Balance control of a 12-DOF mobile manipulator based on two-wheel inverted pendulum robot

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Abstract: Humanoid mobile manipulator which is based on two-wheel inverted pendulum robot has been studied. Balance control is a key problem for this kind of centroid-variable robot. Due to the principle of two wheel inverted pendulum, a timely angle compensation is necessary to make the system keep balance when the centroid changes. In this paper, a method based on coordinate transformation is introduced to get the compensatory angle and a 12-DOF mobile manipulator is also used to check the method. Simulation and experimental results show the effectiveness of the method.

Key words: angle compensation; inverted pendulum; variable centroid; humanoid

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Manipulators have been widely used in factory nowadays. Most of them are installed on a fixed place. Mobile manipulators have attracted more attention in recent years because they can work under uncertain surroundings. Most of this kind of robot have been developed based on a stable platform. In Ref. [1], they introduced a mobile manipulator with a LABNATE platform. A cooperation method between several mobile manipulators was introduced in Ref. [2], and these robots were all with a stable platform. Coordinated task execution of a human and a mobile manipulator was presented in Ref. [3]. After Segway appeared in 2002, more attention was attracted by this kind of platform[4]. Mobile manipulator developed in Keio University was a one-arm manipulator which is based on Segway platform[5]. EMIEW was a two-wheel humanoid service robot with a relatively stable centroid[6]. When a manipulator based on two-wheel inverted pendulum has an obviously time-variant centroid, balance control comes to be a big problem. To do some research on this problem, a 12-DOF mobile manipulator is developed in this paper.

1 Mathematical model of centroid

Establishing the model of centroid is helpful to get the compensatory angle of the body. By given position of centroid, statement control of the robot is available.

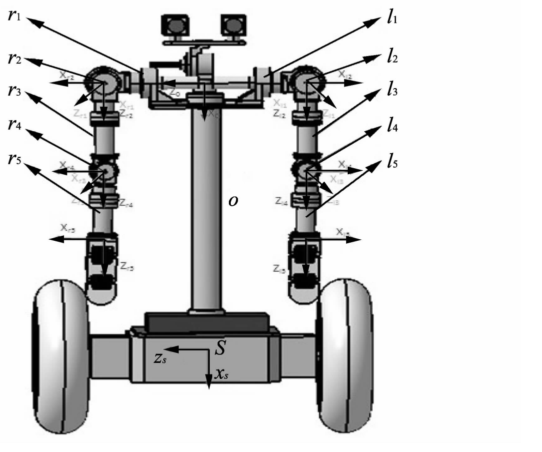

Fig. 1 Model of the robot with 12 coordinate systems

As shown in Fig. 1, the robot introduced in this paper mainly includes two parts: the upper one is a 10-DOF (except camera) dual arm manipulator, and the chassis is a two-wheel inverted pendulum robot. By connecting the manipulator to chassis with a passive joint, the robot gets 12 active joints and 1 passive joint in this robot. Making Cartesian coordinate systems as shown in Fig. 1, then coordinate transformation by D-H method can be got. D-H parameters of the robot are shown in Table 1 and Table 2.

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Table 1 D-H parameters for left arm

L	θ (°)	d(mm)	a(mm)	α(°)
0	θ_0	0	−860	0
l_1	θ_{l1}	−260	0	−90
l_2	θ_{l2}	0	0	90
l_3	θ_{l3}	250	0	−90
l_4	θ_{l4}	0	0	90
l_5	θ_{l5}	200	0	0

Table 2 D-H parameters for right arm

L	θ (°)	d(mm)	a(mm)	α(°)
0	θ_0	0	−860	0
r_1	θ_{r1}	260	0	−90
r_2	θ_{r2}	0	0	−90
r_3	θ_{r3}	250	0	90
r_4	θ_{r4}	0	0	−90
r_5	θ_{r5}	200	0	0

So, transformation matrix for the first line is

$$ {}^{s}\boldsymbol{H}_0 = \begin{bmatrix} c_0 & -s_0 & 0 & -860c_0 \\ s_0 & c_0 & 0 & -860s_0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{1} $$

where c_0 means $\cos\theta_0$; s_0 means $\sin\theta_0$. In this way, ${}^{0}\boldsymbol{H}_{l_1}$, ${}^{l_1}\boldsymbol{H}_{l_2}$, ${}^{l_2}\boldsymbol{H}_{l_3}$, ${}^{l_3}\boldsymbol{H}_{l_4}$, ${}^{l_4}\boldsymbol{H}_{l_5}$, ${}^{0}\boldsymbol{H}_{r_1}$, ${}^{r_1}\boldsymbol{H}_{r_2}$, ${}^{r_2}\boldsymbol{H}_{r_3}$, ${}^{r_3}\boldsymbol{H}_{r_4}$ and ${}^{r_4}\boldsymbol{H}_{r_5}$ can be got.

