## Decentralized supervisory control of continuous timed discrete event systems

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*Abstract*—In this paper, we presented the decentralized supervisory control problem of discrete event system with continuous-time variable. By presenting the definition of coobservability for the timed specification, a necessary and sufficient condition for the existence of decentralized supervisors is obtained. Finally, a numerical example is given.

Keywords — Discrete event systems; Decentralized

supervi sors; Coobservability.

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### **1** Introduction

Discrete event systems (DES) are systems which states are driven by events. Ramadge and Wonham<sup>[1]</sup>, [2] have studied the logical supervisory control theory instantaneously. For distributed systems, such as communication systems and manufacturing systems, decentralized supervisors are more suitable than a centralized one. Decentralized supervisory control has been initiated in [3-7]. Recently, some issues of decentralized supervisory control, such as the state feedback problem<sup>[8-9]</sup>, general architecture<sup>[10]</sup>, reliability<sup>[11-12]</sup> and synthesis problem with communication<sup>[13-15]</sup>, have been studied.

In the range of control problem, real time is more suitable than instantaneity. Supervisory control of discrete timed DES (DTDES) has been studied by Brandin and Wonham, i.e. [16] has considered the time feature of [1] and introduced the event tick to represent 'tick of global clock'. Obviously, the advantage of event tick incorporated into logical DES lies in preserving logical feature, while the disadvantage lies that can cause state explosion. To avoid state explosion, discrete timed DES has been extended by other researchers, i.e. [17] and [18] have introduced dense event and solved supervisory control problem on the base of state space of timed discrete event systems. [19] has eliminated event time and incorporated time information in the state; [20] has constrained the time information of [19] to be in the eligible time bounds; [21] has extended the model of [20] and considered robust supervisory control problem of uncertain DTDES; [22] has extended full observation [16] to partial observation; and [23] has solved robust supervisory control for partially observed DTDES, which is an extension of [24].

In the manufacture cells and logical cells, it always takes some time to operate and handle. In general, the service time of the operation is continuous variable in an interval. Under the circumstances, continuous time and timed control are considered as a new dimension of timed-DES in [25-26]. By using the model, the state explosion can be reduced in dicrete-time and continuous-time models. In this paper, we have developed the model of [25-26] and introduced the synthesis problem of decentralized supervisory control for timed-DES. To solve the synthesis problem, a necessary and sufficient condition for the existence of decentralized supervisors has been presented.

#### 2 Supervisory control of DES

In the model of [1-2], the plant to be controlled is modelled by an automaton  $G = (Q, \Sigma, \delta, q_0)$ , where Qis a countable state space,  $\Sigma$  is a finite event set,  $\delta$  is a partial function from  $Q \times \Sigma$  to Q, and  $q_0 \in Q$  is an initial state. Let  $\Sigma^*$  denote the set of all finite strings on  $\Sigma$ including the empty string  $\varepsilon$ .  $\delta$  can be generalized by  $\delta$ :  $Q \times \Sigma^* \rightarrow Q$ . The language generated by the DES G is defined by  $L(G) = \{s \in \Sigma^* | \delta(q_0, s)!\}$  and means the set of all possible event sequences. Let  $K \subseteq \Sigma^*$  be a language. We denote the set of all prefixes of traces in K by pre(K). K is (prefixed-)closed if K = pre(K).

The event set  $\Sigma$  is divided into an uncontrollable event set  $\Sigma_u$  and a controllable event set  $\Sigma_c$ . A language *K* is controllable if  $\operatorname{pre}(K)\Sigma_u \cap L(G) \subseteq \operatorname{pre}(K)$ . A control input is an event subset  $\gamma$  satisfying  $\Sigma_u \subseteq \gamma$  $\subseteq \Sigma$ . The set of control inputs is denoted by  $\Gamma$ . A supervisor is an enable map  $f: L(G) \rightarrow \Gamma$ . Formally, the

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language generated by the supervised system f/G, denoted by  $L(f/G)^{[1-2]}$ , is defined as follows.

- $\varepsilon \in L(f/G)$ , where  $\varepsilon$  is the empty string.
- $(\forall s \in L(f / G))(\sigma \in \Sigma) s \sigma \in L(f / G) \Leftrightarrow s \sigma \in L(G)$  $\land \sigma \in f(s).$

For a nonempty and closed languages K, there exists a supervisors f for G such that L(f/G) = K if and only if K is controllable.

