# Techniques trend analysis of propagating laser beam quality measurement

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Abstract: Different methods of measuring a propagating laser beam quality are summarized. The disadvantages in traditional way in measuring a laser beam quality is analyzed and the insufficiencies of the Shack-Hartmannin method which is commonly using wave front technique at present is pointed out. Finally, the transmission intensity equation based (TIE-based) measuring way in a laser beam quality evaluation and the corresponding advantages are discussed, which is believed to be a developing trend in laser beam evaluation.

Key words: laser beam; quality measurement; wave front technique;  $M^2$  factor; transmission intensity equation (TIE)

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Since the laser technology emerged, various techniques and methods  $^{[1-6]}$  for quality measurement of laser beams have been explored, both for designing laser beams with high quality in laser manufacturing, and the convenient control of different types of laser beams in a wide range of practical applications. How to accurately describe the quality of laser beams in different fields, including the output beam from a laser oscillator, the beam passing through an optical system and/or the laser beam propagating in space, is always a concern in characterizing the corresponding transverse laser beams properties. For the past decades, different parameters had been used to address the quality of transverse laser beam in different aspects, such as modulation ratio and contrast ratio of near-field, focal spot size, power in bucket, far field divergence angle, Strehl rate,  $M^2$  factor, and so on. In general, these parameters can be divided into three categories: The first is called near-field evaluation and the second is far-field evaluation while the third is the evaluation for propagating laser beam. Among these three categories, the first two types of parameters are more applicably used in the fields of laser designing and manufacturing. On the other hand, in practical laser applications, people are more likely to evaluate the quality of propagating laser beam, as they need to design appropriate optical system to get applicable beams, by transforming corresponding laser beams, according to the quality of laser beams

propagating. That is why, at present marketplace, most of measuring instruments for evaluating laser beam quality focus on the measurement of propagating laser beams. For this reason, in this paper, we are addressing on the quality measurement techniques for propagating laser beam, in which, the main features of the several existing measuring techniques for propagating laser beams are analyzed while the advantages and disadvantages of the corresponding measurement techniques are compared and discussed. Finally, the development trend of measuring techniques for propagating laser beam is explored.

## 1 Measurement technologies for propagating laser beams

## 1.1 Description for propagating laser beam quality

With respect to the quality evaluation of propagating laser beam and/or light beam in and out of an optical system,  $M^2$  factor evaluating has been the main stream since the mid of 1990's, though there were quite different evaluating methods before that.

 $M^2$  factor is defined as the product of actual beam space and beam width divided by the products of ideal Gaussian beam space and the beam width. In fact, the spatial distribution of a laser beam is usual-

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ly expressed in terms of far field divergence angle,  $\theta$ ; while the waist width of a beam is usually called waist radius,  $\omega$ . So  $M^2$  is given by

$$M^2 = \frac{\omega \cdot \theta}{\omega_0 \cdot \theta_0},\tag{1}$$

where  $\omega$  and  $\theta$  are waist radius and the far-field divergence angle of an actual beam respectively, and  $\omega_0$  and  $\theta_0$  are waist radius and the far-field divergence angle of an ideal Gaussian beam respectively. It is demonstrated that, when a Gaussian beam passes through an optical focusing or expanding system (lens or telescope system) without aberration and diffraction effects, the waist size and/or the farfield divergence angle of the actual Gaussian beam may be changed, however, the product of waist radius and the far-field divergence angle keeps constant for a certain beam. Fig. 1 gives schematic diagram of beam changing when a certain beam passes through an optical system, in which,  $\omega_1 \cdot \theta_1 = \omega_2 \cdot$  $\theta_2$ . For a Gaussian beam,  $\omega$  and  $\theta$  are expressed as  $\omega \rightarrow \omega_0$ ,  $\theta \rightarrow \theta_0$ , and the relationship between  $\omega_0$  and  $\theta_0$  is given by

$$\omega_0 \cdot \theta_0 = \lambda/\pi. \tag{2}$$

For actual beams, such as multimode Hermite-Gaussian beam, Laguerre-Gaussian beams, and other mixed mode Gaussian beam, the product of their waist radius  $\omega$  and far-field divergence angle  $\theta$  is bigger than the product of the waist radius  $\omega_0$  and far-field divergence angle  $\theta_0$  of ideal Gaussian beam, namely,

$$M^2 = \frac{\omega \cdot \theta}{\omega_0 \cdot \theta_0} > 1. \tag{3}$$

Generally speaking, the greater the value of the  $M^2$  factor, namely, the larger the product of the  $\omega$  and  $\theta$  (for a certain beam) is, the worse the quality of the actual beam is, as the divergence angle of an actual beam will much deviate from that of the ideal Gaussian beam. On the other hand, when the actual beam is close to ideal Gaussian beam,  $M^2 \rightarrow 1$ . It is obviously that  $M^2$  factor can be a good expression for evaluating the quality propagating beam.

