Signal Separation and Instantaneous Frequency Estimation Based on Multi-scale Chirplet Sparse Signal Decomposition

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Abstract – **An approach based on multi-scale chirplet sparse signal decomposition is proposed to separate the multi-component polynomial phase signals, and estimate their instantaneous frequencies. In this paper, we have generated a family of multi-scale chirplet functions which provide good local correlations of chirps over shorter time interval. At every decomposition stage, we build the so-called family of chirplets and our idea is to use a structured algorithm which exploits information in the family to chain chirplets together adaptively as to form the polynomial phase signal component whose correlation with the current residue signal is largest. Simultaneously, the polynomial instantaneous frequency is estimated by connecting the linear frequency of the chirplet functions adopted in the current separation. Simulation experiment demonstrated that this method can separate the components of the multi-component polynomial phase signals effectively even in the low signal-to-noise ratio condition, and estimate its instantaneous frequency accurately.**

Key words – *multi-scale chirplet base function*; *multi-component polynomial phase signals*; *instantaneous frequency*; *signal-to-noise ratio*

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1 Introduction

According to the Stone-Weierstrass theory, with a large number of signal processing applications such as synthetic aperture, radar, imaging and radio communications, the phase of the observed signals can be modeled as a polynomial function of time[1-5]. Such signals are commonly known as the Polynomial Phase Signals (PPS). Results in the literature on estimation of PPS parameters are mostly limited to the mono-component case such as fast instantaneous frequency estimation[6], the well-known High order Ambiguity Function (HAF)[5-8], etc.

In 1991, Peleg and Porat introduced the High order Ambiguity Function (HAF) to analyze the constant amplitude PPS[5,9]. HAF based techniques reduced the order of PPS successfully by multiplying it with a conjugated lagged copy of itself. In recent years, HAF has proven to be a powerful tool and several of its variations have appeared in the literature[7-8,10]. However, signals appeared in real life often have multiple components, leading to their parameter estimations pose a great challenge. When HAF is applied to multi-component PPS (mc-PPS), it will emerge a large number of cross-terms which are PPS themselves[3]. Consequently, M. Z. Ikram and Zhou Tong proposed the bottom-up PPS parameter estimation algorithm[3]. This algorithm always starts with PHAF of lowest order and increases in PHAF order until a strong peak is observed. To some extent, it can reduce the influence of those cross-terms. However, the computing workload will be enormous if the signals have high order polynomial phase signal components and lots of components.

Another methods available for the study of mc-PPS generally fall into non-parametric techniques that employ time-frequency distributions to track or estimate the unknown Instantaneous Frequency (IF) of the multi-component polynomial phase signals (mc-PPS)[11-12]. However, the conventional time-frequency distributions for multi-component Polynomial Phase Signals (mc-PPS) generally suffer from interference terms, which will obscure the true location of the auto-components in the resulting time-frequency distributions and consequently produce the IF estimation error. In order to reduce the undesired interference terms, various interference reducing distributions and techniques have been introduced such as PWVD, CWD, ZAM[13-15], etc. However, for a large class of signals, there is a trade off between good interference suppression and high auto-component concentration.

The Multi-component polynomial phase signals can be provided with favorable similarity by chirplet signals in a short time interval. Combining the chirplet path pursuit[16] and the sparse signal decomposition[17], we propose a method based on multi-scale chirplet sparse signal decomposition which can separate the multi-component polynomial phase signals and estimate their instantaneous

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frequency. In this method, with different scale coefficients, time span of analytic signals is divided into dynamic time supporting intervals, and then the signals are projected and decomposed by multi-scale chirplet base function. From basis function set which has the maximum projection coefficients in each time supporting intervals, we find out the basis function combinations which can make the energy of the decomposed signals maximum, and chain them to form the signal component adaptively which correlates well with the analysis signals and whose instantaneous frequency is of physical significance. Keep on decomposing until the result satisfies the given terminating principle.