Let  $\Sigma_o$  be the set of observable events. An observable function is a nature projection  $P: \Sigma \to \Sigma_o$ which is satisfied with  $P(\varepsilon) = \varepsilon$ ,  $P(\sigma) = \begin{cases} \sigma & \text{if } \sigma \in \Sigma_o \\ \varepsilon & \text{otherwise} \end{cases}$  and  $P(s\sigma) = P(s)P(\sigma)$  for any s

 $\in \Sigma^*$  and  $\sigma \in \Sigma$ . A partial supervisor which enables and disables any of the controllable events through its observation of the sequence of events as it is generated by *G* is a map  $f : P(L(G)) \rightarrow \Gamma$ . A language *K* is said to be observable if P(s)=P(s'),  $s'\sigma \in L(G)$  and  $s\sigma \in K$  imply  $s'\sigma \in K$  for any  $s, s' \in K$  and  $\sigma \in \Sigma_c$ . For a nonempty and closed languages *K*, there exists a partial supervisors *f* for *G* such that L(f/G) = K if and only if *K* is controllable and observable.

# 3 Supervisory control of continuous timed-DES

Continuous timed-DES is modelled by an automaton  $G_t = (Q, \Sigma_t, \delta_t, q_0)$  in [26], where  $\Sigma_t$  is the set of timed events, and  $\delta_t: Q \times \Sigma_t \to Q$  is a partial function, e.g.  $(\sigma, t_{\sigma})$  is fired in t $\sigma$  time if logical event  $\sigma$  is enable. Let  $\Sigma^*$  be the set of all finite timed strings of elements in  $\Sigma_t$ , including the empty string  $\varepsilon$ . The function  $\delta_t$  can be generalized to  $\delta_t: Q \times \Sigma^* \to Q$  in the natural way. The timed event  $\Sigma_t$  can be divided into controllable event set  $\Sigma_{ct}$  and uncontrollable event set  $\Sigma_{ut}$ . To describe the relations between logical event and timed event, we make the following assumption in [26]:

1) For any timed controllable event ( $\sigma$ ,  $t_{\sigma}$ ), logical event  $\sigma$  is controllable.

2) For any timed uncontrollable event ( $\sigma$ ,  $t_{\sigma}$ ), logical event  $\sigma$  is uncontrollable.

From the assumption, we have  $(\sigma, t_{\sigma}) \in \Sigma_{ct} \Leftrightarrow \sigma \in \Sigma_{c}$  $\Sigma_{c}$  and  $(\sigma, t_{\sigma}) \in \Sigma_{ut} \Leftrightarrow \sigma \in \Sigma_{u}$ .

Let  $TL(G_t)$  be the timed language generated by  $G_t$ , that is  $TL(G_t) = \{s | \delta_t(q_0, s)!, s \in \Sigma^*\}$ , where *s* is a timed string. The traces of  $TL(G_t)$  is defined as  $L(G_t) =$  $tr(TL(G_t))$ , where  $tr(\cdot)$  is the function of logical trace sets for timed language.

**Definition 1:** A timed language *K* is said to be (prefix-)closed <sup>[26]</sup> if pre(K)=K.

Let  $T_K(\sigma/s) = \{[t_{l\sigma}, t_{u\sigma}) | s(\sigma, [t_{l\sigma}, t_{u\sigma})) \in \operatorname{pre}(K)\}$  be the set of service time of  $\sigma$  following the string *s* under the restriction of *K*.

**Definition 2**: A timed language *K* is said to be  $G_t$ -controllable<sup>[26]</sup> if the following conditions are satisfied.

- trace-control:  $\operatorname{pre}(K)\Sigma_u \cap L(G_t) \subseteq \operatorname{pre}(K)$
- time-control: For any  $s \in \operatorname{pre}(K)$  and  $\sigma \in \Sigma_u$ ,  $T_{TL(Gt)}(\sigma/s) \subseteq T_K(\sigma/s)$  holds.

A supervisor *sc* is defined as an ordered pair *sc* = (f, I), where  $f:L(G_t) \rightarrow \Gamma$  is a logical supervisor which is the sets of enable logical events and  $I:L(G_t) \times \Sigma \rightarrow \Omega$  is a timed supervisor which is the enable time-interval of logical events such that  $\Sigma_u \subseteq f(s)$  and  $t\sigma \in I(s, \sigma)$  for any  $s \in TL(G_t)$  and  $\sigma \in \Sigma_u$ , where  $\Omega = \{[R_1, R_2)|R_1 < R_2, R_1 \in R^+, R_2 \in R^+\}$  is the set of time intervals. The closed-loop system under *sc* is denoted by  $sc/G_t$ .

**Definition 3**: The timed language  $TL(sc/G_t)^{[26]}$  generated by  $sc/G_t$  is defined as follows:

- $\varepsilon \in TL(sc/G_t)$ , where  $\varepsilon$  is the empty timed string.
- $s(\sigma,t_{\sigma}) \in TL(sc/G_t) \Leftrightarrow s(\sigma,t_{\sigma}) \in TL(G_t),$  $s \in TL(sc/G_t), (\sigma,t\sigma) \in sc(s).$

**Theorem 1**: If *K* is a closed timed language, there exists a supervisor sc=(f, I) such that  $TL(sc/G_t)=K$  if and only if *K* is  $G_t$ -controllable<sup>[26]</sup>.

