Implementation of Adaptive Wavelet Thresholding Denoising Algorithm Based on DSP

Xue-feng ZHANG(张雪峰), Chun-xia KANG(康春霞), Feng PEI(裴　峰), Zhi-jie ZHANG(张志杰)
(*Key Laboratory of Instrumentation Science & Dynamic Measurement* (*Ministry of Education*),
North University of China, *Taiyuan* 030051, *China*)

Abstract – By utilizing the capability of high-speed computing, powerful real-time processing of TMS320F2812 DSP, wavelet thresholding denoising algorithm is realized based on Digital Signal Processors. Based on the multi-resolution analysis of wavelet transformation, this paper proposes a new thresholding function, to some extent, to overcome the shortcomings of discontinuity in hard-thresholding function and bias in soft-thresholding function. The threshold value can be abtained adaptively according to the characteristics of wavelet coefficients of each layer by adopting adaptive threshold algorithm and then the noise is removed. The simulation results show that the improved thresholding function and the adaptive threshold algorithm have a good effect on denoising and meet the criteria of smoothness and similarity between the original signal and denoising signal.

Key words – ***Mallat algorithm*; *wavelet denoising*; *thresholding function*; *adaptive threshold*; *Digital Signal Processors***

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1 Introduction

In the process of signal acquisition and transmission, noise is inevitably brought in, so it becomes very important to study how to extract the original signal from the noised signal. Wavelet analysis is a new signal processing method developed in recent decades which has a good time-frequency characteristic[1]. The wavelet denoising methods have been applied more and more widely.

The threshold denoising method is used most widely in all wavelet denoising methods, since it can get the approximate optimal estimation of the original signal[2]. The soft-thresholding method and the hard-thresholding method which are used widely at present can obtain a good result, but either of the methods has its own defects which affect the denoising effect. And, another factor affecting the denoising effect is whether it is possible to select the threshold value adaptively according to the wavelet coefficients of each layer after wavelet decomposition[3]. In this paper, an improved thresholding function combined with adaptive threshold algorithm is proposed for denoising. Simulation results based on the DSP platform show the effectiveness of the proposed improved thresholding function and the adaptive threshold algorithm.

2 The wavelet multi-scale decomposition and reconstruction

The well-known Mallat algorithm was proposed while Mallat was constructing on orthogonal wavelet basis. The Mallat algorithm is the fast algorithm of multi-resolution analysis. What Mallat algorithm is to wavelet analysis is what Fast Fourier Transform is to the classical Fourier Transform[4].

The Mallat wavelet decomposition algorithm formula[5] is as follows

$$C_{j+1,k} = \sum_m h(m-2k)C_{j,m}, \tag{1}$$

$$D_{j+1,k} = \sum_m g(m-2k)C_{j,m}, \tag{2}$$

where h and g are the filter coefficients, $C_{j,k}$, $D_{j,k}$ are the smooth coefficient and detail coefficient respectively on the $(j+1)$ th layer. Because the length of the filter is limited, taking the ease of C language programming into account, above the formulas can be converted into the following expressions

$$C_{j+1}[k] = \sum_n h(n)C_j[n+2k], \tag{3}$$

$$D_{j+1}[k] = \sum_n g(n)C_j[n+2k]. \tag{4}$$

The Mallat wavelet reconstruction algorithm formula is

$$C_{j-1,k} = \sum_n [h_{k-2n}C_{j,n} + g_{k-2n}D_{j,n}]. \tag{5}$$

The formula (5) is converted into the expression that is easy to calculate on C language

$$C_{j-1}(k) = \sum_n [h[n]C_j[n+2k] + g[n]D_j[n+2k]]. \tag{6}$$

※ **Received:** 2011-03-05
Corresponding author: Xue-feng ZHANG(zhangxuefeng213@126.com)

3 Wavelet thresholding denoising

Because of the capability of energy concentration, the wavelet transform can make the energy of a signal concentrate on the minority coefficients in the wavelet transform domain. In comparison, these coefficients' values are inevitably greater than those of noise whose energy spreads over the majority coefficients in the wavelet transform domain[6]. That is to say, some methods like cutting, thresholding the wavelet coefficients etc. can be applied to denoising. In general, the denoising process of one dimensional signal can be divided into three steps as follows[7]:

1) Wavelet decomposition. A wavelet is selected and the number N, indicating that how many layers that the noised signal wanted to be decomposed into, is determined. Then, the noised signal is decomposed into N layers based on the wavelet transform.