Simulation analysis shows that the present method can effectively separate PPS components contained in mc-PPS signals, and there is no interference as the quadratic time frequency distribution. It enjoys favorable time frequency gathering and higher precision of instantaneous frequency fitting. Since the correlations of the noise components and the chirplet signals are bad, this method have good anti-noise ability. It is effective even if the signal-to-noise ratio is very low. It is suitable for the separation of multi-component polynomial phase signals with noise. The present method provides a new method for detecting, separating and parameter estimating of mc-PPS.

This paper is organized as follows. In section 2, the method of multi-scale chirplet sparse signal decomposition is introduced in details. In section 3, we present an algorithm that separates the multi-component polynomial phase signals and estimates the instantaneous frequency of the components. In section 4, a three-component fourth order multi-component polynomial phase signal is taken as an example to verify the validity of the present method, and analyze the influence of noise on the present method. Finally, the conclusions are drawn in section 5.

2 Multi-scale chirplet sparse signal decomposition

Based on signal analysis theory, any signal $f(t)$ can be expanded into a linear combination of a group of basis-functions[18-19], that is

$$f(t) = \sum_{n\in Z} a_n h_n. \tag{1}$$

If the group of basic functions is of orthogonal basis, then inner product can be used to compute their expansion coefficients, that is

$$a_n = \langle f(t), h_n\rangle / \| h_n \|. \tag{2}$$

The value of the expansion coefficient reflects the correlation between $f(t)$ and the basis functions. The basis function library adopted in the present paper is the multi-scale chirplet base functions

$$D(h_{a_\mu,b_\mu,I}) = \{h_{a_\mu,b_\mu,I}(t)\} = \{K_{a_\mu,b_\mu,I}\mathrm{e}^{-\mathrm{i}(a_\mu t+b_\mu t^2)} 1_I(t)\}. \tag{3}$$

Where D is the basis function library; $h_{a_\mu,b_\mu,I}(t)$ is the multi-scale chirplet base functions; I is the dynamic analysis time interval, and $I = [kN2^{-j} \sim (k+1)N2^{-j}]$, where j is the analyzing scale coefficient, $j = 0,1,\cdots,\log_2 N-1$, N is the signal size, $k = 0,1,\cdots,2^{j-1}$; $K_{a_\mu,b_\mu,I}$ is the normalization coefficient, set $\| h_{a_\mu,b_\mu,I} \| = 1$; a_μ is the frequency offset coefficient, and b_μ is the rate of frequency modulation. a_μ and b_μ may depend on the scale and the prior information about the objects of interest. According to sampling theorem, $a_\mu + 2b_\mu t$ should be less than $f_s/2$, f_s is the sampling frequency; $1_I(t)$ is the window function; it is 1 when $t \in I$ or it is 0 when $t \overline{\in} I$.

In multi-scale chirplet sparse signal decomposition, it projects the signals onto the multi-scale chirplet base functions. The maximum projection coefficient and its corresponding chirplet base function can be obtained from each supporting interval I, and the linear frequency of that basic function has the most closely correlation with the frequency of the analytic signal in the time supporting interval I.

The formula for the maximum projection coefficient β_I in supporting interval I is

$$\beta_I = \max\langle f(t), h_{a_\mu,b_\mu,I}(t)\rangle. \tag{4}$$

Suppose $c_I(t)$ is the decomposed signal represented by $\beta_I = \max\langle f(t), h_{a_\mu,b_\mu,I}(t)\rangle$, the maximum projection coefficient in dynamic analysis time interval I, then

$$c_I(t) = abs(2\beta_I)\mathrm{e}^{-\mathrm{j}(a_\beta t+b_\beta t^2 - \mathrm{angle}(2\beta_I))} 1_I(t). \tag{5}$$

In dynamic analysis time interval I, $f(t) = C_I(t) + r_I(t)$, where $r_I(t)$ is the decomposed residual signals.

