Research and Simulation on Weak Signal Detection Based on Duffing Oscillator and Damping Ratio Perturbation

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Astract – **The chaotic system is sensitive to the initial value, and this property can be applied in the weak signal detection. There are periodic, critical and chaotic states in a chaotic system. When the system is in the critical state, a small perturbation of system parameter may lead to a qualitative change of the system's state. This paper introduces a new method to detect weak signals by the way of disturbing the damping ratio. The authors choose the duffing equation, using MATLAB to carry on the simulation, to study the changes of the system when the signal to be measured is added to the damping ratio. By means of observing the phase locus chart and time domain chart, the weak signal will be detected.**

Key words – *chaotic system*; *duffing oscillator*; *damping ratio*; *signal detection*

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1 Introduction

Chaos refers to a phenomenon that a process which is very similar to the random process, appears in a deterministic system, which is called intrinsic random phenomenon. The characteristics of chaos are: highly sensitive to initial value, intrinsic random and unpredictability characteristics. In the recent years, chaos and control of chaos have been studied by more and more people in the researches of nonlinear control field[1].

Duffing equation is a chaotic system that has been proved. The nonlinear dynamic system described by duffing equation has complex states, including the periodic oscillation, bifurcation and chaos. Therefore the duffing equation has become one of the important research models of chaos[2]. The steps of detecting weak signal by duffing system are as follows. First, identify the critical value of the system, which is called threshold. Then adjust the system parameter to the vicinity of threshold and make the system go into the critical state, in which state the system is highly sensitive to the weak signal. Add the signal to the system and the phase diagram of the system will greatly changed. In this way the weak signal will be detected.

2 The principle of weak signal detection using duffing system

The duffing equation is a second order differential equation which could produce periodic oscillations and chaotic movements by means of external excitation. Its general form is

$$\ddot{x}(t) + M\dot{x}(t) - x(t) + x^3(t) = F\cos(\omega t) + n(t), \quad (1)$$

where $F\cos(\omega t)$ is the forced periodic term in the equation, M is the damping ratio, $n(t)$ denotes the noise, and the term $-x(t) + x^3(t)$ is the nonlinear recovery force term of the equation[3].

Rewriting the equation as

$$\ddot{x}(t) = -M\dot{x}(t) + x(t) - x^3(t) + F\cos(\omega t) + n(t). \quad (2)$$

According to Eq. (2), we can construct the simulation model based on the duffing system by using MATLAB software. There are many different functional modules in SIMULINK. We can simulate the duffing system by choosing different modules[4], as shown in Fig. 1[5].

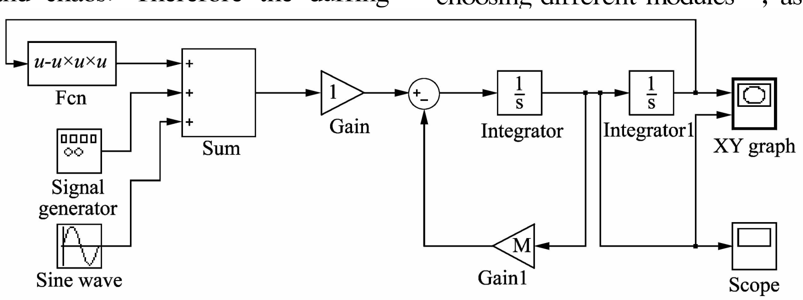

Fig. 1 Simulation model of duffing system

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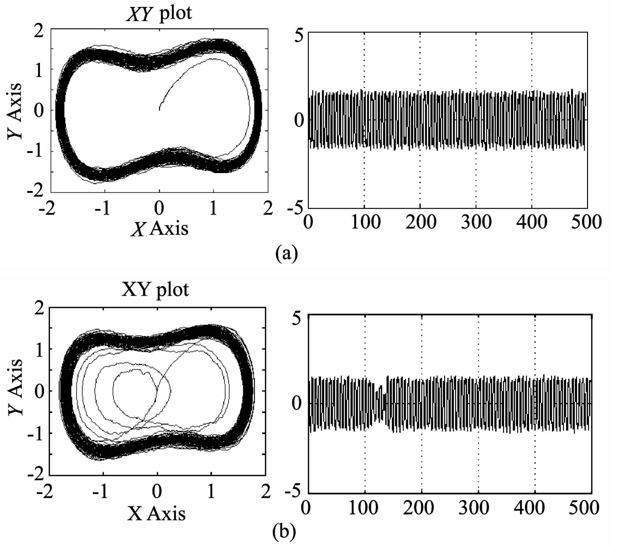
When $M = 0.5$, the phase point is performing a periodic oscillation motion around the focus points $(-1, 0)$ and $(1, 0)$. The system is in a steady state of periodic motion. When $M = 0.6$, the system is between almost period and chaos, that is in the critical state. It is worth noticing that if the simulation time is not long enough, the critical state might be ignored and mistaken for a periodic state. When $M = 0.7$, the chaotic motion of the system occurs. The phase locus charts and time domain charts of the system model are shown in Fig. 2. The horizontal axis of time domain charts in Fig. 2 refers to the simulation process time in seconds, and the vertical axis refers to the amplitude of the output signal.

