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Missile flight simulation and attitude display

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Abstract: Simulation has become an important method for missile flight research to reduce cost and improve accuracy. This paper concerns missile flight trajectory and attitude calculation. As a result, mechanical analysis and modeling are done, the characteristics of missile flight decoding algorithm and attitude parameters are analyzed, the missile trajectory is dynamically simulated in a vertical plane and the missile attitude is real-time displayed. The description of missile flight process in a laboratory has provided a basis for actual missile flight.

Key words: missile flight; simulation; attitude

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0 Introduction

Now, advanced computer technology drives not only rapid development of simulation technology, but also progress of aerospace technology. Special performance requirements of the missile must be fully in line with actual combat, but the requirements of relevant tests are obviously a huge cost. Therefore, simulation technology has become the best method for missile system development. It not only greatly reduces the cost, but also improves the accuracy. The core of missile flight is attitude angle calculation. Accordingly, selection of reasonable attitude solution algorithm is a major factor which affects flight accuracy[1].

Missile flight trajectory and solution of attitude are important research contents in the aerospace field. After analyzing and modeling missile flight mechanics based on missile flight characteristics and attitude parameter calculation algorithm, this paper dynamically simulates vertical plane missile flight trajectory and real-time displays missile attitude at the same time.

1 Basic theory and method

1.1 Coordinate system and interrelation

A missile flight motion in the space is very complicated. In order to describe the motion, missile motion equations need to be established. Of course, some coordinate systems need to be defined first[2-5]. According to missile flight mechanics, we usually use the coordinate system with ground coordinate $Axyz$, body coordinate $ox_1y_1z_1$, ballistic coordinate $ox_2y_2z_2$ and velocity coordinate $ox_3y_3z_3$. Then attitude is built to determine flight movement trend in the air according to angle relationship between the body coordinate system and the various axes. Attitude angles mainly refer to yaw angle, pitch angle and roll angle. The yaw angle is the angle between the horizontal plane projection of the missile longitudinal axis Ax and the ground coordinate axis, denoted as φ. The pitch angle is the angle between the horizontal axis and vertical axis of the body, denoted as θ. The roll angle is the angle between the plane and the vertical axis through the longitudinal axis of the missile, denoted as γ. The missile flight attitude is actually the result of missile rotating around the three-axis coordinate system, and each basic rotation angle is one of them. The results are shown in Fig.1.

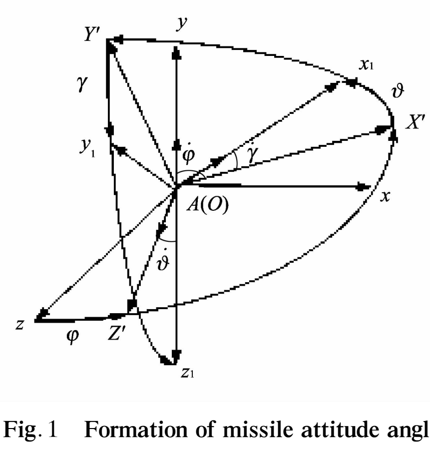

Fig.1 Formation of missile attitude angles

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The ground coordinate system rotates φ around Ay to form coordinate $AX'yZ'$; and then the coordinate system $AX'yZ'$ rotates θ around AZ' to form new coordinate system $Ax_1Y'Z'$; finally the coordinate system $Ax_1Y'Z'$ rotates γ around Ax_1 to get coordinate system $ox_1y_1z_1$[4-8]. This process can be expressed as

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \boldsymbol{L}(\varphi,\vartheta,\gamma)\begin{bmatrix} x \\ y \\ z \end{bmatrix}. \tag{1}$$

where

$$\boldsymbol{L}(\varphi,\vartheta,\gamma) = \begin{bmatrix} \cos\vartheta\cos\varphi & \sin\vartheta & -\cos\vartheta\sin\varphi \\ -\sin\vartheta\cos\varphi\cos\gamma + \sin\varphi\sin\gamma & \cos\vartheta\cos\gamma & \sin\vartheta\sin\varphi\cos\gamma + \cos\varphi\sin\gamma \\ \sin\vartheta\cos\varphi\sin\gamma + \sin\varphi\cos\gamma & -\cos\vartheta\sin\gamma & -\sin\vartheta\sin\varphi\sin\gamma + \cos\varphi\cos\gamma \end{bmatrix}. \tag{2}$$

The conversion relation between other coordinates will not be introduced one by one.