Let  $P_t: \Sigma_t \rightarrow \Sigma_{ot}$  be the observed function, where  $\Sigma_t$  is the timed events set and  $\Sigma_{ot}$  is the observable timed event set which consists of observable event and its service time. For the projection  $P_t$ , we suppose

$$P_{t}(\varepsilon) = \varepsilon, \quad P_{t}((\sigma, t_{\sigma})) = \begin{cases} (\sigma, t_{\sigma}) & \text{if } (\sigma, t_{\sigma}) \in \Sigma_{ot} \\ \varepsilon & \text{otherwise} \end{cases} \quad \text{and} \quad$$

 $P_t(s(\sigma,t_{\sigma})) = P_t(s)P_t((\sigma,t_{\sigma}))$ . A partial supervisor is defined as an ordered pair sc = (f, I), where  $f:P_t(L(G_t)) \rightarrow \Gamma$  and  $I:P_t(TL(G_t)) \times \Sigma \rightarrow \Omega$ , such that  $\Sigma_u \subseteq f(P_t(s))$  and  $t_{\sigma} \in I(P_t(s), u)$  for any  $s \in TL(G_t)$  and  $\sigma \in \Sigma_u$ .

**Definition 4:** For timed languages *K*, *K* is said to be observable<sup>[26]</sup> if  $s(\sigma,t_{\sigma}) \in \operatorname{pre}(K)$ ,  $s'(\sigma,t_{\sigma}) \in TL(G_t)$ and  $P_t(s) = P_t(s')$  imply  $s'(\sigma,t_{\sigma}) \in \operatorname{pre}(K)$  for any  $s,s' \in \operatorname{pre}(K)$  and  $(\sigma,t_{\sigma}) \in \Sigma_{ct}$ .

**Theorem 2**: For closed timed languages *K*, there exists a partial supervisor sc=(f, I) such that  $TL(sc/G_t) = K$  if and only if *K* is  $G_t$ -controllable and observable<sup>[26]</sup> (controllable and observable).

**Theorem 3:** For closed timed languages K, there exists decentralized timed local supervisors  $\{sc_i\}_{i \in I_n}$  such that  $TL(\{\overline{sc_i}\}_{i \in I_n} / G_t\} = K$  if and only if K is  $G_t$ -controllable and observable where  $\overline{sc_i}$  is

the global extension of  $sc_i$ . Proof (Only if)It is assumed that there exists decentralized local supervisors  $sc_1$  and  $sc_2$  such that  $TL\left(\left\{\overline{sc_i}\right\}_{i \in I} / G_i\right) = K$ .

(trace-controllable) Since K is closed, we have  $\operatorname{pre}(tr(K)) = tr(K)$ . For any  $s_1 \in tr(K)$  and  $\sigma \in \Sigma_u$  such that  $s_1 \sigma \in L(G_t)$ , there exists  $s \in K$  and  $t \sigma \in T_{TL(G_t)}(\sigma/s)$ such that  $s_1 \in tr(K)$  and  $s(\sigma, t_{\sigma}) \in TL(G_t)$ . It follows from  $TL\left(\left\{\overline{sc_i}\right\}_{i \in I} / G_i\right) = K$  that  $s \in TL\left(\left\{\overline{sc_i}\right\}_{i \in I} / G_i\right)$ . If  $(\sigma, t_{\sigma}) \in \Sigma_{ut1} \land \Sigma_{ut2}$ , we have  $\sigma \in f_1(P_{t1}(s)) \land$  $f_2(P_{t2}(s))$  and  $t \sigma \in I_1(P_{t1}(s), \sigma) \land I_2(P_{t2}(s), \sigma)$ . So,  $(\sigma, t_{\sigma}) \in sc_1(s) \land sc_2(s)$  holds, and then  $(\sigma, t_{\sigma}) \in sc_1(s)$  $\land \overline{sc_2}$  (s). If  $(\sigma, t\sigma) \in \Sigma_{ut1} - \Sigma_{ut2}$ , we have  $(\sigma, t_{\sigma}) \in sc_1(s)$  $\wedge$  sc<sub>2</sub> (s) by the above proof and the definition of  $\overline{sc_i}$ . So,  $(\sigma, t_{\sigma}) \in \overline{sc_1}(s) \land \overline{sc_2}(s)$  holds. If  $(\sigma, t_{\sigma}) \in \Sigma_{ut2}$  $-\Sigma_{ut1}$ , similarly, we have  $(\sigma, t\sigma) \in \overline{sc_1}(s) \wedge \overline{sc_2}(s)$ . If  $(\sigma, t\sigma) \in \Sigma_{ut} - \Sigma_{ut1} \cap \Sigma_{ut2}$ , similarly, we have  $(\sigma, t\sigma) \in \Sigma_{ut2}$  $t\sigma \in sc_1$  (s)  $\land sc_2$  (s). By the definition of  $TL\left(\left\{\overline{sc_i}\right\}_{i=1}, \left| G_i \right\rangle\right)$ , we have  $s_2(\sigma, t_{\sigma}) \in$  $TL\left(\left\{\overline{sc_i}\right\}_{i\in I} / G_t\right) = K$ . So,  $s_1 \sigma \in tr(K)$  holds.