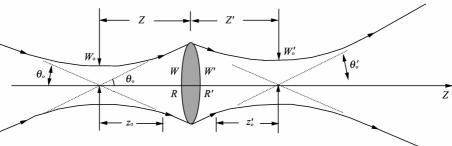


Fig. 1 Schematic diagram of light beam transformation

As above mentioned,  $M^2$  factor can express the quality of propagating laser beam in some degree. However, it only gives some parts of the transverse laser beams properties. Actually, people hope to know more, and even the whole, about the characteristics of a propagating laser beam. Fortunately, it becomes possible for this expectation as the emergence of wave front techniques, with which the phase and intensity of a propagating beam can be restored, so the whole behavior of the propagating beam will be displayed. In other words the transverse laser beams properties are also clear. Obviously, the beam quality can be expressed with wavefront parameters of a laser beam.

Nevertheless, it is very complicated and expensive for the direct wave front measurement of a propagating laser beam, as the phase detection of the beam is involved. Therefore, it is not realistic to directly measure phase information in practical laser beam measuring instrument. At present, with regards to the application of wave-front techniques in beam measurement, instead of directly detecting the phase of a light beam, the phase information is re-

stored mainly via the convenient intensity detection of the beam measured. With both intensity and phase information of the beam, the wave front, the beam profile and even the propagating image of the beam are re-constructed. Consequently, the comprehensive behavior properties, including the transverse properties (beam waist width, far-field divergence angle, etc.) of the measured propagating beam can be easily analyzed via related software. In addition, the wave front parameters can not only be used to measure the quality of a laser beam, but also express the properties of an optical system that a beam passes through, on which a light beam adjustment and collimation can be undertaken in practical application.

Therefore, with wave front techniques being used, the propagating laser beam can be measured, in which not only the parameter  $M^2$  factor can be tested, but also other parameters, such as peak to vally (PV) wave front deviation, root mean square (RMS) wave front error of the beam, as well as the modulation function and system function point spread function, etc. of the optical system that the

beam passes through, can be gained.

### 1.2 Technical features of propagating laser beam quality measurement

In laser beam quality measurement, based on  $M^2$ factor, the related parameters detection are mainly involved in the intensity distribution detecting at different transverse plane of intersection in the light measured, in which the core is accurately detecting the beam intensity distribution at each transversal intersection selected, as shown in Fig. 1, at which at least three planes of intersection needs to be tested, so as to consequently determined the waist radius and the far field divergence angle, and then figure out the corresponding  $M^2$  factor of the light. Obviously, the standard measurement methods and processes as well as the related parameters are strictly required in order to make sure the repeatability and the consistence in the test. For this reason, ISO issued some related instruction on laser beam quality measurement  $\mathsf{method}^{[7]}$ , which becomes the reference standard for  $M^2$  factor based beam measurement in the industry.

As above mentioned, in application of wave front sensing techniques in beam measurement, one of the most practical methods is detecting light beam intensity data to restore the corresponding phase information. In the past ten years, there are two kinds of wave front sensing measurement methods in beam test<sup>[8-10]</sup>: one is the so-called "vector measurement method", and another is the "scalar measurement method".

In vector measurement method, the core is that the laser beam measured is transformed via some kind of optical system, and then the intensity of the transformed light beam is detected in order to calculate the wavefront slope, which is also called wavefront slope measurement. Consequently, the wavefront of the tested beam can be reconstructed via the wavefront slope data by using related wave front restoring algorithm. At present marketplace, there several kinds of such wave front slope based beam measurement techniques, including Shack-Hartmann wave front sensing tecinique and Shearing-Interferometer sensing technique, among which Shack-Hartmann wavefront sensing technique is easy to implement, as the beam transforming is realized via a set of tiny lens, therefore has a wide applications in the beam quality measurement.

On the other hand, in wavefront scalar measurement method, the core is that the beam intensity distribution of two planes in the optical system, through which the beam passes, is detected, and then the wavefront phase information is restored via the tested intensity data by using related algorithms. At present, there are also several wavegront scalar

measurement methods in development, including curvature sensing technique and phase retrieval technique. In curvature sensing technique, the intensity of two focal planes at the tested beam needs to be detected or some kinds of beam transformation need to be undertaken, as a result, the measurement conditions are in high demand and have certain restriction in the application. In contrast, in phase retrieval technique, only the intensity of two planes in the measured beam needs to be measured directly, which is easily implemented in real beam measurement and is getting wide application in beam measurement.