According to the simulation, we could find that disturbing the damping ratio will lead to a great change in the phase locus chart of the system. In this way we can achieve the purpose of weak signal detection. Ref. [6] points out that when the system is in the critical state, the parameters M and F meet a linear relationship which is $F/M = 1.667$. In the above simulation, $F/M = 1/0.6 \approx 1.667$, which is coincident with the linear relationship.



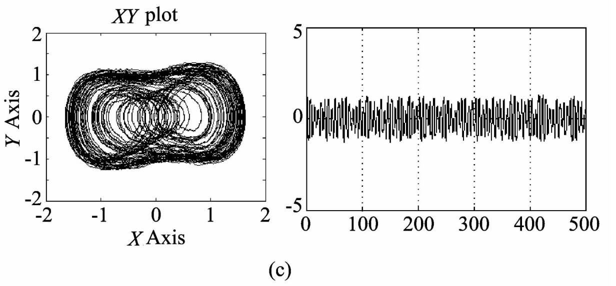


Fig. 2 Phase locus charts and time domain charts of the duffing system: (a) the periodic state; (b) the critical state; (c) the chaotic state

3 The research and simulation on weak signal detection based on damping ratio perturbation

Now disturb the damping ratio of duffing system by using the weak signal δ to be measured, and use the disturbance manner of addition and multiplication[7]. Then the mathematical model of duffing system will become into the following form:

$$\ddot{x}(t) + (M + M * \delta)\dot{x}(t) - x(t) + x^3(t) = F\cos(\omega t) + n(t). \quad (3)$$

When the signal δ has been added into the system, the general damping ratio of duffing system $(M + M * \delta)$ will change with the changes of signal δ. The simulation model of the system is shown in Fig. 3.

If $x' = y$, $M + M * \delta = \varepsilon M'$, $F = \varepsilon F'$, $n = \varepsilon n'$, $0 < \varepsilon \leqslant 1$[8], then the equivalent expression is

$$\begin{cases} x' = y \\ y' = x - x^3 + \varepsilon F + \varepsilon n' - \varepsilon M' y \end{cases} \quad (4)$$

When $\varepsilon = 0$, Eq. (4) is Hamilton system, and the hamilton amount is

$$H(x, y) = \frac{1}{2}y^2 - \frac{1}{2}x^2 + \frac{1}{4}x^4 = h. \quad (5)$$

It has three fixed points, $(0, 0)$ is center, $(-1, 0)$ and $(1, 0)$ are two saddle points.

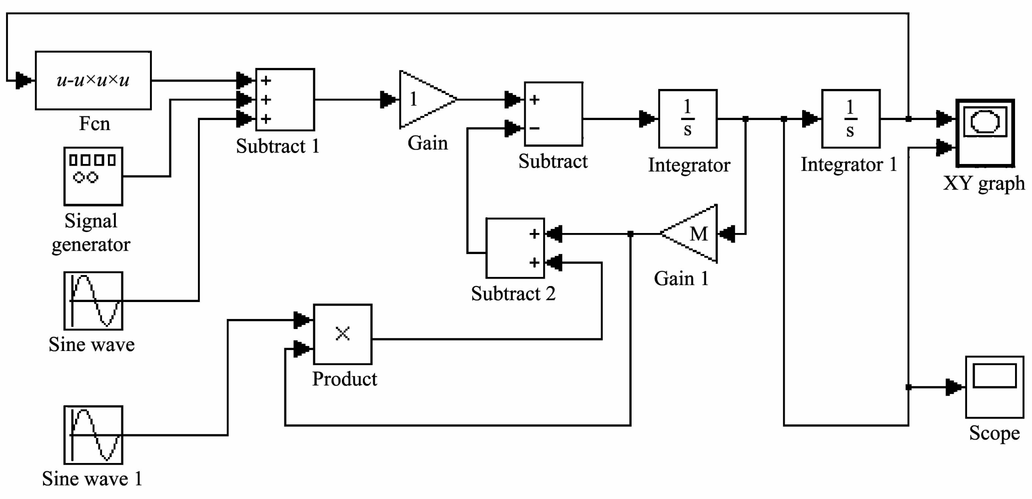


Fig. 3 Simulation model of duffing system based on dampling ratio perturbation

The phase locus charts and time domain charts are shown in Fig. 4. From the shape we can see that the trajectory of the phase space starts from (0,0) and goes to (1,0), then goes back to (0,0). After that it moves from (0,0) to (−1,0), then goes back to (0,0) again. And it repeats to continue. Since the trajectory goes around (1,0) and (−1,0) in a different manner each time, the two points are unstable. When the trajectory is near (0,0), a little change of initial position will lead to a great change of the trajectory in Fig. 4.