(time-controllable)For any  $s \in K$  and  $\sigma \in \Sigma u$ , we need to show  $T_{TL(Gt)}(\sigma/s) \subseteq T_K(\sigma/s)$ .

If  $T_{TL(Gt)}(\sigma/s) = \Phi$ , we have  $T_{TL(Gt)}(\sigma/s) \subseteq TK(\sigma/s)$ .

If  $T_{TL(Gt)}(\sigma/s) \neq \Phi$ , we have  $s(\sigma,t_{\sigma}) \in TL(G_t)$  for any  $t\sigma \in T_{TL(Gt)}(\sigma/s)$ . From the above proof and  $\sigma \in \Sigma_u$ , we have  $(\sigma,t_{\sigma}) \in \overline{sc_i}(P_{ti}(s))$ . It follows from  $s \in K = TL\left(\left\{\overline{sc_i}\right\}_{i \in I_n} / G_t\right)$  that  $s(\sigma,t_{\sigma}) \in TL\left(\left\{\overline{sc_i}\right\}_{i \in I_n} / G_t\right) = K$ . So, we have  $t\sigma \in T_K(\sigma/s)$ . Therefore,  $T_{TL(Gt)}(\sigma/s) \subseteq T_K(\sigma/s)$  holds.

(coobservable timed languages) To show timed language *K* is coobservable, we need to consider the following cases for any  $s_1$ ,  $s_2$ ,  $t \in K = TL\left(\left\{\overline{sc_i}\right\}_{i \in I_n} / G_t\right)$ .

Case 1 If there exists  $(\sigma,t_{\sigma}) \in \Sigma_{ct1} - \Sigma_{ct2}$  such that  $P_{t1}(s_1) = P_{t1}(t), s_1(\sigma,t_{\sigma}) \in K$  and  $t(\sigma,t_{\sigma}) \in TL(G_t)$ , we need to show  $t(\sigma,t_{\sigma}) \in K$ . By the formula  $(\sigma,t_{\sigma}) \in \Sigma_{ct2}$ , we have  $(\sigma,t_{\sigma}) \in \Sigma_{ut2} \cup (\Sigma_t - \Sigma_{t2})$ . So,  $(\sigma,t_{\sigma}) \in \overline{sc_2}(P_{t2}(s))$  holds. Since  $(\sigma,t_{\sigma}) \in \Sigma_{ct1}, s_1(\sigma,t_{\sigma}) \in K = TL\left(\left\{\overline{sc_i}\right\}_{i \in I_n} / G_t\right)$  and  $s_1 \in TL\left(\left\{\overline{sc_i}\right\}_{i \in I_n} / G_t\right)$ , we have  $(\sigma,t_{\sigma}) \in \overline{sc_1}(P_{t1}(s))$ . From  $P_{t1}(s_1) = P_{t1}(t)$ , we have  $(\sigma,t_{\sigma}) \in \overline{sc_1}(P_{t1}(s))$ . Since  $t(\sigma,t_{\sigma}) \in TL(G_t)$ 

and  $t \in TL\left(\left\{\overline{sc_i}\right\}_{i \in I_n} / G_t\right)$ , we have  $t \in TL\left(\left\{\overline{sc_i}\right\}_{i \in I_n} / G_t\right) = K.$  Case 2 If there exists  $(\sigma, t_{\sigma}) \in \Sigma_{ct2} - \Sigma_{ct1}$  such that  $P_{t2}(s_2) = P_{t2}(t), s_2(\sigma, t_{\sigma}) \in \operatorname{pre}(K)$  and  $t(\sigma, t_{\sigma}) \in TL(G_t)$ , we have  $t \in K$  by the proof of case 1.

Case 3 If there exists  $(\sigma, t_{\sigma}) \in \Sigma_{ct1} \cap \Sigma_{ct2}$  such that  $P_{t1}(s_1) = P_{t1}(t)$ ,  $P_{t2}(s_2) = P_{t2}(t)$ ,  $s_1(\sigma, t_{\sigma}) \in \operatorname{pre}(K)$ ,  $s_2(\sigma, t_{\sigma}) \in \operatorname{pre}(K)$  and  $t(\sigma, t_{\sigma}) \in TL(G_t)$ , we need to show  $t(\sigma, t_{\sigma}) \in K$ . By the formula  $TL\left(\left\{\overline{sc_i}\right\}_{i \in I_n} / G_t\right) = K$ , we have  $s_1(\sigma, t_{\sigma}) \in TL\left(\left\{\overline{sc_i}\right\}_{i \in I_n} / G_t\right)$  and  $s_2(\sigma, t_{\sigma}) \in TL\left(\left\{\overline{sc_i}\right\}_{i \in I} / G_t\right)$ . Since