Transformation matrices from each coordinate system to S coordinate are shown as

$$ {}^{s}\boldsymbol{H}_{l_1} = {}^{2}\boldsymbol{H}_0 \times {}^{0}\boldsymbol{H}_{l_1}, \tag{2} $$

$$ {}^{s}\boldsymbol{H}_{l_2} = {}^{s}\boldsymbol{H}_0 \times {}^{0}\boldsymbol{H}_{l_1} \times {}^{l_1}\boldsymbol{H}_{l_2}, \tag{3} $$

$$ {}^{s}\boldsymbol{H}_{l_3} = {}^{s}\boldsymbol{H}_0 \times {}^{0}\boldsymbol{H}_{l_1} \times {}^{l_1}\boldsymbol{H}_{l_2} \times {}^{l_2}\boldsymbol{H}_{l_3}, \tag{4} $$

$$ {}^{s}\boldsymbol{H}_{l_4} = {}^{s}\boldsymbol{H}_0 \times {}^{0}\boldsymbol{H}_{l_1} \times {}^{l_1}\boldsymbol{H}_{l_2} \times {}^{l_2}\boldsymbol{H}_{l_3} \times {}^{l_3}\boldsymbol{H}_{l_4}, \tag{5} $$

$$ {}^{s}\boldsymbol{H}_{l_5} = {}^{s}\boldsymbol{H}_0 \times {}^{0}\boldsymbol{H}_{l_1} \times {}^{l_1}\boldsymbol{H}_{l_2} \times {}^{l_2}\boldsymbol{H}_{l_3} \times {}^{l_3}\boldsymbol{H}_{l_4} \times {}^{l_4}\boldsymbol{H}_{l_5}, \tag{6} $$

$$ {}^{s}\boldsymbol{H}_{r_1} = {}^{s}\boldsymbol{H}_0 \times {}^{0}\boldsymbol{H}_{r_1}, \tag{7} $$

$$ {}^{s}\boldsymbol{H}_{r_2} = {}^{s}\boldsymbol{H}_0 \times {}^{0}\boldsymbol{H}_{r_1} \times {}^{r_1}\boldsymbol{H}_{r_2}, \tag{8} $$

$$ {}^{s}\boldsymbol{H}_{r_3} = {}^{s}\boldsymbol{H}_0 \times {}^{0}\boldsymbol{H}_{r_1} \times {}^{r_1}\boldsymbol{H}_{r_2} \times {}^{r_2}\boldsymbol{H}_{r_3}, \tag{9} $$

$$ {}^{s}\boldsymbol{H}_{r_4} = {}^{s}\boldsymbol{H}_0 \times {}^{0}\boldsymbol{H}_{r_1} \times {}^{r_1}\boldsymbol{H}_{r_2} \times {}^{r_2}\boldsymbol{H}_{r_3} \times {}^{r_3}\boldsymbol{H}_{r_4}, \tag{10} $$

$$ {}^{s}\boldsymbol{H}_{r_5} = {}^{s}\boldsymbol{H}_0 \times {}^{0}\boldsymbol{H}_{r_1} \times {}^{r_1}\boldsymbol{H}_{r_2} \times {}^{r_2}\boldsymbol{H}_{r_3} \times {}^{r_3}\boldsymbol{H}_{r_4} \times {}^{r_4}\boldsymbol{H}_{r_5}. \tag{11} $$