The more similar the signals with the multi-scale chirplet base functions, the larger their projection coefficients, and the bigger the energy of basic functions. Thus, an appropriate connecting method for dynamic analysis time interval is required. With the method, the total energy of basis function signals during the whole analysis time reaches maximum, that is

$$\max\Big(\sum_{I\in\prod^n} \| c_i(t) \|\Big)^2,$$

where

$$\prod{}^n = \{I_1^n, I_2^n, \cdots\} \in \{I\}, \tag{6}$$

$\prod^n$ covers the whole analysis time period, no overlapping, whose maximum projection coefficient and basis function are

$$\beta = \{\beta_{I_1}, \beta_{I_2}, \cdots\}, \tag{7}$$

$$H = \{h_{a_\mu,b_\mu,I_1}, h_{a_\mu,b_\mu,I_2}, \cdots\}. \tag{8}$$

The connecting method of $\prod$ should guarantee that during the entire analysis time period the total energy of connected basis functions is the maximum, while the frequency curve formed by adjoining piecewise linear frequency of the basis functions is the instantaneous frequency estimation of analytic signal′ major frequency component.

The connecting rule of tracking method based on the chirplet path pursuit[16] is as follows:

1) Initialization. Suppose i is the sequence number of time supporting intervals, $d(i)$ is the total energy of decomposed signals before the ith time supporting interval, pre(i) is the sequence number of the proposed time supporting interval which connects with the ith time supporting interval, and $e(i)$ is the energy of the decomposed signals which correspond with the maximum projection coefficient in the ith time supporting interval. When initializing, suppose $d(i) = 0$ and pre $(i) = 0$.

2) For every element I_i in the set of dynamic analysis time interval $\{I_i, i \in Z\}$, find out every next set of dynamic analysis time interval $\{I_j\}$, that is, the starting time of every element in $\{I_j\}$ is adjacent to I_i .

With the sparse signal decomposition method, the signal component with bigger projection coefficient will be decomposed first and the signal component with smaller projection coefficient will be decomposed later. But, when several components are of the same amplitude, they have the identical projection coefficient, which will result in cross-over decomposition. In order to find a solution to the equal amplitude decomposition, a reserve coefficient is introduced in supporting zone connection in the present paper, which is to reserve basis functions with similar projection coefficient in decomposition. If

$$d(i) + e(i) > d(j) \times \delta, \tag{9}$$

then

$$d(j) = d(j) + e(i), \tag{10}$$

$$pre(j) = i. \tag{11}$$

Usually, δ takes 1. But, if the component signal frequencies are still cross modulated, the value of δ will be reduced gradually until the component signals with the identical projection coefficient can be decomposed.

Suppose c^n, r^n are the signal component and the residual signal component respectively from the decomposition, then

$$c^n = \sum_{I_i \in \prod^n} c_{I_i}(t), \tag{12}$$

$$r^n = \sum_{I_i \in \prod^n} r_{I_i}(t) = r^{n-1} - \sum_{I_i \in \prod^n} C_{I_i}(t). \tag{13}$$

Go on decomposition until it satisfies the given terminating principle.

3 Separation and instantaneous frequency estimation of multicomponent polynomial phase signals

Any multi-component polynomial phase signal (mc-PPS) can be described with parametric model as the following formula

$$x(n) = \sum_{k=1}^{K} A_k e^{j\phi_k(n)} = \sum_{k=1}^{K} A_k e^{j\sum_{m=0}^{M_k} a_{k,m} n^m}, n = 0,1,\cdots,n-1. \tag{14}$$

Where k is the number of PPS components, A_k is the amplitude of kth component, M_k is the highest polynomial phase order for the kth component, and $\{a_{k,m}\}_{m=0}^{M_k}$ are the polynomial phase coefficients for the kth component. We allow M_k to be different for different k .

The polynomial phase signal components are always similar to the chirplet signals over a shorter time interval, and the instantaneous frequencies of its components can be well fitted by straight lines as well. In multi-scale chirplet sparse signal decomposition, the chirplet base functions are approximating adaptively to analytic signals in each time interval. Theoretically, it can be used to separate the multi-component polynomial phase signals and estimate their component frequencies.

Suppose the analytic multi-component polynomial phase signals are $x(t)$. With multi-scale chirplet sparse signal decomposition, the decomposition algorithm is as follows:

1) Set a threshold ζ .