 $s_{1} \in TL\left(\left\{\overline{sc_{i}}\right\}_{i \in I_{n}} \middle/ G_{t}\right) \text{ and } s_{2} \in TL\left(\left\{\overline{sc_{i}}\right\}_{i \in I_{n}} \middle/ G_{t}\right), \text{ we}$ have  $(\sigma, t_{\sigma}) \in \overline{sc_{1}}(P_{t_{1}}(s))$  and  $(\sigma, t_{\sigma}) \in \overline{sc_{2}}(P_{t_{2}}(s))$ . It is obvious that  $(\sigma, t_{\sigma}) \in \overline{sc_{1}}(P_{t_{1}}(s)) \land \overline{sc_{2}}(P_{t_{2}}(s))$  by the formulas  $P_{t_{1}}(s_{1}) = P_{t_{1}}(t)$  and  $P_{t_{2}}(s_{2}) = P_{t_{2}}(t)$ . Since  $t \in TL\left(\left\{\overline{sc_{i}}\right\}_{i \in I_{n}} \middle/ G_{t}\right)$  and  $t \in TL(G_{t})$ , we have  $t(\sigma, t_{\sigma}) \in TL\left(\left\{\overline{sc_{i}}\right\}_{i \in I_{n}} \middle/ G_{t}\right) = K.$ 

So, K is coobservable from the definition of coobservability.

(If) Assuming that *K* is closed,  $G_t$ -controllable and coobservable timed language, we need to prove there exists decentralized supervisors  $\{sc_i(P_{ti}(s))\}_{i \in In}$ such that  $TL(\{\overline{sc_i}\}_{i \in I_n} / G_t\} = K$ . For any  $s \in TL(G_t)$ , we construct supervisors as follows.

$$\begin{cases} f_i(P_{ii}(s)) = \sum_{iu} \cup \{\sigma \in \sum_{ic} | \exists s' \in K, P_{ii}(s) = P_{ii}(s'), \\ tr(s')\sigma \in \operatorname{pre}(tr(K))\} \end{cases}$$
$$I_i(P_{ii}(s), \sigma) = \{t_\sigma \mid \sigma \in \sum_{iu}\} \cup \{t_\sigma \mid \sigma \in \sum_{ic}, \exists s' \in K, \\ P_{ii}(s) = P_{ii}(s'), t_\sigma \in T_K(\sigma / s')\} \end{cases}$$

Using the length of tr(s), we can show  $TL\left(\left\{\overline{sc_i}\right\}_{i\in I_n} / G_t\right) = K$  by the methods of mathematics induction. It is obviously that  $\varepsilon \in TL\left(\left\{\overline{sc_i}\right\}_{i\in I_n} / G_t\right) \cap K$ . Assuming that  $s \in TL\left(\left\{\overline{sc_i}\right\}_{i\in I_n} / G_t\right) \cap K$ , we need to prove  $s(\sigma, t_{\sigma}) \in TL\left(\left\{\overline{sc_i}\right\}_{i\in I_n} / G_t\right) \Leftrightarrow s(\sigma, t_{\sigma}) \in K$  for any  $(\sigma, t_{\sigma}) \in \Sigma_t$ .

(We prove  $TL\left(\left\{\overline{sc_i}\right\}_{i\in I_n} \middle/ G_t\right) \subseteq K$ .) Take  $s(\sigma,t_{\sigma}) \in TL\left(\left\{\overline{sc_i}\right\}_{i\in I_n} \middle/ G_t\right)$ . It is obvious that  $s(\sigma,t_{\sigma}) \in TL(G_t)$  from  $TL\left(\left\{\overline{sc_i}\right\}_{i\in I_n} \middle/ G_t\right) \subseteq TL(G_t)$ . So,  $tr(s) \sigma \in L(G_t)$  and  $t\sigma \in T_{TL(Gt)}(\sigma/s)$  hold. If  $(\sigma,t_{\sigma}) \in \Sigma_{ut}$ ,

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we have  $tr(s)\sigma \in tr(K)$  and  $t_o \in T_K(\sigma/s)$  from the trace-controllability and time-controllability of *K*. Therefore,  $s(\sigma,t_o) \in K$  holds. If  $\sigma \in \Sigma_{ct}$ , we have  $(\sigma,t_o) \in \overline{sc_i}(P_{ti}(s))$  from  $s \in TL(\{\overline{sc_i}\}_{i \in I_n} / G_t\}$  for any  $i \in I_n$ . Since  $\Sigma_{ct} = \Sigma_{ct1} \cup \Sigma_{ct2}$ , we consider the following three cases.

