Frequency Domain TOA Estimation Algorithm Based on Cross-HOC

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Abstract – High-Order Cumulants (HOC) and cross-correlation was combined to suppress the Gaussian color noises and the un-related noises in real applications. The cross-HOC TOA estimation model was developed based on the diagonal slice of the forth-cross-cumulant. The eigen analysis was carried out, and the eigen noise space and the eigen signal space was achieved. Then the Frequency Domain TOA estimation algorithm based on Cross-HOC was developed. Different simulation experiments were carried out to draw out the conclusions.

Key words – *wireless location*; *High-Order Cumulant*; *superresolution*

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1 Introdution

The reseach of High-order Cumulants (HOC) is an important topic[1-3] in signal processing. HOC is statistics more than two orders. It's known to all that the cumulants higher than 3 orders of Gaussian signals are zero, that is, HOC can efficiently suppress Gaussian noises, so the spectrum estimation algorithms based on HOC were popular for its suppression ability of color Gaussian noises.

However, HOC can't suppress non-Gaussian noises, so if the cross-spectrum with the excellent suppression ability of non-related noises was combined with HOC, and we call cross-HOC here, then the unrelated non-Gaussian noises and the color Gaussian noises can be both efficiently suppressed. This is important and helpful in the actual application. And it is also demonstrated in paper [3-5] that the unrelated non-Gaussian noises and the color Gaussian noises can be both suppressed to zero in the different communication channels. Based on above, a cross-HOC frequency domain TOA estimation algorithm was developed in this paper.

In this paper, the cross-HOC-based TOA estimation model was developed based on the diagonal slice of the cross-forth-cumulant, and the eigen analysis was carried out, the eigen noise space and the eigen signal space was achieved, then the frequency domain TOA estimation algorithm based on Cross-HOC was developed. Furthermore, research on the bandwidth requirements of the algorithm was carried out, and some conclusions were made.

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2 HOC of the TOA estimation

Here we use the parametric model we adopted in Ref.[6-7]:

$$y(f) = \sum_{i=0}^{L_p-1} a_i \exp\{j(2\pi f\tau_i + \varphi_i)\} + w(n), \quad (1)$$

and L_p is the number of the different paths between the transmitter and the receiver, $\alpha_k = |\alpha_k| e^{j\varphi_n}$, α_k is the normalized amplitude of the k_{th} path, τ_k is the arriving time of the k_{th} path, and the vector summation of the all arriving paths is the total receiving strength of the receiver.

The corresponding discrete model is

$$y(n) = \sum_{i=0}^{L_p-1} a_i \exp\{j(2\pi[f_0 + (n-1)\Delta f]\tau_i + \varphi_i)\} + w(n) = \sum_{i=0}^{L_p-1} a_i' \exp\{j2\pi(n-1)\Delta f \cdot \tau_i\} + w(n), \quad (2)$$

where $a_i' = a_i \cdot \exp(j2\pi f_0 \tau_i)$, φ_i s are independent random variables in uniform distribution in $[-\pi, \pi]$, and we set $\omega_i = 2\pi\tau_i$. According to the cumulant properties, $y(n)$ is the summation of the different and independent paths. so only one single path should be included when we study the cumulant of $y(n)$. Here we adopt the forth-cumulant, and to the zero-mean complex random signals, the forth-cumulant is defined as

$$c_{4x}(m_1, m_2, m_3) = \mathrm{cum}[y^*(n), y^*(n+m_1), y(n+m_2), y(n+m_3)], \quad (3)$$

where, cum stands for cumulant, and the complete expression can be found at Ref.[8].

According to the work of Swami and Mendel[5], to the signal series $y(n)$ described in Eq.(2), the two order auto-correlation function and the forth-cumulant are

$$R_y(m) = \sum_{k=0}^{L_p-1} |a_k|^2 \exp(j\omega_k m), \quad (4)$$

$$\mathrm{cum}_{4y}(m_1, m_2, m_3) = -\sum_{k=0}^{L_p-1} |a_k|^4 \exp[j\omega_k(-m_1 + m_2 + m_3)]. \quad (5)$$

