia of the wheel, J_{mo} is rotational inertia of the motor, J_ψ is rotational inertia of the body and J_ψ is rotational inertia of the disc.

Total kinetic energy is sum of T_1 and T_2. Secondly, the potential energy V is

$$V = M_1gR\cos\theta_R + M_2g(R\cos\theta_R + L\cos\psi\cos\theta_R) + M_3g(R\cos\theta_R + 2L\cos\psi\cos\theta_R). \quad (6)$$

Finally, the Lagrangian is obtained from the early process. Definitely, Lagrangian is difference kinetic energy and potential energy. In this paper, Lag means the Lagrangian.

$$\begin{aligned}\mathrm{Lag} = T - V = {} & \frac{1}{4}(2L^2(M_2 + 4M_3)\dot{\psi}^2 + 4L(M_2 + 2M_3)R\dot{\psi}\dot{\theta}\cos\psi + 2(M_1 + M_2 + M_3)R^2\dot{\theta}^2 + \\ & (L^2M_2 + 4L^2M_3 + 2M_1R^2 + 2M_2R^2 + 2M_3R^2 + 4L(M_2 + M_3)R\cos\psi + \\ & L^2(M_2 + 4M_3)\cos2\psi)\theta_R^2) + \frac{1}{2}J_w\dot{\theta}^2 + \frac{1}{2}J_{mo}n^2(\dot{\theta} - \dot{\psi})^2 + \frac{1}{2}J_\psi\dot{\psi}^2 + \\ & \frac{1}{2}J_d(\dot{\theta}_R + \dot{\theta}_D)^2 - M_1gR\cos\theta_R + M_2g(R\cos\theta_R + L\cos\psi\cos\theta_R) + M_3g(R\cos\theta_R + 2L\cos\psi\cos\theta_R). \quad (7)\end{aligned}$$

Substitute the Lagrangian(Lag) into the Lagrange equation to obtain a dynamic equations. The dynamic equations of the robot are obtained by

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial Lag}{\partial \dot{\boldsymbol{q}}} - \frac{\partial \mathrm{Lag}}{\partial \boldsymbol{q}} = \tau_q, \quad (8)$$

where

$$\boldsymbol{q} = [\theta_R \quad \theta_D \quad \psi \quad \theta]^{\mathrm{T}}.$$

Now, the dynamic equations of the system states of the unicycle robot are obtained from

$$\tau_R = \frac{1}{2}(-4L(M_2+2M_3)R\sin\psi\dot{\psi} - 2L^2(M_2+4M_3)\sin\psi\dot{\psi}\dot{\theta}_R + J_b\ddot{\theta}_R + \frac{1}{2}(L^2M_2 + 4L^2M_3 + 2M_1R^2 + 2M_2R^2 + 2M_3R^2 + 4L(M_2+2M_3)R\cos\psi + L^2(M_2+4M_3)\cos2\psi)\ddot{\theta}_R + J_d(\ddot{\theta}_d + \ddot{\theta}_R) - (gM_1R\sin\theta_R - gM_3(-R\sin\theta_R - 2L\cos\psi\sin\theta_R) - gM_2(-R\sin\theta_R - L\cos\psi\sin\theta_R)), \tag{9}$$

$$\tau_D = J_d(\ddot{\theta}_D + \ddot{V}_R), \tag{10}$$

$$\tau_\psi = J_\psi(\ddot{\psi}) - J_{mo}n^2(-\ddot{\psi}+\ddot{\theta}) + \frac{1}{4}(-4L(M_2+wM_3)R\sin\psi\dot{\psi}\dot{\theta} + 4L^2(M_2+4M_3)\ddot{\psi} + 4L(M_2+2M_3)R\cos\psi\ddot{\theta}) - (gLM_2\cos\theta_R\sin\psi + 2gLM_3\cos\theta_R\sin\psi + \frac{1}{4}(-4L(M_2+2M_3)R\sin\psi\dot{\psi}\dot{\theta} + (-4L(M_2+2M_3)R\sin\psi - 2L^2(M_2+4M_3)\sin2\psi)\dot{\theta}_R^2)), \tag{11}$$

$$\tau_\theta = J_w\ddot{\theta} + J_{mo}n^2(-\ddot{\psi}+\ddot{\theta}) + \frac{1}{4}(-4L(M_2+2M_3)R\sin\psi\dot{\psi}^2 + 4L(M_2+2M_3)R\cos\psi\ddot{\psi} + 4(M_1+M_2+M_3)R^2\ddot{\theta}) = \tau_\theta. \tag{12}$$

3 Design control

So far dynamic equations of the robot are obtained, which are used to design the sliding control. The unicycle robot has two actuators which are separately located in roll and pitch and use these actuators to control the attitude. Because the sliding mode controller has single-input single-output structure, the two sliding mode controllers are needed to control these actuators properly.