1.2 Algorithm of attitude

The solution method of flight attitude parameters is the core of inertial navigation system. The choice of suitable method has an important effect on precision of inertial navigation system[9]. In this paper, the quaternion is adopted.

The quaternion is the simplest hyper-complex, which has been adopted in recent years to express attitude angles and widely used in practical engineering. The quaternion can be defined as

$$\boldsymbol{n} = q_0 + q_1\boldsymbol{i} + q_2\boldsymbol{j} + q_3\boldsymbol{k}, \tag{3}$$

where q_0, q_1, q_2 and q_3 are real numbers; $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are imaginary unit vectors orthogonal to each other.

The rotation vector attitude matrix in Eq.(2) can be expressed as the following form with the quaternion,

$$\boldsymbol{C}_n^a = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 + q_0q_3) & 2(q_1q_3 - q_0q_2) \\ 2(q_1q_2 - q_0q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 + q_0q_1) \\ 2(q_1q_3 + q_0q_2) & 2(q_2q_3 - q_0q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}. \tag{4}$$

We can get the relationship between the rotation quaternion and the attitude angles as

$$\begin{cases} \varphi = \arctan\dfrac{2(q_1q_2 + q_0q_3)}{q_0^2 + q_1^2 - q_2^2 - q_3^2}, \\ \vartheta = -\arcsin 2(q_1q_3 - q_0q_2), \\ \gamma = \arctan\dfrac{2(q_2q_3 + q_0q_1)}{q_0^2 - q_1^2 - q_2^2 + q_3^2}. \end{cases} \tag{5}$$

It can be obtained by quaternion of the multiplication formula.

According to initial values φ, ϑ and γ, we can obtain q_0, q_1, q_2 and q_3. Substituting them into Eq.(5), the attitude angles are obtained. And then subsituting these attitude angles into Eq.(6), the quaternion and the attitude angles are constantly updated.

$$\begin{cases} q_0 = \cos\dfrac{\varphi}{2}\cos\dfrac{\vartheta}{2}\cos\dfrac{\gamma}{2} + \sin\dfrac{\varphi}{2}\sin\dfrac{\vartheta}{2}\sin\dfrac{\gamma}{2}, \\ q_1 = \cos\dfrac{\varphi}{2}\cos\dfrac{\vartheta}{2}\sin\dfrac{\gamma}{2} - \sin\dfrac{\varphi}{2}\sin\dfrac{\vartheta}{2}\cos\dfrac{\gamma}{2}, \\ q_2 = \cos\dfrac{\varphi}{2}\sin\dfrac{\vartheta}{2}\cos\dfrac{\gamma}{2} + \sin\dfrac{\varphi}{2}\cos\dfrac{\vartheta}{2}\sin\dfrac{\gamma}{2}, \\ q_3 = \sin\dfrac{\varphi}{2}\cos\dfrac{\vartheta}{2}\cos\dfrac{\gamma}{2} - \cos\dfrac{\varphi}{2}\sin\dfrac{\vartheta}{2}\sin\dfrac{\gamma}{2}. \end{cases} \tag{6}$$

2 Mathematical modeling of missile motion

The equations of missile motion are a mathematical model to characterize the movement of missiles, as well as the basis for analyzing, calculating or simulating missile motion. The missile motion equations are based on Newton theorem, which also relates to variable mass mechanics, aerodynamics, propulsion principle and automatic control theory. It is used to describe the relationship between the forces acting on the missile, the missile torque and the motion parameters. It is composed of the dynamics equations which describe the missile centroid motion, the kinematic equations, the mass change equations, the angle geometry equations and the equations which describe the work of control system[10].