The position of each centroid in $X_sY_sZ_s$ coordinate system is expressed by

$$ \begin{bmatrix} {}^{s}x_i \\ {}^{s}y_i \\ {}^{s}z_i \\ 1 \end{bmatrix} = {}^{s}\boldsymbol{H}_l \begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix}, \tag{12} $$

where $i \in (0, l1, l2, l3, l4, l5, r1, r2, r3, r4, r5, s)$; $({}^{s}x_i, {}^{s}y_i, {}^{s}z_i)$ means the i-th centroid coordinate expressed in $X_sY_sZ_s$ system. And then the centroid position of the whole body will be

$$ {}^{s}x_c = \frac{\sum {}^{s}x_i * m_i}{\Sigma m_i}, \quad {}^{s}y_c = \frac{\sum {}^{s}y_i * m_i}{\Sigma m_i}, $$
$$ {}^{s}z_c = \frac{\sum {}^{s}z_i * m_i}{\Sigma m_i}, \tag{13} $$

where $({}^{s}x_c, {}^{s}y_c, {}^{s}z_c)$ denotes the coordinate of centroid expressed in $X_sY_sZ_s$ system. So it is possible to get the compensatory angle in X_sOY_s plane, namely,

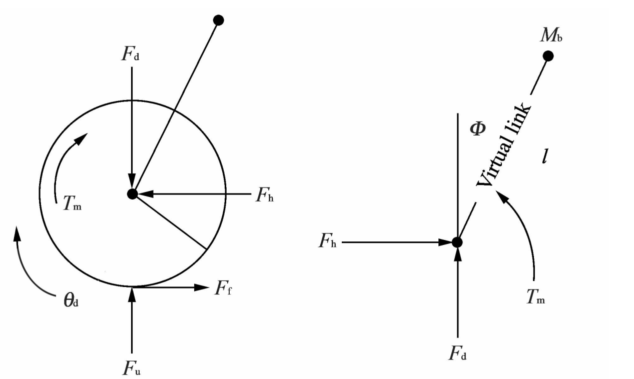
$$ \theta = \arctan\left(\frac{{}^{s}y_c}{{}^{s}x_c}\right). \tag{14} $$

And the length from centroid to zero which belongs to X_sOY_s plane should be

$$ l = \sqrt{{}^{s}x_c^2 + {}^{s}y_c^2}. \tag{15} $$

2 Dynamic model

Dynamic model can help analyze and control the system. A dynamic model of the robot introduced is shown in Fig. 2. In this model, the robot is divided into two parts. In Fig. 2(a), force analysis of wheel is proposed and Fig. 2(b), the main body of the robot is seen as a virtual body which includes the centroid and a virtual link.



(a) Force analysis of wheel (b) Virtual body of robot

Fig. 2 Dynamic model of the robot

In Fig. 2, F_d is the vertical force between wheel and body, F_h is the horizontal force between them, T_m is the torque produced by motors, r is the radius of wheel, θ_w is the rotation angle of wheel, M_b is the mass of robot body, Φ is the tilt angle of centroid − F_u and F_f represent the force given by ground vertically and friction, respectively.

Movement of wheel can be divided into horizontal movement and rotation movement. For horizontal! movement, its kinematic equation is

$$ m\dot{v} = F_f - F_h, \tag{16} $$

where v and m denote velocity and mass of wheel, respectively. For rotation movement, the kinematic equation is wirtten as

$$J\dot{\omega} = T_{\mathrm{m}} - F_{\mathrm{f}} r, \tag{17}$$

where J and ω represent moment of inertia and angle velocity of wheel, respectively.

Taking Eq. (17) into Eq. (16), then Eq. (18) can be got as

$$m\dot{v} = -\frac{J}{r}\dot{\omega} + \frac{T_{\mathrm{m}}}{r} - F_{\mathrm{h}}. \tag{18}$$

Assume that there is no sliding happened between wheel and ground, and then Eq. (18) can be rewritten as

$$mr\dot{\omega} = -\frac{J}{r}\dot{\omega} + \frac{T_{\mathrm{m}}}{r} - F_{\mathrm{h}}, \tag{19}$$

where $v = \omega r$.

Robot body is seen as a centroid and a virtual link. Movement of body also can be divided into two parts as wheel. For vertical direction, kinematic equation is

$$M_{\mathrm{b}}\ddot{l}_{\mathrm{v}} = F_{\mathrm{d}} - M_{\mathrm{b}} g, \tag{20}$$

where l_{v} is vertical position of centroid.