Therefore, the two controllers called pitch controller and roll controller are designed. Pitch control uses ψ as state variable and roll control uses θ_R as state variable[4].

3.1 Pitch control

Sliding surface is established to design the sliding controller. Sliding surface of pitch is

$$s_{\text{pitch}} = K_{\text{pitch}}e_{\text{pitch}} + \dot{e}_{\text{pitch}}, \tag{13}$$

where K_{pitch} is constant and e_{pitch} is difference between reference input and current pitch angle.

$\dot{s}_{\text{pitch}}$ is obtained to find the equivalence control input, $u_{\text{eq\_pitch}}$, which is

$$\dot{s}_{\text{pitch}} = K_{\text{pitch}}\dot{e}_{\text{pitch}} + \ddot{e}_{\text{pitch}}, \tag{14}$$

where $\ddot{e}_{\text{pitch}} = \ddot{\psi} - \ddot{\psi}_{\text{ref}}$, which is the second order equation of e_{pitch}; $\ddot{\psi}$ is obtained from dynamic equations.

$$\dot{s}_{\text{pitch}} = K_{\text{pitch}}\dot{e}_{\text{pitch}} + \ddot{\psi} - \ddot{\psi}_{\text{ref}}. \tag{15}$$

When $\dot{s}$ becomes zero, e_{pitch} and $\dot{e}_{\text{pitch}}$ also become zero. At this point, the control input is called equivalent input.

Fig. 4 indicates the structure of pitch control.

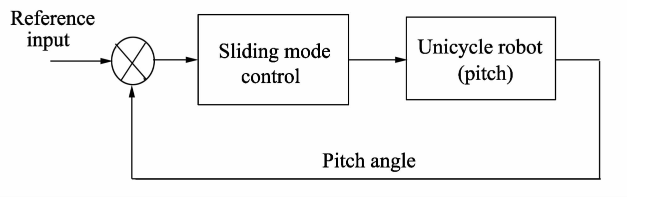

Fig. 4 Pitch control block diagram

The control input is the sum of equivalent input and robust control input. Control input u_{pitch} is

$$u_{\text{pitch}} = u_{\text{eq\_pitch}} + \gamma_{\text{pitch}}\tanh(s), \tag{16}$$

where $\gamma_{\text{pitch}} > 0$. Instead of using sgn function, tanh function is used as switching function to eliminate the chattering phenomenon.

3.2 Roll control

The sliding surface of roll is also considered to design the sliding controller. Sliding surface of roll is

$$s_{\text{roll}} = K_{\text{roll}}e_{\text{roll}} + \dot{e}_{\text{roll}}, \tag{17}$$

where K_{roll} is constant; e_{roll} denotes difference between reference input and current roll angle, $e_{\text{roll}} = \theta_R - \theta_{\text{R\_ref}}$.

To find the equivalent control input, $\dot{s}_{\text{roll}}$ is given by

$$\dot{s}_{\text{roll}} = K_{\text{roll}}\dot{e}_{\text{roll}} + \ddot{e}_{\text{roll}}, \tag{18}$$

where $\ddot{e}_{\text{roll}} = \ddot{\theta}_{\text{R}} - \ddot{\theta}_{\text{R\_ref}}$ and it is the second order equation of e_{roll}, and $\ddot{\theta}$ is obtained from dynamic equations.

When $\dot{s}$ becomes zero, e_{roll} and $\dot{e}_{\text{roll}}$ also become zero. At this point, the control input is called equivalence control input.

The control input is the sum of equivalent input and robust control input. Control input u_{roll} is

$$u_{roll} = u_{eq\_roll} + \gamma_{roll} \tanh(s), \tag{19}$$

where $\gamma_{roll} > 0$. Sigmoid function is applied as switching function to remove the chattering phenomenon.

Fig. 5 shows the block diagram of sliding mode control of roll. This block diagram is the same as block diagram of pitch.

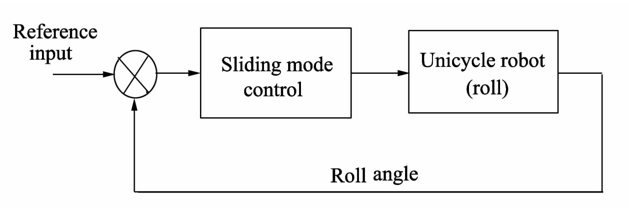

Fig. 5 Roll control block diagram

4 Simulation

The Matlab Simulink is used to check whether the chattering phenomenon is removed and to confirm that the system which is expressed by dynamic equations is controllable.

