Method of Weak Signals Detection Based on Chaos Suppression in the Nonresonant Parametric Drive

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Abstract – **The evolution of chaotic state of Lorenz system on the familiar parameter space orbit is analyzed. Based on the principle of chaos suppression with nonresonant parametric drive, the model of detecting weak periodic signals in strong noise is built. According to the parametric equivalent relationship obtained using averaging method and renormalization method, the critical values of detection parameters are determined, which lead to a sudden change of system dynamical behavior from periodic orbit to stable equilibrium point. Simulation results show that weak periodic signals in strong noise can be detected accurately with the proposed system. The method can obtain accurate range of parameter threshold through theoretical analysis, and the detection criterion is rather simple, which is more convenient for automatic detection.**

Key words – *Lorenz system*; *weak signal detection*; *nonresonant parametric drive*; *chaos suppression*

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1 Introduction

Last decade, there are mainly two directions of detecting weak signals in the application of nonlinear dynamics: Chaos and Stochastic Resonance. By contrast, however, the approaches based on the Chaos have particular advantages. There is no request on background noise or the length of measured data. In the recent years, domestic scholars achieved remarkable progress on weak signals detection by applying nonlinear dynamics chaotic theory[1-3]. Firstly, detected signals through the forcing driven bifurcation of Duffing-Holmes system, and then proposed modified Duffing-Holmes system, which can reach the lower Signal-Noise-Radio(SNR) work low limit. In a word, existing methods are mainly based on chaos suppression theory of nonautonomous chaotic system.

Resently, some disadvantages of the detection method based on nonautonomous Duffing-like system were uncovered: results affected obviously by critical value, judging criteria according to the mutation of phase diagram is not effective when the intensity of background noise is not stationary. Consequently, it is necessary to study new chaotic detection system.

Accordingly, some autonomous systems, such as Lorenz system, have been researched deeply. Some new progresses about controlling and application of chaos are gotten[4-5], and global dynamics behavior characteristics are verified rigorously[6]. In which chaos suppression in the nonresonant parametric drive is a new way for signal detection[4].

The paper shows the chaotic evolution of Lorenz system, builds the model of detecting weak periodic signals based on chaos suppression in the nonresonant parametric drive, and performs direct numerical simulations to verify the theory.

2 The evolution of chaotic state of Lorenz system

Lorenz system is a classical autonomous chaotic system which global dynamics behavior has been analyzed thoroughly and proved strictly. Its system equations are

$$\begin{cases} \dot{x} = \sigma(y - x), \\ \dot{y} = rx - y - xz, \\ \dot{z} = xy - bz. \end{cases} \quad (1)$$

With parameters:

$$\sigma = 10, b = 8/3, r \in [24.74, 2\,000],$$

set initial point $(x_0, y_0, z_0) = (1,1,1)$, step $h = 0.001$, and create attractor by solving system (1). We computed 20 000 times and discard first 9 000 times to make sure the system converged to the attractor. Fig. 1 shows a group of representative result.

Observe the changes of phase space: As r declines, we would see successively the limit cycle (See Fig. 1(a)), period-doubling bifurcation (See Fig. 1(b)), Lorenz-like attractors (See Fig. 1(c)). And the process repeats three times[7].

when $r < 24.74$ system (1) has three equilibrium points

$$S_0 = (0,0,0),$$

$$S_{\pm} = (\pm\sqrt{b(r-1)}, \pm\sqrt{b(r-1)}, r-1).$$

When $0 < r < 1$, system is stable on the origin; $1 < r < 24.74$, the origin turns to unstable, the other two equilibrium points are stable; $r > 24.74$, system is chaotic or periodical state[8].

Analyzed particularly $r = 1$, forked bifurcation appeared at the origin; $r > 1$, the origin turns to unstable and becomes saddlefocus point of one-dimensional unstable manifold; $1 < r < r_h$, the other two equilibrium points

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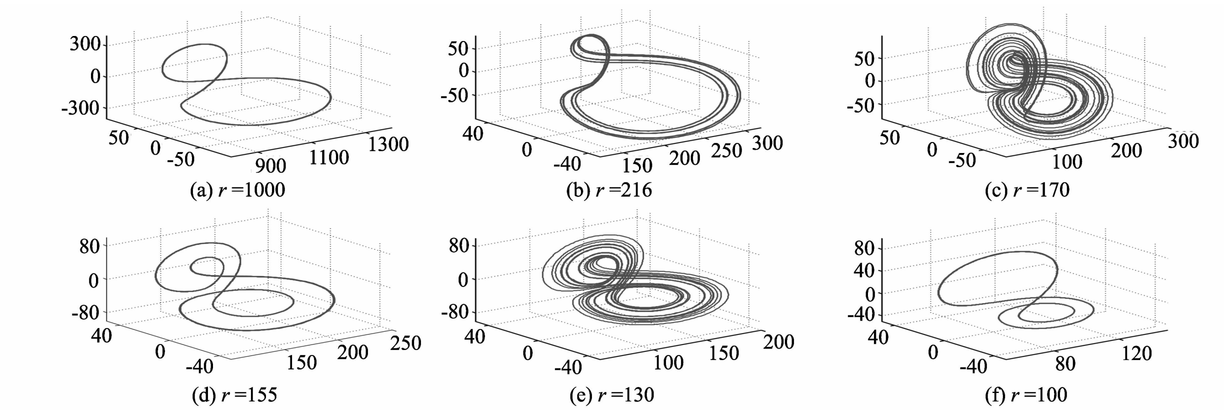