Specially, we focus on the one dimesion diagonal slice of the forth-cumulant. We set $m_1 = m_2 = m_3 = m$ in Eq.(5), and we can describe the diagonal slice of the complex harmonic process as

$$\mathrm{cum}_{4y}(m) = \sum_{k=0}^{L_p-1} |a_k|^4 \exp[j\omega_k(m)]. \tag{6}$$

And Eq.(6) gives us a special and important information that the diagonal slice completely keeps the all parameters such as paths count L_p , amplitude a_k and frequency ω_k . What's more, if Eq.(6) was compared to Eq.(4), we can find that the diagonal slice of the forth-cumulant of $y(n)$ is almost the same to the auto-correlation function of the complex harmonic process described by Eq.(7), only with the sign different:

$$\bar{y}(n) = \sum_{k=0}^{L_p-1} |a_k|^2 \exp[j\omega_k n + \varphi_k]. \tag{7}$$

It can be deduced from Eq.(7) that if the auto-correlation function in the traditional TOA algorithms was replaced by the forth-cumulant, the eigen-decomosition based TOA estimation method can still work, of course the cross-HOC too, and this is the idea of the cross-HOC based TOA algorithm.

3 The cross-HOC based TOA algorithm

Firstly, supposing two frequency domain sample series $x(n)$, $y(n)$ in different bands are in ready, then according the channel model in Ref.[6,7], we have

$$y(n) = \sum_{i=0}^{L_p} a_i \exp[j(\omega_i n + \varphi_{xi})] + \xi_y(n) + \eta_y(n), \tag{8}$$

$$x(n) = \sum_{i=0}^{L_p-1} \beta_i \exp[j(\omega_i n + \varphi_{yi} + \theta_i)] + \xi_x(n) + \eta_x(n), \tag{9}$$

where, $a_i' = a_i \cdot \exp(j2\pi f_0 \tau_i)$, $\beta_i' = \beta_i \cdot \exp(j2\pi f_0' \tau_i)$, in which f_0 and f_0' is the starting frequency of the sampling bands, are the amplitude information, $\omega_i = 2\pi\tau_i$ shows the time delay, and φ_{xi} , φ_{yi} are the phases of the different paths uniformly distributed in $[0,2\pi]$, θ_i is the phase difference between $x(n)$, $y(n)$, L_p is the path count, ξ_x , ξ_y , η_x , η_y are zero-mean noises, among which ξ_x , ξ_y are independent Gaussian or non-Gaussian noises, η_x , η_y related Gaussian noises, furthermore, ξ_i and η_j are independent. Then according to the definition of the forth-cumulant, the slice of the cross -forth-cumulant can be deduced as

$$c_{yxxx}(m) = \mathrm{cum}\{y(n)x(n+m)x^*(x+m)x^*(x+m)\} = \sum_{i=0}^{L_p-1} \alpha_i\beta_i^3 \exp[-j(\omega_i m + \theta_i)]. \tag{10}$$

Therefore, we can develop the cross-forth-cumulant matrix $(q \gg p)$ as

$$\boldsymbol{C}_{yxxx} = \begin{bmatrix} c(0) & c(-1) & \cdots & c(-q+1) \\ c(1) & c(0) & \cdots & c(-q+2) \\ \vdots & \vdots & \ddots & \vdots \\ c(q-1) & c(q-2) & \cdots & c(0) \end{bmatrix} = APA^H, \tag{11}$$

where

$c(i) = c_{yxxx}(i) = c_{xyyy}(i,0,0,0)$, $i = 0, \pm 1, \cdots, \pm(q-1)$,

$A = [a(\omega_1), a(\omega_2), \cdots a(\omega_P)]$,

$a(\omega_1) = [1 \quad e^{j\omega_i} \quad e^{j2\omega_i} \cdots e^{j(q-1)\omega_i}]^{\mathrm{T}}$,

$P = \mathrm{diag}[-\alpha_1\beta_1^3\exp(j\theta_1), -\alpha_2\beta_2^3\exp(j\theta_2), \cdots, -\alpha_p\beta_p^3\exp(j\theta_p)] = \mathrm{diag}[a_1, a_2, \cdots, a_p]$.