## 2 Development trend of propagating laser beam measurement

### 2. 1 Disadvantages of existing measurement techniques

As we see, at present marketplace, the measuring instruments for propagating laser beam measurement are mainly based on two kinds of techniques: is the direct beam profile measurement based on the  $M^2$  factor, the other is the wave front sensing measurement technique, which is becoming the main stream in beam measuring techniques.

The direct beam profile measurement technique is no longer the first choice as it appears to have the following disadvantages:

- 1) The intensity distribution detection of a light beam needs to repeat for a few times, according to ISO standard, at least five times of measuring need to be undertaken within the beam Rayleigh length, which makes the measurement process complicated;
- 2) Anthropic factor increases in the beam measurement because of the complicated measurement process, which may cause different results measured by different persons.
- 3)In order to detect the intensity distribution of a light beam at different intersection of the beam, a mobile platform is needed, which probably introduces an extra measurement error.
- 4) As  $M^2$  factor is mainly reflecting the diffraction properties of the measured beam, it is not appropriate for evaluating the beam quality in some practical applications.

On the other hand, for the past ten year, among the wave front techniques, which include Shack -Hartmann wavefront sensor, phase retrieval method, pyramid wavefront sensor, and curvature wavefront sensing, Shack-Hartmann based sensing technique becomes more and more applicable in beam measurement.

However, the Shack-Hartmann based sensing technique, also has some insufficiencies in the fol-

lowing aspects:

- 1) In Shack-Hartmann wavefront sensor, there needs a set of tiny lens array, which are used to transform the beam.
- 2) The aberration of the lens arrays is also the source of the measurement error.
- 3) In present calculation model for lens arrays in Shack-Hartmann wavefront sensor, each lens is regarded to be independent, with the phase grating effect of each lens not being considered, which will result in measuring errors for most cases in Shack-Hartmann wavefront sensors<sup>[11]</sup>.
- 4)In the measurement, the wave front slope of a light beam, instead of directly detecting from the beam intensity, is calculated from the delta-displacement data, transferred via the intensity detection of the transformed beam by the lens arrays. Therefore the accuracy of a light beam measurement can not be determined by the numbers of detected charge-coupled device (CCD) pixels.

### 2.2 Development trend of beam measurement

It is clear that wavefront sensing based technique is the main trend for propagating light beam measurements, in which phase retrieval algorithm is one of the developing direction for wavefront sensing based laser beam measurement in the future. In the various sorts of phase retrieval algorithms, different kinds of iterative algorithms are continuously being improved, and phase solving algorithm based on transmission intensity equation (TIE) [15-18] is much being concerned, among which, as the iterative algorithm is quite complicated due to its convergence properties, the phase solving algorithm based on TIE is getting more and more applicable, because of its simplicity, which, we believed, is one of the most important measurement technique for the beam measurement.

In wavefront measurement based on TIE technique, the core is to solve the corresponding phase information via the transmission intensity equation.

Set a monochromatic light beam is propagating along the z axis and the corresponding electric field can be expressed as

$$E(\mathbf{r}, \phi) = u(\mathbf{r})e^{ikz} = I^{\frac{1}{2}}(\mathbf{r})e^{ikz+i\phi(\mathbf{r})}$$
. (4)

where r = xi + yj + zk; I(r) is the intensity of the beam;  $\phi(r)$  is beam phase, k is wave number of the beam and z is transmission distance. Get equation (4) substituted into the wave equation, it can be got as

$$k \frac{\partial I}{\partial z} = - \nabla [I \cdot \nabla \phi] = - \nabla I \cdot \nabla \phi - I \nabla^2 \varphi,$$
(5)

where  $\nabla^2 = \partial_x^2 + \partial_y^2$  is the two-dimensional Laplacian.

In the Fresnel diffraction conditions, the change of beam intensity I(x, y, z) transmitting along the z axis is linked with corresponding beam space phase by Eq. (5), which is called TIE. It is known from Eq. (5) that phase information can be got through solving TIE (solving second order differential equation). In this way, the wavefront of the beam is reconstructed and measured for the beam.

In Eq. (5), the key in restoring the phase is solving  $\partial I/\partial z$  accurately.

Setting that the beam is propagating along the z axis, the beam intensity propagating in three planes of intersection ( $z=z_0$ ,  $z_0-\Delta z$ ,  $z_0+\Delta z$ ) are  $I(x,y,z_0)$ ,  $I(x,y,z_0-\Delta z)$  and  $I(x,y,z_0+\Delta z)$ , respectively. If is focal plane, the beam intensity  $I(x,y,z_0-\Delta z)$  and  $I(x,y,z_0+\Delta z)$  can be expressed by the beam intensity in focal plane, and the corresponding delta-changes, namely,

$$I(x, y, z_0 - \Delta z) = I(x, y, z_0) - \Delta z \cdot \frac{\partial I}{\partial z} + \frac{(-\Delta z)^2}{2!} \cdot \frac{\partial^2 I}{\partial z^2} + \varepsilon (-\Delta z)^3,$$
(6)  
$$I(x, y, z_0 + \Delta z) = I(x, y, z_0) + \Delta z \frac{\partial I}{\partial z} + \frac{\partial$$

$$I(x, y, z_0 + \Delta z) = I(x, y, z_0) + \Delta z \frac{\partial}{\partial z} + \frac{(\Delta z)^2}{2!} \cdot \frac{\partial^2 I}{\partial z^2} + \varepsilon (\Delta z)^3.$$
 (7)