Fig. 1 Attractors of Lorenz system

turn to saddlefocus points of two-dimensional unstable manifold, Hopf bifurcation occurs, eigenvalues are $\lambda = -(\sigma + b + 1), \lambda = \pm i\sqrt{2\sigma(\sigma+1)/(\sigma - b - 1)}$, system turns into chaos[9]. Where

$$r_h = \sigma(\sigma + b + 3)/(\sigma - b - 1) \approx 24.74. \qquad (2)$$

$r = 14.546\,2$ is a homoclinic branch point. When $1 < r \leqslant 14.546\,2$, unstable manifold tends spirally to equilibrium point $S+$ or $S-$ on the same side, the spiral turns to bigger as r increasing; when $r > 14.546\,2$, unstable manifold tends spirally to equilibrium point $S+$ or $S-$ on the other side.

The evolution of system including: the conversions between the attractor and limit cycle, the attractor and periodic bifurcation, the attractor and stable equilibrium points, limit cycle and periodic bifurcation. When $r = 1 \to r < 1$, system changed from bifurcation to stable at the origin. Equilibrium point and r would not vary as other parameters changing. Therefore, we built detection model according to the state conversion.

3 Detecting model based on chaos suppression in the nonresonant parametric drive

In Ref. [4], chaos suppression of Lorenz system was carried out by using periodic signal which frequency is much higher than the system characteristic frequency as parameter drive. System tended to periodic state or stable at an equilibrium point, and the results were verified with an analog electronic circuit.

The frequency of Lorenz system can not analytically express the external drive without apparent time and frequency, so we carried out open-loop control of chaos suppression with nonresonant parametric drive.

Assumed that the detected signals include weak periodic component with frequency ω, and ω was much bigger than system characteristic frequency ω_0, we designed detection system[8]

$$\begin{cases} \dot{x} = \sigma(y - x), \\ \dot{y} = r\{1 + k\cos(\omega t + \varphi) + aF_{bp}[u(t)]\}x - y - xz, \\ \dot{z} = xy - bz. \end{cases} \qquad (3)$$

Where $\sigma = 10$; $b = 8/3$; r is adjustable, $k\cos(\omega t + \varphi)$ is periodic driven signal which controls system state, adjustable phase φ can match the phase of driven and detected signals.

Input $u(t) = s(t) + n(t)$, where $s(t)$ is weak periodic signal, $n(t)$ is noise, F_{bp} is bandpass filter function, and y is output of detection system. Considering $a = 0$, i.e. with no input, integrated the system variables and neglected the higher orders element by using averaging method, we obtained the renormalization system

$$\begin{cases} \dot{x}_1 = \sigma(y_1 - x_1), \\ \dot{y}_1 = r_{\mathrm{eff}}x_1 - y_1 - x_1 z_1, \\ \dot{z}_1 = x_1 y_1 - bz_1. \end{cases} \qquad (4)$$

where

$$r_{\mathrm{eff}} = r\left(1 - \frac{r\sigma k^2}{2\omega^2}\right), \qquad (5)$$

Relations (1) and (3) of two systems have same dynamical character for $r_{\mathrm{eff}} = r$, That is for $r_{\mathrm{eff}} < 1$, the Jacobi Matrix eigenvalues λ of Eq. (3) at the origin are three negative real number, and for $r_{\mathrm{eff}} > 1$, an eigenvalue turning to positive, bifurcation occurs. Because the origin of Eq. (3) is global stable equilibrium point, k should agree the condition

$$k > k_c = (\omega/r)\sqrt{2(r-1)/\sigma}, \qquad (6)$$

The output of detection system converging to zero for $k > k_c$, stable limit cycle periodic for k is a slightly less than k_c. We set $k = k_c$, what the amplitude of internal drive of system (3), and adjust the coefficient a to control the power of input. The output would converge to zero from periodic state as long as any weak periodic signals $s(t)$ exist.

