Gray weighted algorithm for variable voltage CT reconstruction

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Abstract: In conventional computed tomography (CT) reconstruction based on fixed voltage, the projective data often appear overexposed or underexposed, as a result, the reconstructive results are poor. To solve this problem, variable voltage CT reconstruction has been proposed. The effective projective sequences of a structural component are obtained through the variable voltage. The total variation is adjusted and minimized to optimize the reconstructive results on the basis of iterative image using algebraic reconstruction technique (ART). In the process of reconstruction, the reconstructive image of low voltage is used as an initial value of the effective projective reconstruction of the adjacent high voltage, and so on until to the highest voltage according to the gray weighted algorithm. Thereby the complete structural information is reconstructed. Simulation results show that the proposed algorithm can completely reflect the information of a complicated structural component, and the pixel values are more stable than those of the conventional.

Key words: variable voltage computed tomography (CT) reconstruction; total variation-algebraic reconstruction technique (TV-ART) algorithm; gray weight; effective projection

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X-ray computed tomography (CT) is a technique for imaging cross-sections of an object using a series of projection taken from different angles around the object. It is wildly used in industrial, medical and other fields[1]. However, for CT imaging of a complicated structural component, due to the effective thickness variation and limited dynamic range of detector system, conventional fixed single voltage imaging mode cannot get the entire image of the complicated structural component at the same time. The projective data often appear overexposed or underexposed and the reconstructive results are poor[2-3]. Therefore, the research on variable voltage CT reconstruction is of great practical significance. The variable voltage CT reconstruction makes use of the effective projection data of different voltages to reconstruct the complete information of the object.

At present, the main iteration method to reconstruct the incomplete projective data of the variable voltage conditions is algebraic reconstruction technique (ART), but the results are pure. In 2006, Sidky et al. applied total variation minimization method and projection onto convex sets (POCS) method to the CT reconstruction of incomplete projection data, and the results are very good[4-5]. Based on the research on traditional total variation-algebraic reconstruction technique (TV-ART) algorithm[6], this article applies it and gray weighted algorithm to the variable voltage CT reconstruction. In the process of reconstruction, the reconstructive image of the low voltage is used as an initial value of the effective projective reconstruction of the adjacent high voltage, and so on until to the highest voltage according to the gray weighted algorithm, thus the complete structural information is reconstructed.

Finally, the experiment shows that the proposed algorithm can completely reflect the information of a complicated structural component, and the pixel values are more stable.

1 Theoretical principle

1.1 Effective information extraction

For X-ray imaging of variable voltage, high dynamic imaging of the variable voltage X-ray is put forward in Institute of Signal Capturing & Processing Technology, North University of China[7]. In the imaging process, due to the limited dynamic range of detector system, any voltage can match the effective thickness information except the overexposure and underexposure. In this article, we extracted the effective information from projected sequences of variable voltage recursively based on the best gray level. Specific steps are as follows:

1) Read the projected sequences from low to high voltages, and generate the initialized image segmen-

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tation template with the same size;

2) Extract the available data, starting from the lowest voltage data, according to the best gray belt, and record the location and edge information of extracted data subset in the segmentation template;

3) In next extraction, the location information of previous available data is utilized to extract the rest of the location of the valid data subset, and so on, to avoid that the available data is extracted by mistake;

4) At last, determine whether the projection information can be extracted completely, according to the location information of available data subset of segmentation template;

5) Export the available data subset sequences.

1.2 TV-ART algorithm

CT reconstruction is of inverse problem

$$wf = p. \tag{1}$$

In Eq. (1), f is unknown features of the model, w is a system and p is the experimental observations. The inverse problem solves the unknown quantity f from the known w and p. When the variable voltage projection data is incomplete, CT reconstruction is the ill-posed problem[8].

The method to solve the ill-posed problem is typical total variation regularization, which has good noise resistance, strong artifact inhibition ability and facility to handle interceptive projective data and a prior knowledge of the object. It can be represented as the following optimization problem

$$\min_{f\in H(f)}\left[\frac{1}{2}\parallel wf - p\parallel^{2} + \partial TV(f)\right]. \tag{2}$$

Considering the noise influence, Eq. (2) can be equivalent to the following optimization problem

$$\min_{f\in H(f)} TV(f),$$
$$st.\ \parallel wf - p\parallel \leqslant \varepsilon. \tag{3}$$