Obviously, with the detected data of  $I(x, y, z_0 - \Delta z)$  and  $I(x, y, z_0 + \Delta z)$ , the phase of the beam can be restored via the measurement of beam strength, by solving the differential equation.

With Zernike polynomial,  $\phi(x,y)$ , the wavefront of the measured beam can be expressed as

$$\phi(x,y) = \sum_{n=0}^{\infty} a_n z_n = \sum_{n=0}^{\infty} a_i z_i + E,$$
 (8)

where  $z_n$  is the *n*-th Zernike polynomial,  $a_n$  is *n*-th coefficient, and E denotes the total error after nth term.

Assuming there is a set of discrete detected points with the total number of m. It is obvious that each detected data satisfies Eq. (8) and there have m sets of such equations, in which, the equation can be expressed as matrix form as

$$\mathbf{\Phi} = \mathbf{Z} \cdot \mathbf{A} + \mathbf{E}. \tag{9}$$

where  $\boldsymbol{\Phi} = (\varphi_1, \varphi_2, \dots, \varphi_n)^T$ ,  $\boldsymbol{Z} = (z_{ij})$  is  $(\boldsymbol{m} \times \boldsymbol{n})$ order matrix, and  $\boldsymbol{A} = (a_1, a_2, \dots, a_n)^T$ ,  $\boldsymbol{E} = (\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n)^T$ .

With regards to the wavefront measured, the least squares conditions for M sets of discrete sampling points can be expressed as

$$\delta = \sum_{i=1}^{M} \left[ a_n Z_n(x_i, y_i) - S(x_i, y_i) \right]^2 \rightarrow \min.$$
(10)

In matrix formalism, it can be expressed as

$$\mathbf{Z}a = s. \tag{11}$$

So, Eq. (11) can be writen as

$$\boldsymbol{a} = (\boldsymbol{Z}^T \boldsymbol{Z})^{-1} \boldsymbol{Z}^T \boldsymbol{S}. \tag{12}$$

Parameter A can be estimated with corresponding algorithm.

From the above analysis, the accuracy of propagating laser beam measuring via TIE based method is upon the accuracy of the laser light detecting system. If a light detecting system is based on CCD technique, then the measuring accuracy of the detecting laser beam is directly related to the pixel number of the CCD system. For this reason, the TIE based wavefront measurement technique is called "digital measuring technique", whose measuring accuracy is decided by the number of digits (CCD pixel number). Figs. 2 and 3 are measuring results corresponding to the Shack-Hartmann and TIE technique, respectively, in which the measuring accuracy in Fig. 3 is much higher than that in Fig. 2.

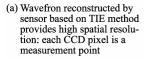


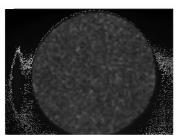
(a) Wavefront reconstructed by (Shack-Hartmann sensor: finite number of microslenses limits the spatial resolution

(b) Beam intensity as captured by Shacke Hartmann sensor: low resolution in the intensity image is due to data sampling by microlenses

Fig. 2 Measuring results by Shack-Hartmann sensor







(b) Beam intensity as captured by sensor based on TIE method, featuring native camera resolution

Fig. 3 Measuring results by TIE method

#### 3 Conclusion

It is indispensable for the measuring and evaluating of a propagating laser beam in practical laser application system. In this paper, the developing process of the measurement and instrumentation for a propagating laser beam quality is summarized: From the 1990's of last century, the method of  $M^2$  factor had been used to evaluate the quality of a propagating laser, in which  $M^2$  is calculated via measuring the light distribution of a laser beam in different transversal intersections. With the emerging of wavefront techniques, the phase information is restored and the corresponding image of a propagating laser beam can be obtained, as a results, related parameters, besides  $M^2$  factor, are used to evaluate the quality of a propagating laser beam. In addition, the disadvantages of direct measuring light distribution in different transversal intersections are discussed, and at the same time, the insufficiency of the Shack-Hartmann is pointed out, which is one of the most commonly used wavefront techniques at present marketplace. Finally, the TIE based method for phase restoring and the corresponding advantages are analyzed. We believe that the TIE based wavefront technique will be the mainstream technique in the measuring and evaluating a propagating laser beam.

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