2) Initialization. i is the number of decomposition, and suppose the initial value is 1;

3) Get component c_i from the ith decomposition, whose corresponding frequency is f_i .

4) Carry out differential operator on f_i , and ∇f_i denotes the derivative of f_i . If the sign of ∇f_i is not changed, and $\max(|\nabla f_i|) < \zeta$, then c_i, f_i from this decomposition is valid, do $i = i + 1$,return to step (3). Otherwise, the decomposition is invalid, do $i = i - 1$, and the decomposition terminates.

In the method, the terminating principle is aiming at the monotonous continuity of multi-component polynomial phase signals. $\max(|\nabla f_i|) < \zeta$ means that the frequency difference between two adjacent sampling points should not be too large.

Suppose the decomposed signal component is $c_1, c_2, \cdots, c_K$, then

$$x(t) = \sum_{k=1}^{K} A_k e^{j\phi_k(t)} = \sum_{k=1}^{K} A_k e^{j\sum_{m=0}^{M_k} a_{k,m} t^m} = c_1 + c_2 + \cdots + c_K, \tag{15}$$

$c_1, c_2, \cdots, c_K$ just correspond with K (the number of PPS components) PPS components whose multi-component polynomial phase signal $x(t)$ energy ranks from high to low, which fulfills the separation of multi-component polynomial phase signal. Meanwhile, the instantaneous frequency estimation of the corresponding components is accomplished by adjoining the linear frequency of the basic functions which are employed to separate the mc-PPS.

4 Simulation example

Take a three-component fourth order multi-component polynomial phase signal as an example. The simulation signal is expressed as

$$s(n) = \cos(2\pi \times (n^4 + n^3 + n^2 + 5n)) + 0.8\cos(2\pi \times (3n^3 + n^2 + 20n)) + 0.6\cos(2\pi \times (5n^2 + 30n)). \quad (16)$$

Where $n = 0, 1, \cdots, 511$, and the sampling frequency f_s = 200 Hz, namely the sampling time is 2.555 s. The simulation signal and its frequency are shown in Fig. 1. Decompose the signal with multi-scale chirplet sparse signal decomposition method. The results are shown in Fig. 2, Fig. 3 and Fig. 4.

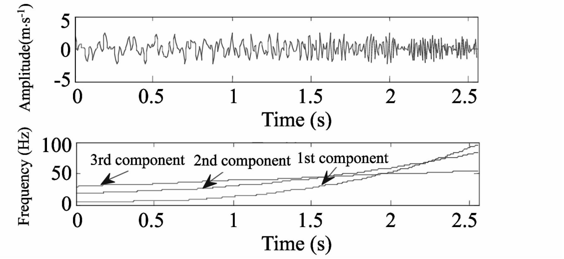

Fig. 1 Simulation signal and its frequency

From Fig. 2～Fig. 4, we can see that the components′ estimation errors and the frequencies′ estimation errors are all very small. Moreover, if numerical analysis is carried out on the decomposition results, the RMSEs of their corresponding components′ estimations are 0.148 8, 0.090 4 and 0.081 6 respectively, and the RMSEs of their corresponding frequencies′ estimations are 0.469 2, 0.352 5 and 0, respectively. These results prove that the present method can not only separate the multi-component polynomial phase signal components effectively, but also estimate their instantaneous frequencies accurately.