The motion equations of missile flight can be expressed by Eq.(7), where P is thrust; X is resistance; Y is lift; Z is lateral force; M_X, M_Y and M_Z are torques; φ_v is trajectory deflection angle; α is angle of attack; β is side-slip angle; γ_v is roll angle of speed; φ is yaw angle; θ is pitch angle; γ is roll angle; δ is rotation angle; x, y and z are distances on each axis; and V is speed. Given the initial conditions of the parameters above, they can be solved by numerical integration method to obtain variation of the controlled trajectory and corresponding parameters.

$$\begin{cases} m\dfrac{\mathrm{d}V}{\mathrm{d}t} = P\cos\alpha\cos\beta - X - mg\sin\theta, \\ mV\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = P(\sin\alpha\cos\gamma_v + \cos\alpha\sin\beta\sin\gamma_v) + Y\cos\gamma_v - Z\sin\gamma_v - mg\cos\theta, \\ -mV\cos\dfrac{\mathrm{d}\varphi_v}{\mathrm{d}t} = P(\sin\alpha\sin\gamma_v - \cos\alpha\sin\beta\cos\gamma_v) + Y\sin\gamma_v + Z\cos\gamma_v, \\ J_X\dfrac{\mathrm{d}\omega_X}{\mathrm{d}t} + (J_Z - J_Y)\omega_Z\omega_Y = M_X, \\ J_Y\dfrac{\mathrm{d}\omega_Y}{\mathrm{d}t} + (J_X - J_Z)\omega_X\omega_Z = M_Y, \\ J_Z\dfrac{\mathrm{d}\omega_Z}{\mathrm{d}t} + (J_Y - J_X)\omega_Y\omega_X = M_Z, \\ \dfrac{\mathrm{d}x}{\mathrm{d}t} = V\cos\theta\cos\varphi_v, \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = V\sin\theta, \\ \dfrac{\mathrm{d}z}{\mathrm{d}t} = -V\cos\theta\sin\varphi_v, \\ \dfrac{\mathrm{d}\theta}{\mathrm{d}t} = \omega_Y\sin\gamma + \omega_Z\cos\gamma, \\ \dfrac{\mathrm{d}\varphi}{\mathrm{d}t} = \dfrac{1}{\cos\theta}(\omega_Y\cos\gamma - \omega_Z\sin\gamma), \\ \dfrac{\mathrm{d}\gamma}{\mathrm{d}t} = \omega_X - \tan\theta(\omega_Y\cos\gamma - \omega_Z\sin\gamma), \\ \dfrac{\mathrm{d}m}{\mathrm{d}t} = -m_s(t), \\ \sin\beta = \cos\theta[\cos\gamma\sin(\varphi - \varphi_v) + \sin\vartheta\sin\gamma\cos(\varphi - \varphi_v)] - \sin\theta\cos\theta\sin\gamma, \\ \cos\alpha = [\cos\vartheta\cos\theta\cos(\varphi - \varphi_v) + \sin\vartheta\sin\theta]/\cos\beta, \\ \cos\gamma_v = [\cos\gamma\cos(\varphi - \varphi_v) + \sin\vartheta\sin\gamma\sin(\varphi - \varphi_v)]/\cos\beta, \\ \varepsilon_i = 0,\ i = 1,2,3,4. \end{cases} \tag{7}$$

3 System design

3.1 System architecture

The mathematical model of missile simulation is mainly used to realize the missile flight trajectory and attitude simulation. According to the idea of modular modeling, the software system is designed to be multiple subsystem modules based on functions. Each subsystem module is abstracted into more specific functional sub-modules, and finally combined into a complete system model. The missile simulation system hierarchy model is shown in Fig. 2.