Case 1 If  $(\sigma,t_{\sigma}) \in \Sigma_{ct1} - \Sigma_{ct2}$ , there exists  $s_1 \in K$ such that  $P_{t1}(s_1) = P_{t1}(s)$  and  $t\sigma \in T_K(\sigma/s_1)$  by the formula  $(\sigma,t_{\sigma}) \in \overline{sc_1}(P_{t1}(s))$ . So,  $s_1(\sigma,t_{\sigma}) \in K$  holds. Since  $s(\sigma,t_{\sigma}) \in TL(G_t)$ , we have  $s(\sigma,t_{\sigma}) \in K$  from the definition of coobservability.

Case 2 If  $(\sigma, t_{\sigma}) \in \Sigma_{ct2} - \Sigma_{ct1}$ , similarly, we have  $s(\sigma, t_{\sigma}) \in K$  from the proof of case 1.

Case 3 If  $(\sigma,t_{\sigma}) \in \Sigma_{ct1} \cap \Sigma_{ct2}$ , there exists  $s_1$ ,  $s_2 \in K$  such that  $P_{t1}(s_1) = P_{t1}(s)$ ,  $P_{t2}(s_1) = P_{t2}(s)$  and  $t\sigma \in T_K(\sigma/s_1) \cap T_K(\sigma/s_1)$  by the formula  $(\sigma,t_{\sigma}) \in \overline{sc_1}(P_{t1}(s)) \land \overline{sc_2}(P_{t2}(s))$ . So,  $s_1(\sigma,t_{\sigma}) \in K$  and  $s_2(\sigma,t_{\sigma}) \in K$ . Since  $s(\sigma,t_{\sigma}) \in TL(G_t)$ , we have  $s(\sigma,t_{\sigma}) \in K$ by the definition of coobservability.

So, 
$$TL\left(\left\{\overline{sc_i}\right\}_{i\in I_n}/G_i\right)\subseteq K$$
 holds

(We prove  $K \subseteq TL\left(\left\{\overline{sc_i}\right\}_{i \in I_n} / G_i\right)$ .) Take  $s(\sigma, t_{\sigma})$ 

 $\in K. \text{ If } (\sigma, t_{\sigma}) \in \Sigma_{ut1} \cap \Sigma_{ut2}, \text{ it is obvious that } \sigma \in f_1(P_{t1}(s)) \land f_2(P_{t2}(s)) \text{ and } t\sigma \in I_1(P_{t1}(s)) \land I_2(P_{t2}(s)), \text{ and then } (\sigma, t_{\sigma}) \in sc_1(P_{t1}(s)) \land sc_2(P_{t2}(s)). \text{ If } (\sigma, t_{\sigma}) \in \Sigma_{ut1} \cdot \Sigma_{ut2}, \text{ we have } (\sigma, t_{\sigma}) \in sc_1(P_{t1}(s)) \land \overline{sc_2}(P_{t2}(s)) \text{ by the above proof and definition of } \overline{sc_2} \cdot \text{If}(\sigma, t_{\sigma}) \in \Sigma_{ut2} \cdot \Sigma_{ut1}, \text{ we have } (\sigma, t_{\sigma}) \in \overline{sc_1}(P_{t1}(s)) \land sc_2(P_{t2}(s)). \text{ If } (\sigma, t_{\sigma}) \in \Sigma_{ut2} - \Sigma_{ut1}, \text{ we have } (\sigma, t_{\sigma}) \in \overline{sc_1}(P_{t1}(s)) \land sc_2(P_{t2}(s)). \text{ If } (\sigma, t_{\sigma}) \in \Sigma_{ut1} - \Sigma_{ut1} \cap \Sigma_{ut2}, \text{ we have } (\sigma, t_{\sigma}) \in \overline{sc_1}(P_{t1}(s)) \land \overline{sc_2}(P_{t2}(s)) \text{ by the definitions of } \overline{sc_1} \text{ and } \overline{sc_2} \text{ . By the formulas } s \in TL\left(\left\{\overline{sc_i}\right\}_{i \in I_n} / G_t\right) \text{ and } s \in K \subseteq TL(G_t), \text{ we have } s(\sigma, t_{\sigma}) \in TL\left(\left\{\overline{sc_i}\right\}_{i \in I_n} / G_t\right) \text{ . If } (\sigma, t_{\sigma}) \in \Sigma_{ct}, \text{ we consider the following three cases. }$ 