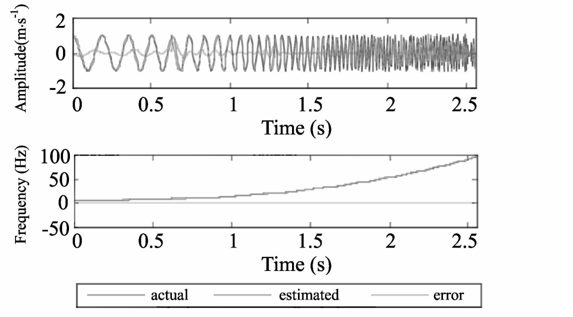

Fig. 2 Decomposition results and differences of the first component

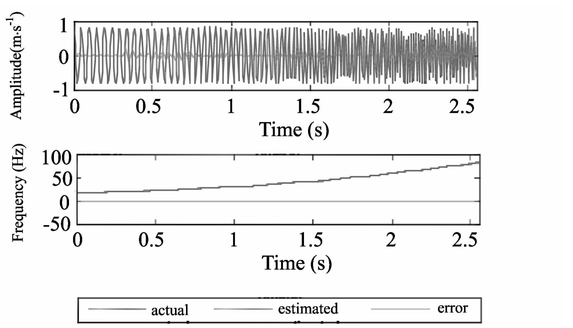

Fig. 3 Decomposition results and differences of the second component

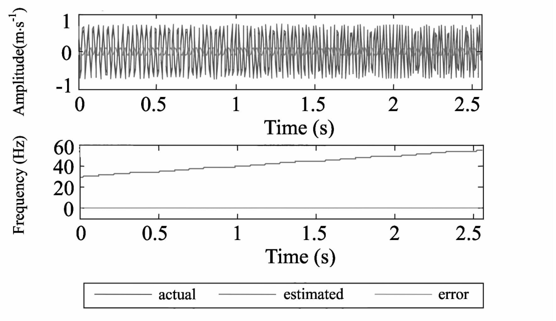

Fig. 4 Decomposition results and differences of the third component

Because of the bad relation between the noise components and the chirplet signals, the present method have good anti-noise ability theoretically. Adding the white noise to the above analysis signals makes the value of the SNR vary from 0 dB to 10 dB with an interval of 1 dB. We obtain the RMSEs of their corresponding components′ estimations and frequencies' estimations which can be seen from Fig. 5 and Fig. 6. The third component is a chirplet component itself, and it is correlated well with the chirplet function. Thus from Fig. 5 and Fig. 6, we can observe that the thirddecomposition component error and frequency error are smaller than the first and second components. Even The accuracy rate of frequency estimation can be 100%. While within the signal-to-noise ratio interval [0 dB 10 dB], all the RMSEs of the three frequencies′ estimations are no more than 0.9, and the RMSEs of the three signal components' estimations are all lower than 0.4. Thereby, the present method performs well in resistance of noise interference and is suitable for the separation and instantaneous frequency estimation of multicomponent polynomial phase signal with noise.

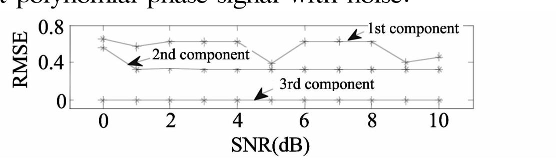

Fig. 5 Influence analysis of noise on signal frequency estimation

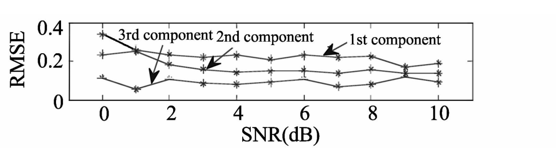

Fig. 6 Influence analysis of noise on signal separation

5 Conclusion

1) The simulation analysis shows that the Polynomial Phase Signal component (PPS component) contained in the multi-component Polynomial Phase Signal can be effectively separated by the multi-scale chirplet sparse signal decomposition method. The frequency curve, which is formed by connecting the linear frequency of the basic

chirplet functions used in the separation, can be regarded as the frequency estimation of PPS components. Furthermore, the frequency estimation is of higher precision.

2) The present method provides a new idea for analyzing and processing of the multi-component polynomial signals. Firstly, the separation of mc-PPS component benefits the searching of its component number k and the estimation of amplitude parameter A_k. Secondly, the instantaneous frequency estimation of the PPS component benefits the searching of its phase polynomial parameter $a_{k,m}$ and the estimation of the highest order M_k .

3) Since the noise signals badly correlate with the chirplet signals, the proposed method has good anti-noise ability and is valid even though the signal-to-noise ratio of multi-component Polynomial Phase Signals (mc-PPS) is very low, which makes it suitable for the analysis of actual engineering signals.

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