Obviously, the order of $\boldsymbol{C}_{yxxx}$ is p, $\boldsymbol{C}_{yxxx}$ is a $q \times q$ square matrix, and its exigent-decomposition is

$$\boldsymbol{C}_{yxxx} = U \begin{bmatrix} \sum & 0 \\ 0 & 0 \end{bmatrix} V^H. \tag{12}$$

Where, U, V are the left and right singular vector of $\boldsymbol{C}_{yxxx}$, and $\sum = \mathrm{diag}\{\sigma_1, \sigma_2, \cdots, \sigma_p\}$ with $\sigma_1 \geqslant \sigma_2 \geqslant \cdots \geqslant \sigma_p$. From Eq.(12), we can see that $\boldsymbol{C}_{yxxx}$ completely suppress the influence of the independent noises and the related Gaussian noises. Similar to the eigen-decomposition of the auto-correlation matrix, U, V can be decomposed to signal subspace and noise subspace as $U = [U_1 \quad U_2]$, $V = [V_1 \quad V_2]$, here the signal subspace vectors $U_1 = [u_1 \; u_2 \cdots u_p]$, $V_1 = [v_1 \; v_2 \cdots v_p]$ and the noise subspace vectors $U_2 = [u_{p+1} \; u_{p+2} \cdots u_q]$, $V_2 = [v_{p+1} \; v_{p+2} \cdots v_q]$, so it can be concluded that:

1) the signal space $S = \mathrm{span}\{u_1 \; u_2 \cdots u_p\}$ or $S = \mathrm{span}\{v_1 \; v_2 \cdots v_p\}$. And $\mathrm{span}\{u_1 \; u_2 \cdots u_H\} = \mathrm{span}\{v_1 \; v_2 \cdots v_H\} = \mathrm{span}\{a(\omega_1) \; a(\omega_2) \cdots a(\omega_H)\}$, that is U_1, V_1, A are three different base vectors of the signal subspace, with the dimension L_p .

2) the noise subspace $G = \mathrm{span}\{v_{H+1} \; v_{H+2} \cdots v_m\}$ or $g = \mathrm{span}\{u_{H+1} \; u_{H+2} \cdots u_m\}$.

U_2, V_2 are two different base vectors of the noise subspace, with the dimension $m - L_p$.

3) the total space $I = [SE] = S \oplus E$, the subspace are orthogonal S and E complementary subspaces of I .

Based on above, we can deduce the frequency domain TOA estimation Algorithm Based On Cross-HOC as the following: $\forall u_i \in U_2$(or $\forall v_i \in V_2$) , we have

$$A(\omega)^H u_i = 0. \tag{13}$$

Furthermore, the psesudo MUSIC curve of the cross-forth-cumulant can be deduced as

$$f^{x\mathrm{cum}}(\omega) = \frac{1}{\| U_2^H A(\omega) \|^2}. \tag{14}$$

And by searching the peaks of $f^{x\mathrm{cum}}(\omega)$, the time delays of the different arriving paths can be estimated by $\hat{\omega}_k = 2\pi p\hat{\tau}_k$. This is the Frequency Domain TOA estimation algorithm based on Cross-HOC proposed in this paper.

4 Experiments about the bandwidth requirements

In the small sample conditions, the estimation error of the HOCs is very large, and if we want to achieve accurate TOA estimation, large dimension of the HOC matrix is required, which is usually far beyond that of the two order auto-correlation matrix. For example, a 80×80 cumulant matrix was adopted in paper [8] to estimate the two frequency components in a harmonic process. To the dimension requirements or bandwidth requirements in TOA estimation, we analyze from simulation. Here we suppose a channel with three arriving paths (115, 1), (150,0.2) and (175,0.3), and the first value was the arriving time of the different paths in ns and the second value was the normalized strength amplitude. Firstly, we analyzed the fundamental requirements to realize precise TOA estimation in the perfect condition with no noise.

And concerning the 20 MHz bandwidth 802.11 a/g can offer, we carried out simulation experiments if the 20 MHz bandwidth is enough to realize accurate forth-order-cumulant estimation. In this simulation, we set, $BW = 20$ MHz, $m\Delta f = 6$ MHz and chose different values of Δf, (L, m), then adopted equation (14) to estimate the different TOAs of the paths. And the parameter values and experiment results were shown in Tab.1.