2) Threshold quantization of wavelet coefficients for high-frequency coefficients of each layer. The wavelet coefficients of every layer are thresholding respectively after a suitable threshold value is selected. And then the new wavelet coefficients of each layer are obtained.

3) Wavelet reconstruction. The wavelet reconstruction is done based on the low frequency coefficients of the Nth layer decomposed by the Mallat wavelet decomposition algorithm and the wavelet coefficients by threshold quantization of each layer.

The key of the above three steps is to select a wavelet basis, determine the decomposition layers, construct a thresholding function and select a suitable threshold value.

3.1 The selection of thresholding function

Among the wavelet thresholding denoising algorithms, the nonlinear processing of the wavelet coefficients is implemented by the thresholding function. Different thresholding functions reflect the different treatment strategies for the wavelet coefficients, and affect the denoising results to a large extent. Currently, the commonly used thresholding functions are the hard-thresholding function and the soft-thresholding function.

The hard-thresholding function retains the wavelet coefficients greater than the threshold value, while the values of the other coefficients are zeros. The soft-thresholding function shrinks the wavelet coefficients by t (threshold value) for those wavelet coefficients whose values are greater than the threshold values, while the values of the other coefficients are zeros[8]. Although the above two methods are widely used in practical applications and have achieved good results, some shortcomings inherently exist in both methods. For example, for the hard-thresholding method, the wavelet coefficients $\hat{W}_{j,k}$ disposed of each layer are not continuous at the threshold value t , and as a result, the reconstructed signal based on $\hat{W}_{j,k}$ may produce turbulence; Although the $\hat{W}_{j,k}$ overall has good continuity estimated by the soft-thresholding method, there is always a constant bias between $\hat{W}_{j,k}$ and $W_{j,k}$ when $|\hat{W}_{j,k}| > t$, which directly affects the degree of approximation between the reconstructed signal and the real signal[9]. In view of these shortcomings, this paper proposes a new thresholding function as follows

$$\hat{W}_{j,k} = \begin{cases} \operatorname{sgn}(W_{j,k})((W_{j,k})^2 - t^2)^{1/2}, & |W_{j,k}| > t; \\ 0, & |W_{j,k}| \leqslant t. \end{cases} \tag{7}$$

where $\hat{W}_{j,k}$ is the wavelet coefficients disposed of by wavelet thresholding, $W_{j,k}$ is the wavelet coefficients of each layer after wavelet decomposition, and t is the threshold value.

Although this improved thresholding function curve is within a range between the soft-thresholding function and hard-thresholding function, the curve of this function approaches to the curve of the hard-thresholding function as $W_{j,k}$ increases. As a result, the constant deviation between $W_{j,k}$ and $\hat{W}_{j,k}$ is narrowed. At the same time, this improved function overcomes the discontinuity of the hard-thresholding function at the threshold value t, and avoids the generation of pseudo-Gibbs phenomena.

3.2 Adaptive threshold algorithm

The adaptive threshold algorithm is an algorithm that takes the wavelet transform coefficients corresponding to the minimum risk value as the threshold value[10]. According to the Parserval's theorem[11], the square of the wavelet coefficients by wavelet decomposition has the dimension of energy. Therefore, the wavelet coefficients are sorted after the decomposition, and the risk value is found corresponding to a given threshold value. Then the likelihood estimation is obtained corresponding to the wavelet coefficients. After the execution of the minimizing non-likelihood, the desired threshold value is obtained, which is a soft-thresholding estimator[12]. The specific algorithm procedure is as follows:

1) The amplitudes of the wavelet transform coefficients of each layer after decomposition are sorted in an ascend order, and then a vector is obtained

$$\boldsymbol{p} = [p_1, p_2, \cdots, p_N],$$

where $p_1 \leqslant p_2 \leqslant \cdots \leqslant p_N$, N is the number of the wavelet coefficients.