Case 1 If  $(\sigma, t_{\sigma}) \in \Sigma_{ct1} - \Sigma_{ct2}$ , we have  $(\sigma, t_{\sigma}) \in \Sigma$  $_{ct1}$   $\cap$  (  $\Sigma_{ut2}$   $\cup$  (  $\Sigma_{t-}$   $\Sigma_{t2}$ )). It is obvious  $(\sigma, t_{\sigma}) \in sc_2(P_{t_2}(s))$ . Since  $s \in K$  and  $s(\sigma, t_{\sigma}) \in K$ , there exists  $s_1 \in K$  such that  $s_1(\sigma, t_{\sigma}) \in K$  and  $P_{t1}(s) = P_{t1}(s_1)$ by the definition of coobservability. So,  $t_{\sigma} \in T_{K}(\sigma/s_{1})$ ,  $tr(s_1) \sigma \in tr(K)$  and  $P_{t1}(s) = P_{t1}(s_1)$  hold. By the construction of  $sc_1$ , we have  $\sigma \in f_1(P_{t1}(s))$  and  $t\sigma \in I_1(P_{t1}(s), \sigma)$ . So,  $(\sigma, t_{\sigma}) \in sc_1(P_{t1}(s))$ , and then  $(\sigma, t_{\sigma}) \in sc_1(P_{t1}(s)).$ By the formulas S  $\in TL\left(\left\{\overline{sc_i}\right\}_{i=1}, /G_i\right)$  and  $s(\sigma, t_{\sigma}) \in K \subseteq TL(G_i)$ , we have  $s(\sigma, t_{\sigma}) \in TL\left(\left\{\overline{sc_i}\right\}_{i \in I} / G_i\right).$ 

Case 2 If  $(\sigma, t_{\sigma}) \in \Sigma_{ct2} - \Sigma_{ct1}$ , we have  $s(\sigma, t_{\sigma}) \in TL\left(\left\{\overline{sc_i}\right\}_{i \in I_n} / G_i\right)$  from the proof of case 1.

Case 3 If  $(\sigma,t_{\sigma}) \in \Sigma_{ct1} \cap \Sigma_{ct2}$ , there exists  $s_1, s_2 \in K$  such that  $P_{t1}(s) = P_{t1}(s_1)$ ,  $P_{t2}(s) = P_{t2}(s_2)$ ,  $s_1(\sigma,t_{\sigma}) \in K$  and  $s_2(\sigma,t_{\sigma}) \in K$  by the assumption of  $s \in K$ and  $s(\sigma,t_{\sigma}) \in K$ . So, we have  $tr(s_1)\sigma \in tr(K)$ ,  $tr(s_2)\sigma \in tr(K)$ ,  $t\sigma \in T_K(\sigma/s_1)$  and  $t\sigma \in T_K(\sigma/s_2)$ . By the construction of  $sc_1$  and  $sc_2$ , we have  $\sigma \in f_1(P_{t1}(s))$ ,  $\sigma \in f_2(P_{t2}(s))$ ,  $t\sigma \in I_1(P_{t1}(s))$  and  $t\sigma \in I_2(P_{t2}(s))$ . So,  $(\sigma,t_{\sigma}) \in sc_1(P_{t1}(s)) \land sc_2(P_{t2}(s))$ , and then  $(\sigma,t_{\sigma}) \in \overline{sc_1}(P_{t1}(s)) \land \overline{sc_2}(P_{t2}(s))$ . By the definition of  $TL\left(\left\{\overline{sc_i}\right\}_{i\in I_n}/G_t\right)$ , we have  $s(\sigma,t_{\sigma}) \in TL\left(\left\{\overline{sc_i}\right\}_{i\in I_n}/G_t\right)$ . Therefore,  $K \subseteq TL\left(\left\{\overline{sc_i}\right\}_{i\in I_n}/G_t\right)$  holds.

**Example 1**: A continuous Timed-DES  $G_t$  considered is shown in Figure 1, where timed event set  $\Sigma_t = \{(a, [3, 8)), (\beta, [2, 7)), (\gamma, [3, 5)), (\mu, [0, 5))\}$  and controllable timed event set  $\Sigma_{ct} = \Sigma_t$ . Obviously,  $\Sigma_c = \{a, \beta, \gamma, \mu\}$ . Let n = 2. The observed function  $P_{t1}$  and  $P_{t2}$  is presented by the projections  $P_{t1}: \Sigma_t \rightarrow \Sigma_{t1}$  and  $P_{t2}: \Sigma_t \rightarrow \Sigma_{t2}$ . In the local systems, we suppose  $\Sigma_{t1} = \Sigma_{ct1} = \{(a, [3, 5)), (\beta, [2, 5)), (\gamma, [3, 5))\}$  and  $\Sigma_{t2} = \Sigma_{ct2} = \{(a, [4, 8)), (\beta, [2, 7)), (\mu, [0, 5))\}$ . Obviously, we have  $\Sigma_{c1} = \{a, \beta, \mu\}$ .



For timed system  $G_t$ , we have  $TL(G_t) = \text{pre}([((a, [5, 8)) + (Y, [0, 7)))(a, [3, 5)) + ((\beta, [2, 7)) + (\mu, [0, 5)))(\beta, [3, 4))]^*)$  by Figure 1. Let  $K = \text{pre}([(Y, [2, 5))(a, [3, 3.5)) + (\mu, [0, 5))(\beta, [3, 4))]^*)$  be the timed specification shown in Figure 2. By the definition of closeness,  $G_t$ -controllability and coobservability, we can show K is closed, Gt-controllable and co-observable. So, there exists decentralized supervisors  $\{sc_1, sc_2\}$  such that

sc<sub>2</sub>:

 $TL\left(\left\{\overline{sc_i}\right\}_{i \in \{1,2\}} / G_i\right) = K$ , where  $\{sc_1, sc_2\}$  can be constructed followed the above proof

constructed followed the above proof.