Tab.1 TOA estimation results with $BW = 20$ MHz, $m\Delta f = 6$ MHz

$BW = 20$ MHz, $m\Delta f = 6$ MHz, $\Delta f, (L, m)$	$\tau_1 = 115$ ns		$\tau_2 = 150$ ns		$\tau_3 = 175$ ns	
	$\hat{\tau}_1$	$\hat{\tau}_1 - \tau_1$	$\hat{\tau}_2$	$\hat{\tau}_2 - \tau_2$	$\hat{\tau}_3$	$\hat{\tau}_3 - \tau_3$
$\Delta f = 0.1$ MHz, (201,60)	101.1	−14.9	151.6	1.6	*	*
$\Delta f = 0.2$ MHz, (101,30)	102.7	−12.3	173.7	23.7	*	*
$\Delta f = 0.4$ MHz, (51,15)	109.4	−5.6	144.7	−5.3	*	*
$\Delta f = 1$ MHz, (21,6)	119.6	4.6	129.3	30.7	*	*

* stands for unresolved

From Tab.1 we can see that tie 20 MHz bandwidth can't realize forth-order-cumulant estimation, neither can accurate TOA estimation. Then we expanded the bandwidth to 40 MHz, and chose to 15 MHz, the experiment results were shown in Tab.2.

Tab.2 TOA estimation results with $BW = 40$ MHz, $m\Delta f = 15$ MHz

$BW = 40$ MHz, $m\Delta f = 15$ MHz, $\Delta f, (L, m)$	$\tau_1 = 115$ ns		$\tau_2 = 150$ ns		$\tau_3 = 175$ ns	
	$\hat{\tau}_1$	$\hat{\tau}_1 - \tau_1$	$\hat{\tau}_2$	$\hat{\tau}_2 - \tau_2$	$\hat{\tau}_3$	$\hat{\tau}_3 - \tau_3$
$\Delta f = 0.1$ MHz, (401,150)	130.6	15.6	154.5	4.5	*	*
$\Delta f = 0.2$ MHz, (201,75)	130.9	15.9	154.7	4.7	*	*
$\Delta f = 0.4$ MHz, (101,48)	132.1	17.1	152.0	2.0	*	*
$\Delta f = 1$ MHz, (41,15)	132.9	17.9	152.9	2.9	*	*

From Tab.2 we can see that with the 40 MHz bandwidth and the 15 MHz $m\Delta f$, accurate TOA estimation results still can't be realized. We can solve the problem from two ways, one is to increase the cumulative average calculating times M, and the second is to increase the dimension of the cumulant matrix m. So we increased to a 50 MHz bandwidth which directly will increase the value of M, but with the same $m\Delta f = 15$ MHz, and the experiment results were shown in Tab.3.

Tab.3 Estimation results with $BW = 50$ MHz, $m\Delta f = 15$ MHz

$BW = 50$ MHz, $m\Delta f = 15$ MHz, $\Delta f, (L, m)$	$\tau_1 = 115$ ns		$\tau_2 = 150$ ns		$\tau_3 = 175$ ns	
	$\hat{\tau}_1$	$\hat{\tau}_1 - \tau_1$	$\hat{\tau}_2$	$\hat{\tau}_2 - \tau_2$	$\hat{\tau}_3$	$\hat{\tau}_3 - \tau_3$
$\Delta f = 0.1$ MHz, (501,150)	137.6	12.6	162.1	12.1	*	*
$\Delta f = 0.2$ MHz, (501,75)	140.7	25.7	164.8	4.8		*
$\Delta f = 0.4$ MHz, (126,48)	108.7	−6.3	150.6	0.6	*	*
$\Delta f = 1$ MHz, (51,15)	137.7	17.9	162.1	2.9	*	*

From Tab.3 we can see that the increment of M can't produce better accuracy, and even worse in sme conditions. So we adopted the second solution, that is, to increase to 20 MHz. The experiment results were shown in Tab.4.