For CT reconstruction problem, f represents the discrete image gray, $f_{s,t}$ represents the pixel gray value of the first s line and the first t column of the image. The TV algorithm is the l_1 norm of its gradient image. It can be represented as

$$TV(f) = \iint\left|\frac{\partial f}{\partial x}\right| + \left|\frac{\partial f}{\partial y}\right| \mathrm{d}x\mathrm{d}y \approx$$
$$\sum_{s,t}(\mid f_{s,t} - f_{s-1,t}\mid + \mid f_{s,t} - f_{s,t-1}\mid). \tag{4}$$

In the actual TV algorithm norm calculation, mostly l_2 norm is used to approximate the norm l_1. And a small positive parameter $\tau = 10^{-8}$ is introduced in case of that TV algorithm is infinite after derivation

$$TV(f) = \iint\sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 + \tau^2}\,\mathrm{d}x\mathrm{d}y \approx$$
$$\sum_{s,t}\sqrt{(f_{s,t} - f_{s-1,t})^2 + (f_{s,t} - f_{s-1,t})^2 + \tau}. \tag{5}$$

In 2006, based on discrete CT system[9], Sidky gave the model iterative solution

$$\min_{f\in H(f)} TV(f),$$
$$st.\ wf = p,\ f \geqslant 0. \tag{6}$$

In this article, we call this method as TV-ART method, and specific steps are as follows:

1) Initialize the reconstruction parameter: ART iterative number M, relax factor λ, TV declined relax factor ∂, and the original image $f^{(0)} = 0$;

2) ART iteration:

$$f_j^{(n)} = f_j^{(n-1)} + \lambda\frac{p_i - \tilde{p}_i}{\sum_{k=1}^{N} a_{ik}^2}a_{ij},$$
$$n = 1,2,\cdots,M. \tag{7}$$

where $\tilde{p}_i = \sum_{k=1}^{N} a_{ik} f_k^{(n-1)}$.

3) Introduce the nonnegative constraints:

$$f_j^{(\text{ART-POCS})} = \begin{cases} f_j^{(M)} & f_j^{(M)} \geqslant 0, \\ 0 & \text{else}. \end{cases} \tag{8}$$

4) TV decline:

$$f^{(\text{TV-GRAD})} = f^{(\text{ART-POCS})},$$
$$d = \parallel f^{(0)} - f^{(\text{ART-POCS})}\parallel. \tag{9}$$

5) TV minimizes the derivative and iteration:

$$v_{s,t} = \frac{\partial TV(f^{(\text{TV-GRAD})})}{\partial f_{s,t}^{(\text{TV-GRAD})}},$$

$$f^{(\text{TV-GRAD})} = f^{(\text{TV-GRAD})} - \partial\mathrm{d}\frac{v}{\parallel v\parallel}. \tag{10}$$

6) If iteration meets the termination conditions, it returns f_{res}; Otherwise, let $f^{(0)} = f^{(\text{TV-GRAD})}$, it turns to 2).

2 Gray weighted algorithm

Gray weight proposed based on the imaging process of any voltage can match the different effective thickness information. Weighted data is effective projection in the best gray belt based on experience. If we do not take effective projection data, the same point projective data information is repeated reconstructed, thus image artifacts and unsmooth edge would appear.

Due to the mass attenuation coefficient associated with voltage, different projective data between the voltages can not restore in the same function. If the projection model is transformed as logarithmic transformation, the effective projective gray between different voltages appears linear relationship

$$p = \ln \frac{I_0}{I} = ux. \tag{11}$$

According to the gray level diagram, the linear relationship between gray levels of the neighboring voltage projective data is determined, and the linear parameter is calculated by fitting. Then the projection of each low voltage data is recovered to the adjacent gray range of high voltage based on this relationship. In the process of reconstruction, the reconstructive image of the low voltage is used as an initial value of the effective projective reconstruction of the adjacent high voltage, and so on until to the highest voltage according to the gray weighted algorithm; thus the complete structural information is reconstructed. In this way, image has been reconstructed here.

3 Simulation

3.1 Simulation based on variable voltage CT

In order to verify feasibility of the proposed algorithm, we make the simulated reconstruction in computer whose configuration is memory at 2 GB, CPU at 2.5 GHz. In this simulated model, its main ingredients are outer magnesia elliptic parts and inner aluminum circular material composition. Its concrete model and structure dimension are shown in Fig. 1 and Table 1, respectively.