Considering Eq. (6), we analyzed the effect of k on eigenvalues λ for $r_{\mathrm{eff}} = 1$, and obtained the Jacobi Matrix at the origin. Three eigenvalues are $\lambda_1 = 0$, $\lambda_2 = -b$, $\lambda_3 = -(\sigma + 1)$. λ_1, λ_2 are constants or unrelated to k, so

$$p = \left.\frac{\partial \lambda_3}{\partial k}\right|_{r_{\mathrm{eff}}=1} = \frac{\partial(-\sigma - 1)}{\partial k}, \qquad (7)$$

From Eq. (6) and Eq. (7), we obtained

$$p = \frac{2\sigma r}{\omega}\sqrt{\frac{\sigma}{2(r-1)}}. \qquad (8)$$

The absolute value of p is bigger, and the change of λ is more remarkable as the change of $k.\omega$. So a bigger r and a less ω (for $\omega \gg \omega_0$) can make the system output

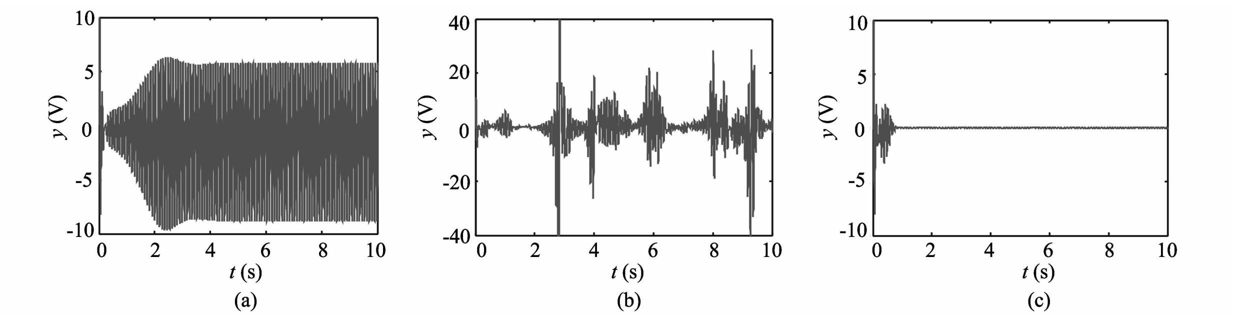

Fig. 2 System output: (a) When there is no signal, (b) When input is white noise, (c) when input is weak perodic signal and white noise

more sensitive to external signals. Because of the renormalization, the estimate of critical parameter k_c would be slightly bigger than practical value, so it needs to be modified through simulation. Time scale transformation $t = \omega_1 * t_1/\omega$ should be taken to Eq. (3) if detecting signals which frequency $\omega_1 \neq \omega$.

4 Numerical simulation

According to the analysis above, there are three model parameters need to be determined ω, r, k_c . From Eq. (6) we see that three parameters restrict each other, which should be determined simultaneously. The frequency (which is the mean-time derivative of the phase) of the Lorenz system can be defined as[4]

$$\omega_0 = \lim_{T\to\infty} \frac{2\pi N(T)}{T}. \qquad (9)$$

where $N(T)$ is the number of turns performed in T .

In Eq. (3), the characteristic frequency of the system is $\omega_0 \approx 8.33$ rad/s for $r = 28$. After simulation and comprehensive consideration, we obtained $k_c = 2.408$ in Eq. (3), for determining $r = 168$, $\omega = 70$ rad/s, and then determined the correction $k_c = 2.751$ through system simulation. Fig. 2 shows that when system input is $u(t) = 0$, the output y take on critical periodic state for $k = k_c, \varphi = 0, a = 1$, the central frequency of bandpass filter ω , and the gain approximate 5 dB.

When input signals are white noise with power $P = 0.04$ W, random fluctuation appeared in the wave envelop of system output (Fig. 2(b)). For $s(t) = \mu\cos(\omega t)$, system output changed suddenly as μ increased from 0. When $\mu = 0.002$ V, output converges rapidly to zero (Fig. 2(c)). Therefore we can make judging criteria on weak periodic signals exist or not to calculate the mean power of output after specific time.

We computed SNR low limit SNR $= -26$ dB, which would be lower if system parameters are optimized according to the analysis result of Eq. (8).

5 Conclusion

1) The model of weak signals detection under strong noise based on Lorenz system was built. The system output would change suddenly from periodic to zero as long as system input include weak periodic signals. The theoretical analysis has been found to agree well with the results of numerical simulations.

2) There is room for optimizations on reducing SNR work low limit, advancing detecting ability through choosing more appropriate parameters. Some potential problems of practice signals detection should be analyzed thoroughly, such as the relationship of phase information between model and signals.

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