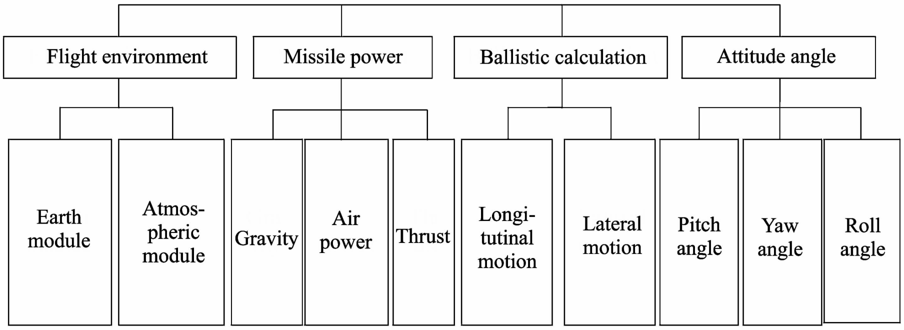

Fig. 2 Simulation system hierarchy model chart

3.2 Software design

In this paper, the Matlab was used to solve missile flight trajectory and attitude angle. The program was composed of different functions with logical relationship and the relationships between functions are shown in Fig. 3.

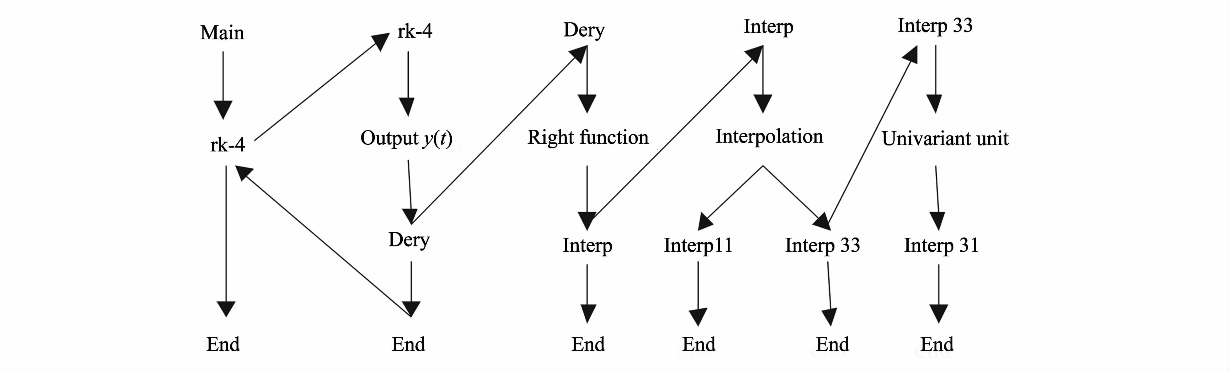

Fig. 3 Relationship between functions in the program

3.3 Software simulation

Under standard atmospheric pressure, assume that the initial flying height of the missile is 20 m, the initial velocity is 20 m/s, the missile aerodynamic reference area is S and the length is L, the mass of the missile is 50 kg, the attack angle ranges from 0° to 10°, the initial pitch angle is 5°, the yaw angle is 0°, the roll angle is 0°, the step is $h=0.005$, the simulation sampling period is $t=0.001$ s and the simulation time is 10 s. Using Matlab to simulate the missile flight path, the flight path in a vertical plane and the attitude change of the missile can be derived. The results are shown in Fig. 4.