By the observed functions, we have the local systems  $G_{t1}$  and  $G_{t2}$  shown in Figure 3 and Figure 4. For all  $s \in TL(G_t)$ , we can construct decentralized local supervisors  $\{sc_1, sc_2\}$  as follows.

 $sc_1$ :

If  $P_{t1}(s) = [(Y, [3, 5))(a, [3, 3.5)) + (\beta, [3, 4))(\beta, [3, 4))]^*, f_1(P_{t1}(s)) = \{a, \beta, Y\}, I_1(P_{t1}(s), a) = [3, 3.5), I_1(P_{t1}(s), \beta) = [3, 4) \text{ and } I_1(P_{t1}(s), Y) = [2, 5).$ 



Fig 3: Observed Timed-DES  $G_{t1}$ 



Fig 4: Observed Timed-DES  $G_{t2}$ 

If  $P_{t1}(s) = [(Y, [3, 5))(a, [3, 3.5)) + (\beta, [3, 4))(\beta, [3, 4))]^*(Y, [3, 5)), f_1(P_{t1}(s)) = \{a\}$  and  $I_1(P_{t1}(s), a) = [3, 3.5).$ 

If  $P_{t1}(s) = [(Y, [3, 5))(a, [3, 3.5)) + (\beta, [3, 4))(\beta, [3, 4))]^*(\beta, [3, 4)), f_1(P_{t1}(s)) = \{a, \beta, Y\},$ 

 $I_1(P_{t1}(s), \alpha) = [3, 3.5), I_1(P_{t1}(s), \beta) = [3, 4)$  and  $I_1(Pt1(s), \gamma) = [2, 5).$ 

If  $P_{t1}(s) = TL(G_{t1}) - \text{pre}([( \forall, [3, 5))( \alpha, [3, 3.5)) + (\beta, [3, 4))(\beta, [3, 4))]^*), sc_1(P_{t1}(s)) = \emptyset.$ 

If  $P_{t2}(s) = [((\beta, [3, 4)) + (\mu, [0, 5)))(\beta, [3, 4))]^*$ ,  $f_2(P_{t2}(s)) = \{ \alpha, \mu \}, I_2(P_{t2}(s), \alpha) = [3, 3.5) \text{ and} I_2(P_{t2}(s), \mu) = [0, 5).$ 

If  $P_{t2}(s) = [((\beta, [3, 4)) + (\mu, [0, 5)))(\beta, [3, 4))]^*$ ( $\beta$ , [3, 4)),  $f_2(P_{t2}(s)) = \{\alpha, \mu\}, I_2(P_{t2}(s), \alpha) = [3, 3.5)$  and  $I_2(P_{t2}(s), \mu) = [0, 5).$ 

If  $P_{t2}(s) = [((\beta, [3, 4)) + (\mu, [0, 5)))(\beta, [3, 4))]^*$  $(\mu, [0, 5))), f_2(P_{t2}(s)) = \{\beta\}$  and  $I_2(P_{t2}(s), \beta) = [3, 4).$ 

If  $P_{t2}(s) = TL(G_{t2})$  -pre( [(( $\beta$ , [3, 4)) + ( $\mu$ , [0, 5)))( $\beta$ , [3, 4))]<sup>\*</sup>),  $sc_2(P_{t2}(s)) = \Phi$ .

**Remark 1**: If the service time of any event is discrete, definition 5 and theorem 3 hold.

**Remark 2**: If the service time of any event is 0, definition 5 is the coobservable language of [7] and theorem 3 can be got from [7].

### **5** Conclusions

In this paper, decentralized supervisory control of discrete event system with continuous service time is considered. With the continuous time feature incorporated into DES, timed-DES with continuous-time variable is suitable to model. To solve synthesis problem of distributed systems, decentralized supervisory control for timed-DES is considered.

(Continued on P.81)

The main work of realizing configuration is to establish level control objects and make animating display scenes. Controlled objects include inletting water flow, exporting water flow and the numerical object of the boiler level. When animation connection is established, the basic graphic elements and animation component library are called in the user window to construct configuration diagram. Graphic objects and data objects defined by the state are set in the state of the corresponding attribute and animation connection is defined. Having finished the design of the developing system, you can switch to run mode to carry on the real-time monitoring to the control system and test configuration.

#### 6 Conclusions

This paper has introduced the composition and running of EFPT process control system based on ControlLogix5550 PLC control, the mathematical

#### (From P.61)

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model establishing of controlled object and the parameter tuning of PID. The use of configuration software extends the communication function. Through experimental testing, the control curve's overshoot is small and the transition time is short, so the control effect is quite ideal. This device being reliable and intuitive is suitable for scientific research and teaching, and has important application value in the actual industrial production.

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