2) The calculation of the risk vector

$$\boldsymbol{R} = [r_1, r_2, \cdots, r_N], \tag{8}$$

$$r_i = \frac{N - 2i + (N - i)p_i + \sum_{k=1}^{i} p_k}{N}. \tag{9}$$

The minimum element r_i of vector $\boldsymbol{R}$ is taken as the risk value, and then the p_i corresponding to r_i is figured out.

3) The calculation of the threshold value

$$\delta = \sigma(p_i)^{1/2}, \tag{10}$$

where σ is the variance of noise, it can be defined by

$$\sigma = \frac{1}{0.674\,5N}\sum_{i=1}^{N} |d_i^k|, \tag{11}$$

where d_i^k is the ith wavelet coefficient on the kth layer; N is the number of wavelet coefficients of this layer.

4 The implementation of algorithm based on DSP and simulation results

The self-designed TMS320F2812DSP storage test system is used as the experimental platform. The DSP program flowchart is shown in Fig. 1. The adaptive wavelet thresholding denoising algorithm program is simulated in Code Studio CCS2.0. The wavelet thresholding denoising algorithm is divided into three main parts: wavelet decomposition, threshold quantization of wavelet coefficients and wavelet reconstruction. In the wavelet decomposition, the noised signal is decomposed into many layers, the wavelet coefficients and scale coefficients of each layer are obtained; In threshold quantization of the wavelet coefficients, the wavelet coefficients of each layer are dealt with adaptively by using the adaptive threshold algorithm and the improved thresholding function, and then the new wavelet coefficients of each layer are obtained; In the wavelet reconstruction, the denoising signal is extracted by reconstructing the scale coefficients and the new wavelet coefficients of each layer.

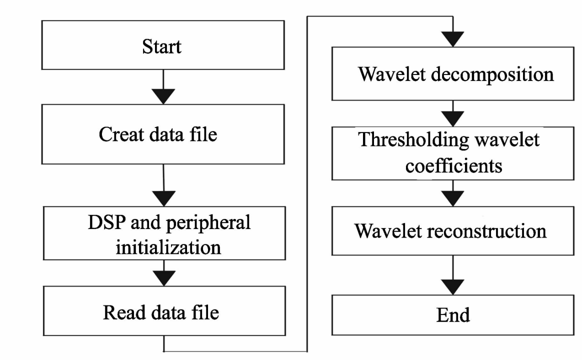

Fig. 1 Flowchart of wavelet thresholding denosing

In order to verify the performance of the improved threshold function and the proposed adaptive threshold algorithm, simulation experiments are conducted. The length of the data file to be processed is 128 bytes in the simulation. The original signal (no noise) is shown in Fig. 2, the abscissa represents the length of data file whose length is 128 bytes in this figure, and the ordinate respents the amplitude of signal. After adding noise to the original signal, we get the noised signal whose SNR is 20 dB, as shown in Fig. 3.