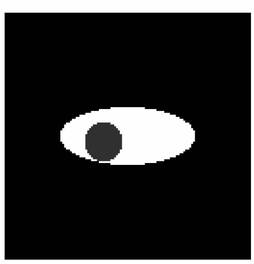


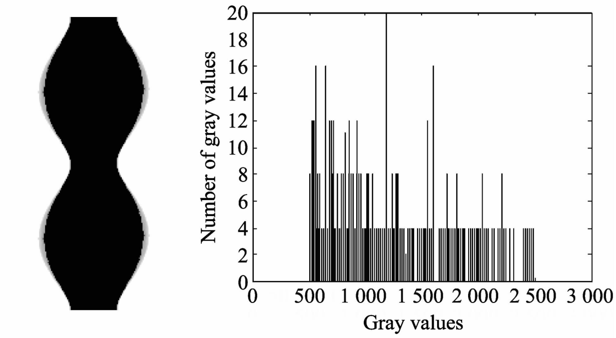
Fig. 1 Structure diagram of simulated model

Table 1 Structural size of simulated model

Parts	Outer layer		Inner layer
	Long axis	Short axis	Radius
Size (m)	70	15	10
Material	Magnesium		Aluminum

Variable voltages are 30, 40, 50, 60, 80 and 100 kV. Set the effective gray scale of the projection under variable voltages at [500,2 500]. When the gray value is less than 500, it is set at zero; or else, 4 000.

The simulated parameters are as follows: Distance of the center of rotation to X-ray source: 1 000 mm; Distance of X-ray source to the detector: 1 300 mm; Scanning mode: fan beam circular trajectory; Projected angle: 360 degrees; Detector array: 1×160. Some projected data with the limit of the dynamic range and the corresponding gray histogram are shown in Fig. 2. The object is not fully penetrated and has many dark areas in low voltage; otherwise the object is exposed and has many highlighted areas in high voltage.



(a) 30 kV projection (b) Gray histogram of 30 kV projection

(c) 100 kV projection (d) Gray histogram of 100 kV projection

Fig. 2 Effective projected data graph of partial voltages

3.2 Fixed voltage CT reconstruction

To verify the necessity of variable voltage CT relative to fixed voltage CT, the article selected the incomplete projective data under 80 kV for reconstruction using TV-ART algorithm, and the results are shown in Fig. 3.

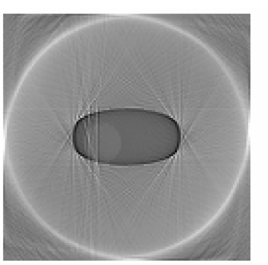


Fig. 3 Reconstructive results of 80 kV single voltage

Due to the low X-ray source, the radiation energy is weak, and incomplete reconstructive quality is poor, whose information is missing and contrast is low. Therefore, the variable voltage CT reconstruction is put forward.

3.3 Variable voltage CT reconstruction

Given the limitations of fixed voltage imaging, we reconstruct image using the projective sequences of the variable voltage. Firstly, we do unweighted gray reconstruction, and the accumulative reconstructive results under different voltages are shown in Fig. 4.

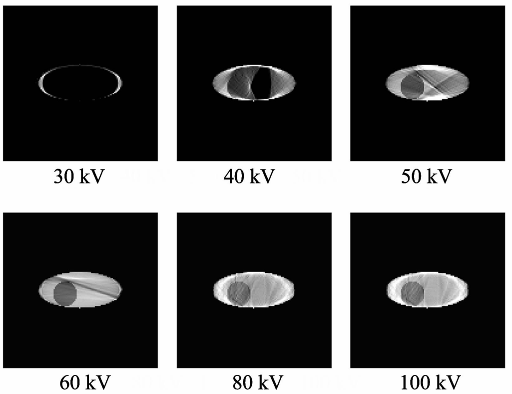


Fig. 4 Reconstructive results with gray unweighted algorithm

In unweighted gray reconstructive results, artifacts and unsmooth edge appear from the fifth image in Fig. 4. In order to improve the reconstructive results, the following gray weighted reconstruction is realized.

By section gray weighted analysis and calculation, the parameters of linear function $y = kx + b$ are presented in Table 2.

Table 2 Linear parameters between adjacent voltage projected data

Voltage (kV)	30	40	50	60	80
k	1.136	1.146	1.393	1.142	1.054
b	877.0	485.9	129.5	31.53	9.928

Again gray weighted reconstruction is done using recovery effective projective data, the accumulative reconstructive results under different voltages are shown in Fig. 5.