For horizontal direction, kinematic equation is

$$M_{\mathrm{b}}\ddot{l}_{\mathrm{h}} = F_{\mathrm{h}}, \tag{21}$$

where l_{h} is horizontal position of centroid.

From Fig. 2, l_{v} and l_{h} can be derived as

$$l_{\mathrm{v}} = r + l\cos\Phi, \tag{22}$$

$$l_{\mathrm{h}} = p_0 + l\sin\Phi, \tag{23}$$

where p_0 is horizontal position of wheel axis.

Then $\ddot{l}_{\mathrm{v}}$ and $\ddot{l}_{\mathrm{n}}$ can be got as

$$\ddot{l}_{v} = -l(\ddot{\Phi}\sin\Phi + \dot{\Phi}^2\cos\Phi), \tag{24}$$

$$\ddot{l}_{h} = \dot{v} + l(\ddot{\Phi}\cos\Phi - \dot{\Phi}^2\sin\Phi), \tag{25}$$

where $\Phi = \theta + \theta_{\mathrm{tilt}}$ and θ_{tilt} is the tilt angle of platform.

Kinematic equation of rotation of body is

$$I\ddot{\Phi} = F_{\mathrm{d}} l\sin\Phi - F_{\mathrm{h}} l\cos\Phi - T_{\mathrm{m}}, \tag{26}$$

where I is moment inertia to axis produced by robot body.

Then taking Eqs. (24) and (25) into Eqs. (20) and (21), $\boldsymbol{F}_{\mathrm{d}}$ and $\boldsymbol{F}_{\mathrm{h}}$ can be got, taking $\boldsymbol{F}_{\mathrm{d}}$ and $\boldsymbol{F}_{\mathrm{h}}$ into Eq. (26), then

$$\left(\frac{I}{M_{\mathrm{b}} l} + l\right)\ddot{\Phi} + \dot{v}\cos\Phi - g\sin\Phi + \frac{T_{\mathrm{m}}}{M_{\mathrm{b}} l} = 0, \tag{27}$$

where l is just seen as a constant.

Taking $v = \omega r$ into Eq. (27), and then taking T_{m} into Eq. (19), when Φ just changes from $-5°$ to $5°$, linearization is made around 0. Taking $\sin\Phi = \Phi$ and $\cos\Phi = 1$ into consideration, dynamic equation of the robot is

$$\left(m\boldsymbol{r} + \frac{J}{r} + M_{\mathrm{b}} l + M_{\mathrm{b}} r\right)\dot{\omega} = -\left(\frac{1}{r}(I + M_{\mathrm{b}} l^2) + M_{\mathrm{b}} l\right)\ddot{\Phi} + gM_{\mathrm{b}} l\,\frac{1}{r}\Phi. \tag{28}$$

This differential equation between angle velocity and centroid tilt angle can be got. Let $r(t) = \bar{\omega}$ and $c(t) = \Phi$, the transfer function is got as

$$G(s) = \frac{C(s)}{R(s)} = \frac{mr + \frac{J}{r} + M_{\mathrm{b}} l + M_{\mathrm{b}} r}{-\left(\frac{1}{r}(I + M_{\mathrm{b}} l^2) + M_{\mathrm{b}} l\right)s^2 + gM_{\mathrm{b}} l\,\frac{1}{r}}. \tag{29}$$

Eq. (29) can be rewritten as

$$G(s) = \frac{C(s)}{U(s)} = \frac{a}{bs^2 + c}, \tag{30}$$

where

$$a = mr + \frac{J}{r} + M_{\mathrm{b}} l + M_{\mathrm{b}} r, \tag{31}$$

$$b = -\left(\frac{1}{r}(I + M_{\mathrm{b}} l^2) + M_{\mathrm{b}} l\right), \tag{32}$$

$$c = gM_{\mathrm{b}} l\,\frac{1}{r}. \tag{33}$$

3 Simulation of centroid

Simulation about centroid can check the accuracy of equations got above and get influence of each joint to centroid. The simulation results of joint and influence are not included in this paper. The first and fourth joints of each arm have more obvious influence to centroid, these 4 joint will be taken out to finish the experiment. Fig. 3 shows partial simulation results.

Mathematic simulation shows that when the first joint moves forward, the centroid will also move forward, and so do the other joints, respectively. In Fig. 4(a), it shows the relation between θ_{l1}, θ_{r1} and compensatory angle, Fig. 4 (b) shows the relation between θ_{l1}, θ_{l4} and compensatory angle. These results are used to compare with the experiment data.