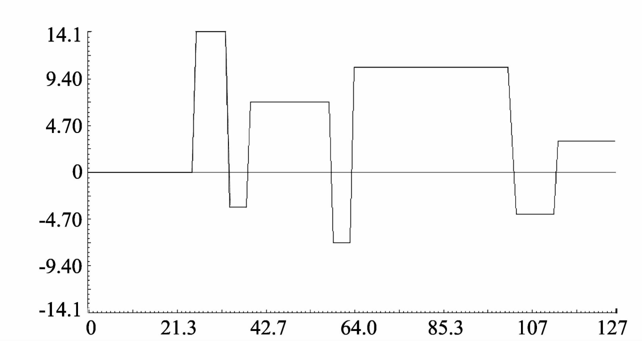

Fig. 2 Original signal

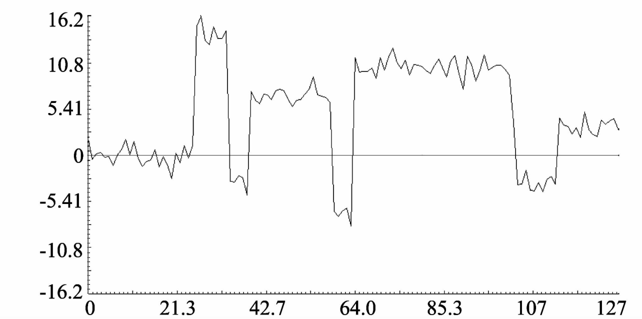

Fig. 3 Noised signal

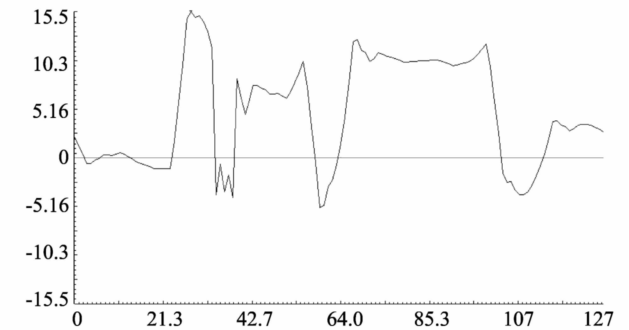

Fig. 4 Hard-thresholding denoising result

The noised signal is decomposed into 3 layers based on db3 wavelet basis. The wavelet coefficients of each layer are handled by the hard-thresholding function, soft-thresholding function and the new thresholding function respectively. After a suitable threshold is selected by the adaptive threshold algorithm, the wavelet reconstruction is carried out. The reconstructed signal waveform is shown in Fig. 4,

Fig. 5 and Fig. 6 respectively. Among these figures, the signal to noise ratio of the reconstructed signal is 20.5 dB, 20.7 dB and 21.4 dB respectively. The improved thresholding gunction improves the SNR obviously. It is clear that the improved thresholding function and the adaptive threshold algorithm have good effects on denoising and! the proposed method is feasible.

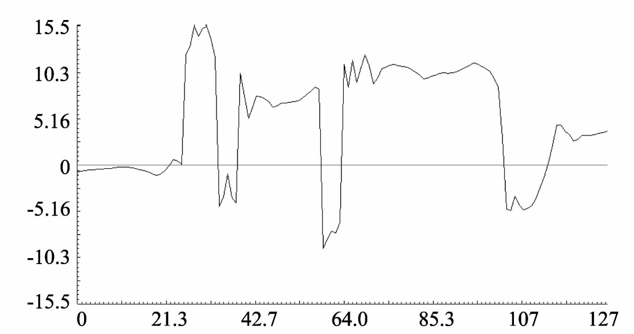

Fig. 5 Soft-thresholding denoising result

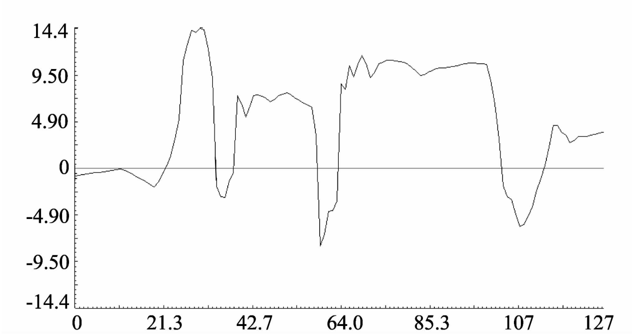

Fig. 6 Improved thresholding denoising result

5 Conclusions

This paper describes the basic idea of wavelet thresholding denoising and Mallat's decomposition and reconstruction algorithm. For the deficiency of the hard-thresholding and soft-thresholding, new thresholding function is proposed, which, to some extent, overcomes the shortcomings of discontinuity in hard-thresholding and bias in soft-thresholding. The threshold value can be obtained adaptively according to the characteristics of wavelet coefficients of each layer and then the noise is removed. The resultant simulation figures and the SNR values from calculation show that the improved thresholding function and adaptive threshold algorithm have good effects on denoising and meet the criteria of smoothness and similarity between the original signal and the denoised signal, reflecting the superiority of this algorithm.

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