It can be seen from Fig. 5 that the effective information sequences of variable voltage are reflected in the the complete structure of objects enormously according to the gray weighted reconstruction. Compared to the unweighted gray algorithm, the gray weighted algorithm can make effective reconstructive information of variable voltage link in an image smoothly, therefore, the reconstructive results improve significantly after gray weighting, whose edge tends to smooth and the contrast also increases.

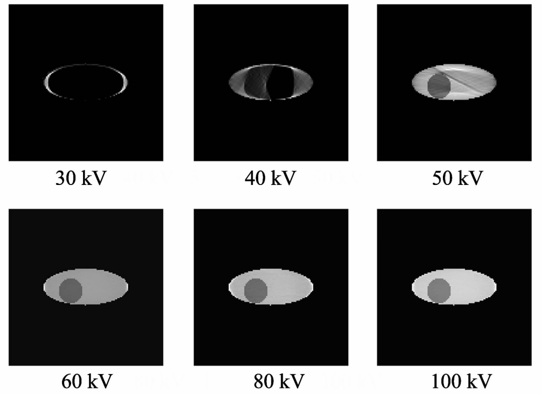


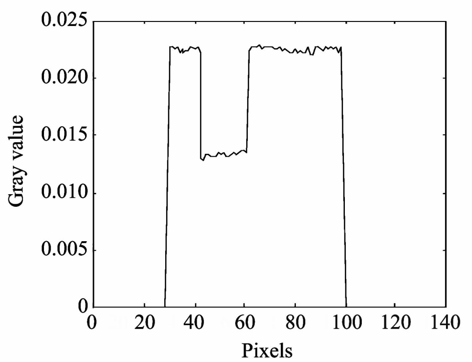
Fig. 5 Reconstruction results of gray weighted algorithm

In order to reflect the difference of those two algorithms, the normalized mean square distance of the image, the normalized mean absolute distance[10] of the image and gray curve comparing with the results of the above are shown in Table 3.

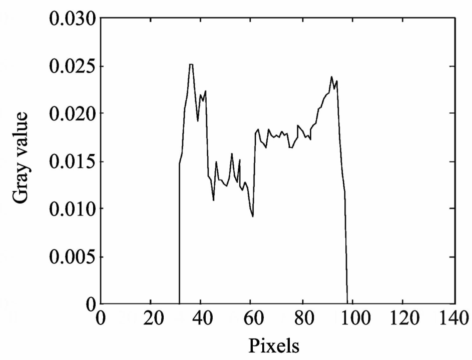
Table 3 Evaluation parameters of reconstruction results

Algorithm	Normalized mean square distance	Normalized absolute distance
Gray unweighted	0.086 9	0.052 1
Gray weighted	0.047 0	0.022 6

It can be seen from Fig. 6 that pixel gray value of unweighted gray algorithm is not so stable. While the gray weighted algorithm not only improves the reconstruction image, but also makes the gray value tend to be smoother. It indicates that the proposed algorithm has obtained the good experimental results to reduce the shortcoming that the pixel is not smooth effectively.



(a) The 64th line gray value curve of gray weighted reconstruction results



(b) The 64th gray value curve of gray unweighted reconstruction results

Fig.6 Gray value curve of reconstruction results in *X* axis

4 Conclusion

The gray weighted algorithm is proposed in this article to achieve the reconstruction of incomplete projection data of variable voltages. The simulation shows that the image quality using the proposed algorithm has been improved compared with that using the gray unweighted algorithm. Furthermore, the normalized mean square distance has been reduced effectively. The experiment can clearly show the artifacts of reconstructed results, namely the simulated data is variable voltage, attenuation coefficient is associated with energy, and gray weighted coefficient is estimated.

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变电压 CT 重建的灰度加权算法

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摘 要: 常规固定电压的 CT 重建, 因成像系统动态范围受限, 投影数据易出现过曝光和欠曝光共存现象, 造成信息缺失多, 成像质量差, 为此提出变电压 CT 重建。通过变电压获得跟工件有效厚度相匹配的有效投影序列, 在 ART 迭代图像的基础上, 调整全变差使其最小化, 从而优化重建。在重建过程中, 依据灰度加权, 把低电压的重建图像作为初值, 应用在相邻高电压有效投影重建中, 得到相邻高电压的重建图像, 依次类推直至最高电压。至此, 工件的全部结构信息重建完毕。仿真结果表明, 灰度加权算法不仅实现了变电压图像信息的完整重建, 而且像素值更加稳定。

关键词: 变电压 CT 重建; TV-ART 算法; 灰度加权; 有效投影

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