Simulation results are used to check the relation between position of centroid and angles of each joint. Follow these results, compensatory angle for whole body can be checked.

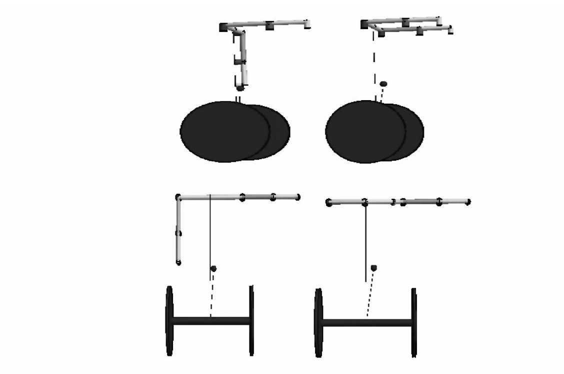
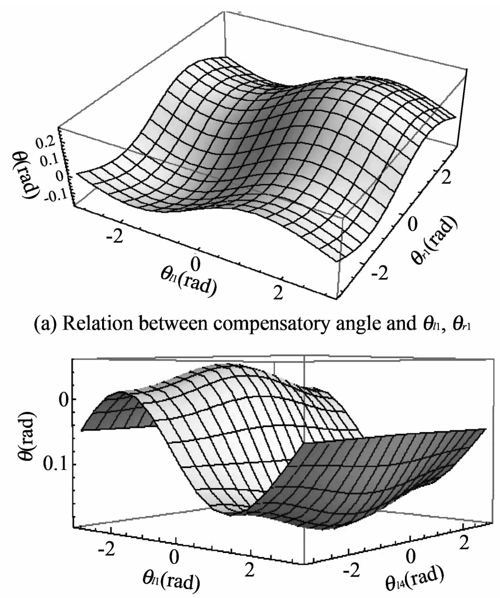


Fig. 3 Simulation results of centroid. The dot is centroid of whole body, first two pictures are about side face, and another two are about front side



(a) Relation between compensatory angle and θ_{l1}, θ_{r1}

(b) Relation between compensatory angle and θ_{l1}, θ_{l4}

Fig. 4 Mathematics simulation results

4 Communication structure, controller and experimental results

4.1 Communication structure of the whole body

In this system, there are ten AVR for manipulator and one ARM for mobile platform. In order to calculate the position of centroid, another singlechip is necessary. So in this system, the 11th AVR is used as a centroid calculator and data collector. Basic method of communication is call and answer. The calculator can get 10 sets of data from each AVR in a second. This will make sure that the calculator can get the position of centroid in time.

As shown in Fig. 5, L_1 to L_5 and R_1 to R_5 represent the controller of each joint and C means the calculator for position of centroid, which is connected to ARM by an UART port. RS232 is used to connect C with other AVR because only UART communication can not make it well.

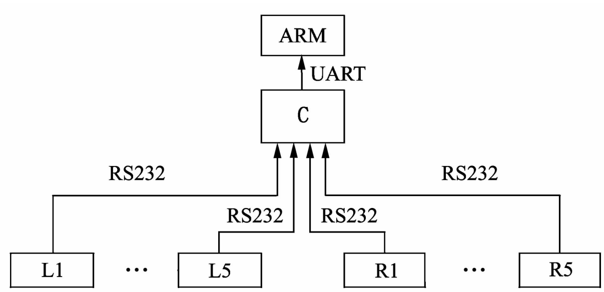


Fig. 5 Communication structure of the system

4.2 Controller illustration

Normally, PID control is widely used in this kind of inverted pendulum robot. But we can not get the position of centroid so exactly, the PD control is used in this paper, as shown in Fig. 6.