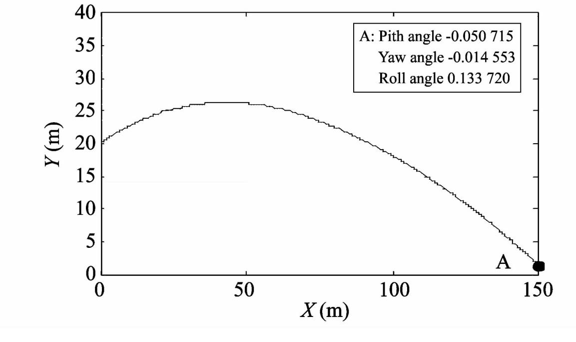

Fig. 4 Missile trajectory simulation in vertical plane flight

Fig. 4 is the missile trajectory simulation in the vertical plane flight based on the two-dimensional inertial coordinate system (ground coordinate system), where X represents displacement away from the launch point and Y represents the height away from the launch point. The length of per unit in the abscissa is 1, the ordinate unit is m and the attitude angle is arc system. As can be seen from Fig. 4, due to the thrust action, the distance in the direction of flight continues to increase. However, due to drag and gravity, the increase manner does not appear in a line. As the fuel consumes, the thrust decreases and the flight acceleration decreases. To the highest point, the vertical velocity is zero. As the downward force is greater than the upward lift, the missile starts fall, and its speed gradually accelerates until constant speed. The results above are consistent with the regulation of missile flight.

The simulation curves of missile flight attitude angle are shown in Figs. 5 − 7.

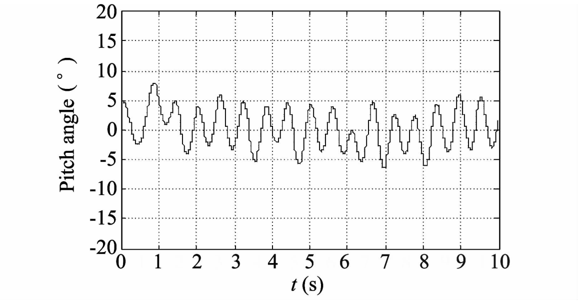

Fig. 5 Pitch angle curve

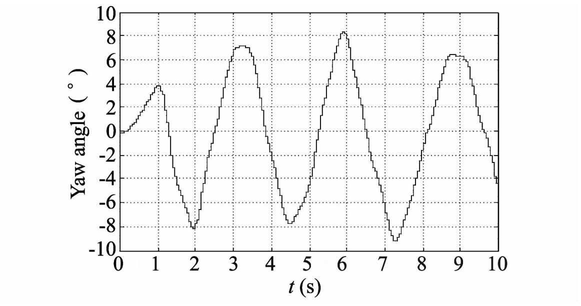

Fig. 6 Yaw angle curve

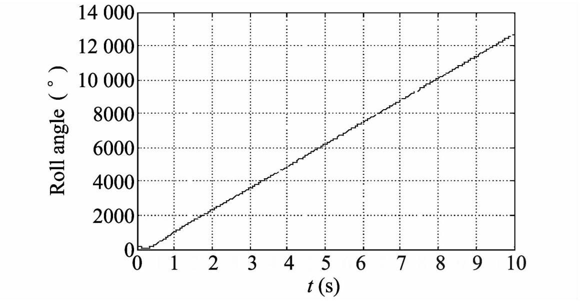

Fig. 7 Roll angle curve

According to the missile's attitude angle curves, the changes of pitch angle, yaw angle and roll angle can be seen during the process of missile flight in 10 s. Due to the error and interference, the pitch angle fluctuates up and down. The yaw angle stays about sine condition, indicating that the missile sways around during the flight. The roll angle stays increasing. The missile in flight has been in high speed rotating and the roll angle has been accumulated. As a result, numerical simulations above are basically in compliance with relevantly real flight rules.

4 Conclusion

Experiments and numerical simulation were carried out for modeling design of real-time flight motion system. The model of missile flight motion has been established. Based on initial parameters such as the pitch angle, the speed and the mass of missile, the missile flight trajectory has been drawn and the missile attitude parameters have been visually displayed at different time points, which has laid a good foundation for actual missile flight.

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弹体飞行模拟及姿态显示

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摘 要： 为降低成本、提高准确度，仿真技术成为导弹飞行研究的重要方法。本文研究了导弹飞行轨迹及姿态的解算，对飞行导弹进行了力学分析与建模。在分析导弹飞行特点和姿态参数求解算法的基础上，动态模拟了垂直平面内弹体飞行轨迹，并实时显示了弹体的姿态。本文实现了实验室环境下对导弹飞行过程的描述，为导弹实际飞行研究提供了参考。

关键词： 弹体飞行；仿真；姿态

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