Fig. 6 shows the chattering phenomenon that switching function is signum function.

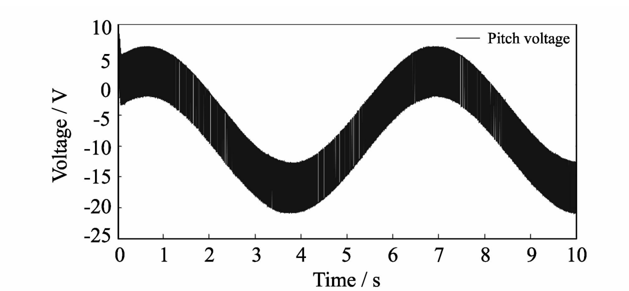

Fig. 6 Chattering phenomenon of input voltage

Fig. 7 zooms in Fig. 6 to show the chattering more clearly, Time interval is 0 − 0.5 s.

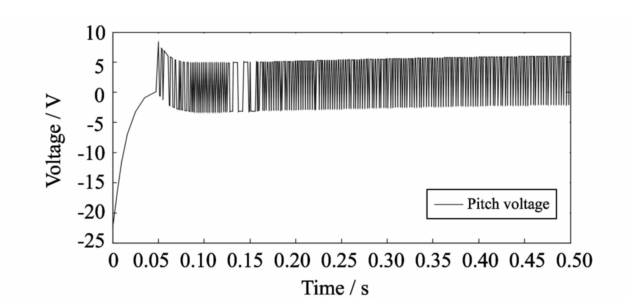

Fig. 7 Zoomed in chattering phenomenon of input voltage

Fig. 8 indicates the effect of the sigmoid function. From Fig. 8, it can be seen that the method eliminating the chattering by using sigmoid function as switching function is reasonable.

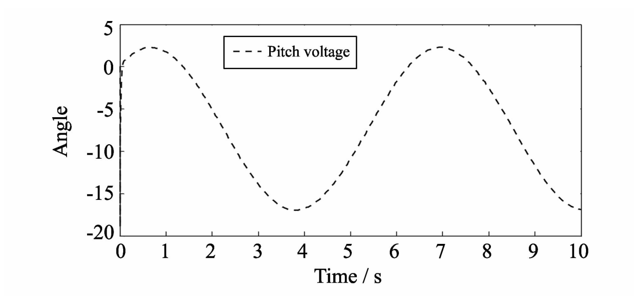

Fig. 8 Eliminated chattering by using sigmoid function

Fig. 9 is the measured roll angle which is controlled by sliding mode controller. The reference input is sin-wave amplitude is ±5, as reference angle. And at 3 s, there is some disturbance. But we confirm that the measured roll angle follows the reference input.

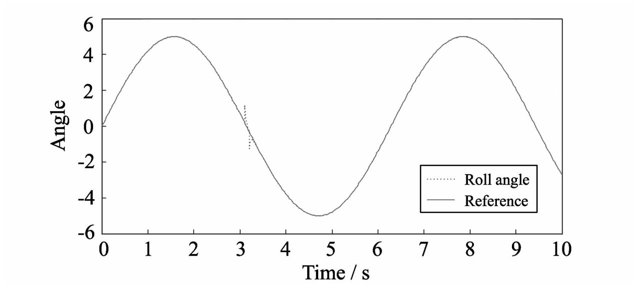

Fig. 9 Measured output pitch angle

Fig. 10 is the measured pitch angle which is controlled by sliding mode controller. The reference input is also sin-wave that amplitude is ±5, as reference angle. And at 3 s, there is also some disturbance. But we confirm that the measured pitch angle follows the reference input.

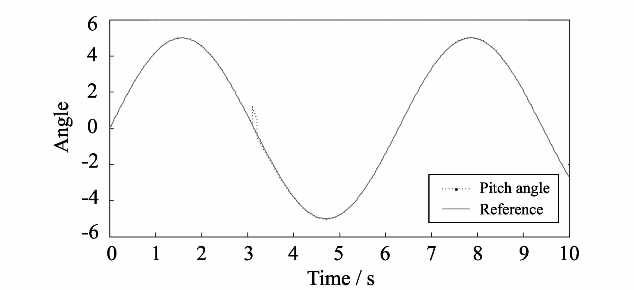

Fig. 10 Measured output roll angle

5 Conclusion

The simpler dynamic equations are obtained from the robot without the actuator in yaw axis. Sigmoid function as switching function does not generate chattering phenomenon. Those controllers and hardware are used to realize the unicycle robot. After that the driving control of unicycle robot can be conducted.

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