Tab.4 Estimation results with $BW = 50$ MHz, $m\Delta f = 20$ MHz

$BW = 50$ MHz, $m\Delta f = 20$ MHz, $\Delta f, (L, m)$	$\tau_1 = 115$ ns		$\tau_2 = 150$ ns		$\tau_3 = 175$ ns	
	$\hat{\tau}_1$	$\hat{\tau}_1 - \tau_1$	$\hat{\tau}_2$	$\hat{\tau}_2 - \tau_2$	$\hat{\tau}_3$	$\hat{\tau}_3 - \tau_3$
$\Delta f = 0.1$ MHz, (501,200)	108.1	−7.1	150.7	0.7	182.9	7.9
$\Delta f = 0.2$ MHz, (256,100)	108.1	−7.1	150.7	0.7	182.7	7.7
$\Delta f = 0.4$ MHz, (126,50)	108.0	−7.1	150.7	0.7	182.5	7.5
$\Delta f = 1$ MHz, (51,20)	107.9	0	150.8	0.8	181.8	6.8

From Tab.4 we can see that with $m\Delta f = 20$ MHz, the algorithm in this paper can resolve the three paths, but the error is big. Based on above, we can conclude that $m\Delta f = 20$ MHz is the turning point of the cross-HOC based algorithm, and if $m\Delta f < 20$ MHz, the information included in the cumulant matrix is not enough to resolve the paths, even with large Ms. At the same time, in the $m\Delta f = 20$ MHz condition, the value of m is neither important and the estimation result is almost the same with different m values, and the only requirement here is $m > L_p$. In high sample rate conditions, in order to reach the requirement of $m\Delta f$, large M, m have to be adopted, so the computation consumption is large. Consequently, for computation consumption purpose, we should choose large sample intervals to reduce the dimension of the cumulant matrix, which is the same to the two order auto-correlation matrix. Furthermore, we can see that the dimension and bandwidth requirements of the cumulant matrix estimation is much higher, and the algorithm in this paper should be applied in conditions with large bandwidth.

Finally, we expanded $m\Delta f$ to 30 MHz, and made simulations in different parameter choices. The experiment results were shown in Tab.5. From Tab.5 we can see that in the experiments with *BWs* lower than 200 MHz, the error is still big, and as *BW* reaching to 300 MHz, the estimation accuracy seems to be stable. In brief, the Frequency Domain TOA estimation Algorithm Based On Cross-HOC has the ability to suppress mixture noises, but this ability should be supported by large bandwidth.

Tab. 5 Estimation results with $BW = 70 \sim 1000$MHz, $m\Delta f = 30$ MHz

$BW = 70,140,200,300,400,1\,000$ MHz,	$\tau_1 = 115$ ns		$\tau_2 = 150$ ns		$\tau_3 = 175$ ns	
$m\Delta f = 30$ MHz, $\Delta f,(L,m)$	$\hat{\tau}_1$	$\hat{\tau}_1 - \tau_1$	$\hat{\tau}_2$	$\hat{\tau}_2 - \tau_2$	$\hat{\tau}_3$	$\hat{\tau}_3 - \tau_3$
$BW = 70$, $\Delta f = 1$ MHz, (71,30)	117.6	2.6	147.9	−2.1	181.1	6.1
$BW = 140$, $\Delta f = 1$ MHz, (141,30)	109.7	−5.3	150.8	0.8	175.30	0.3
$BW = 200$, $\Delta f = 1$ MHz, (201,30)	113.2	−1.8	149.6	−0.4	178.0	2
$BW = 200$, $\Delta f = 1$ MHz, (301,30)	115.0	0	149.9	−0.1	176.3	1.3
$BW = 200$, $\Delta f = 1$ MHz, (401,30)	114.1	−0.9	149.8	−0.2	176.8	1.8
$BW = 200$, $\Delta f = 1$ MHz, (1001,30)	114.6	−0.4	149.9	−0.1	175.8	0.8

5 Conclusion

A requency domain TOA estimation algorithm based on Cross-HOC was developed in this paper. And simulation experiments were carried out to study the bandwidth requirements of the algorithm. According to the simulation experiments above, we came to such conclusions as:

1) $m\Delta f$ is the key factor to the forth-cumulant-matrix estimation, and to include the basic characteristics of the signal $m\Delta f$ have to satisfy a certain bandwidth.

2) In the condition of a fixed $m\Delta f$, small sample intervals means a large dimension of the cumulant matrix, and a large computation consumption.

3) The bandwidth requirement to realize accurate cumulant matrix estimation is much higher than that of the auto-correlation matrix, and accurate TOA estimation can't be realized in small bandwidth conditions.

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