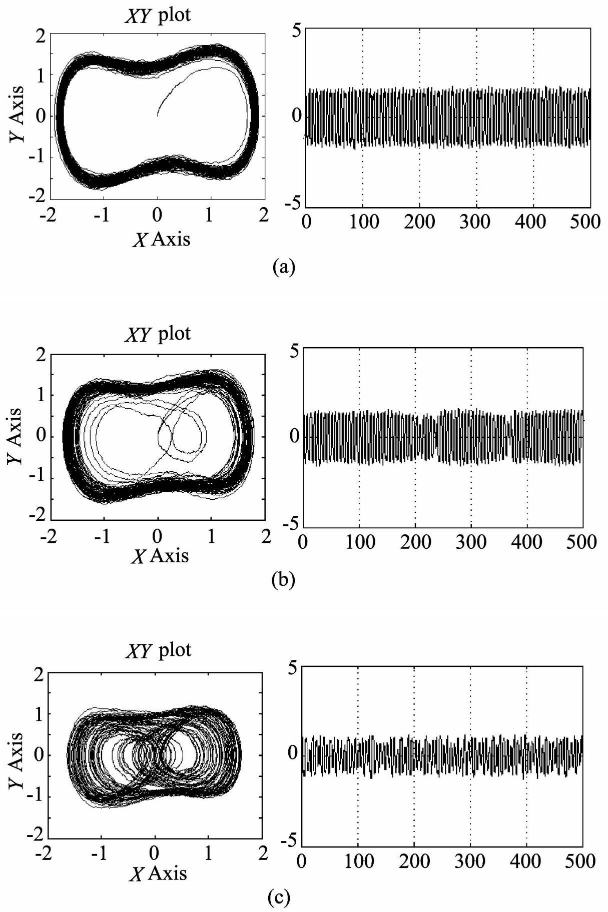


Fig. 4 The phase locus charts and time domain charts of the duffing system based on damping ratio perturbation: (a) The periodic state; (b) The critical state; (c) The chaotic state

Although the overall look of the phase trajectory remains unchanged, the ways in which the trajectory goes around (1,0) and (1,0) are obviously random and unpredictable. And this is the sensitive dependence on initial conditions of the chaotic system[9]. The horizontal axis of time domain charts in Fig. 4 refers to the simulation process time in seconds, and the vertical axis refers to the amplitude of the output signal. By means of observing the time domain charts, we found that the signal amplitude in periodic state is two times as large as the signal amplitude in chaotic state. Therefore we can achieve the purpose of detecting the weak signal by observing the changes of signal amplitude.

4 Conclusion

Though comparing the above simulation results, we come to the conclusions that:

For a nonlinear dynamic system, such as the duffing system, we can control the states of a chaotic system by means of disturbing the system parameters. The traditional duffing oscillator used to detecting the weak signal by disturbing the forced periodic term, that makes the frequency of the weak signal to be measured is limited. The damping ratio perturbation is a method to disturb the characteristic of the system itself, which is not limited by the signal's frequency.

Detecting weak signal by using duffing system based on damping ratio perturbation is a new idea for chaos detection, which needs further optimization in order to improve the precision.

References

[1] Da-shen Xu, Guo-dong Li, Hong-yan Zang, 2004. Briefness description on chaos control. *Journal of Qinghai Normal University (Natural Science)*, (4): 25-30.

[2] Chun-yan Nie, 2006. Research on the Weak Signal Detection Method Based on the Chaos Theory and Specific Chaotic System. Jilin University, Jilin.

[3] Jian Zhang, Zhu-ping Wang, Ning Lv, Xiao-yang Yu, De-yun Chen, 2005. Development of detection technology based on chaos theory. *Chinese Journal of Electron Devices*, 28: 307-311.

[4] Jing Zhang, Fang-fang Han, 2007. Simulation and analysis of duffing-equation. *Science Mosaic*, (9): 29-31.

[5] Yue Li, Bao-jun Yang, Yao-wu Shi, et al, 2001. The application of the chaotic oscillator to the weak sine signal detection in strong noise based. *Acta Scientiarum Naturaliaum Universitatis Jilin Ensis*, (1): 75-77.

[6] Guan-yu Wang, 1998. Research and Practice on the Theory of Chaos Vibrator Feeble Signal Measure. Zhejiang University, Hangzhou.

[7] Hua Zhao, 2005. Study on Weak Square Wave Detection Method by Duffing Oscillator. North China Electric Power University, Baoding.

[8] Ya-jun Li, Yue Li, Jin Lu, Bao-lin Lv, 2004. Determination of chaotic threshold for chaos-based weak signal detection system. *Journal of Jilin University (Information Science Edition)*, 22(2): 106-110.

[9] Run-sheng Huang, 2005. Chaos and Its Application. Wuhan University Press, Wuhan, p. 146-157.