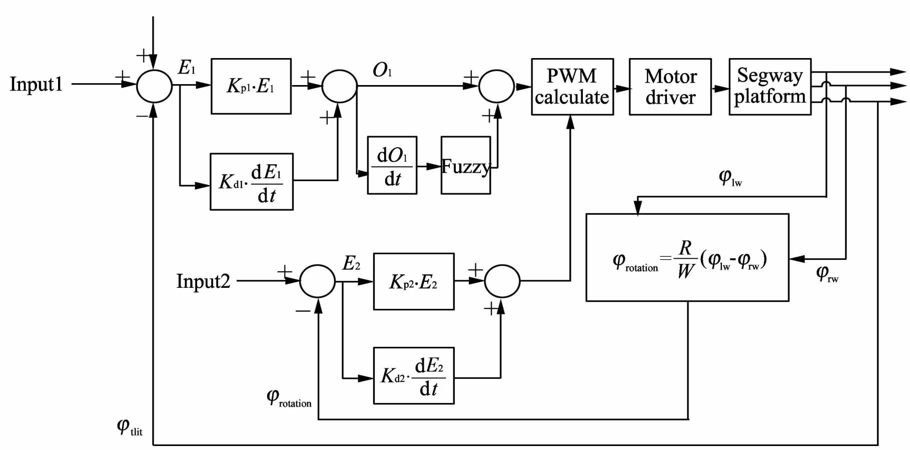


Fig. 6 Controller diagram

In Fig.6 input1 is the desired tilt angle for platform without manipulator. Input2 is the desired rotation angle produced by difference of both sides. To keep the robot balance, input1 and input2 are set to be zero. Rotation angle can be got as

$$\varphi_{\mathrm{rotation}} = \frac{r}{W}(\varphi_{\mathrm{lw}} - \varphi_{\mathrm{rw}}), \tag{34}$$

where φ_{lw} and φ_{rw} mean the rotation of left and right wheel, respectively. r is the radius of one wheel and W is the distance of both wheels.

The first PD controller is used to control tilt angle of platform to keep the robot balance and the second PD controller is used to keep both sides move the same distance.

In Fig.6, a fuzzy controller is used to compensate the output of the first PD controller so that the output will not change sharply.

Fuzzy controller receives the differential of O_1 as its input, where O_1 is output of the first PD controller. Both IF-THEN rule and rule table of fuzzy controller are used to determine the compensation quantity. Fuzzy membership is shown in Fig.7.

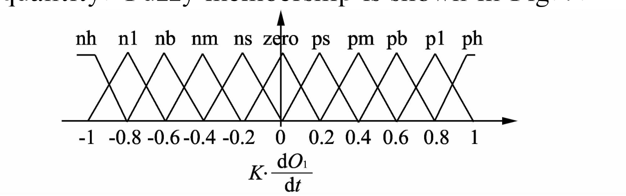


Fig.7 Fuzzy membership

Table 3 is the fuzzy rule, where ph, pb, pm, ps, 0, ns, nm, nb, nl, nh are positive huge, positive large, positive big, positive medium, positive small, zero, negative small, negative medium, negative big, negative large, negative huge.

Table 3 Fuzzy rule

	dO_1/dt										
Input	nh	nl	nb	nm	ns	0	ps	pm	pb	pl	ph
Output	nh	nl	nb	nm	ns	0	ps	pm	pb	pl	ph

Output of fuzzy controller is defined as

$$F_{\mathrm{out}} = \frac{\sum_i b_i \int u_i}{\sum_i \int u_i}, \tag{35}$$

where b_i denotes the center of the membership function for the implied fuzzy set for the i-th rule. And $\int u_i$ denotes the area under the membership function u_i.

4.3 Experimental results

From the simulation results, compensatory angle is easily got. When the manipulator only moves its first two joints, the minimum compensatory angle is

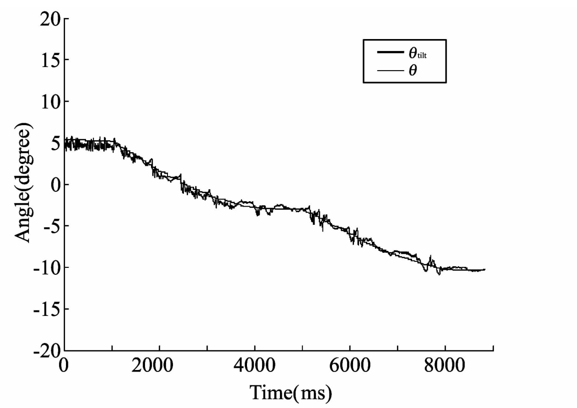
$$-0.18 \times 180/\pi = -10.31°. \tag{36}$$

And the maximum compensatory angle is

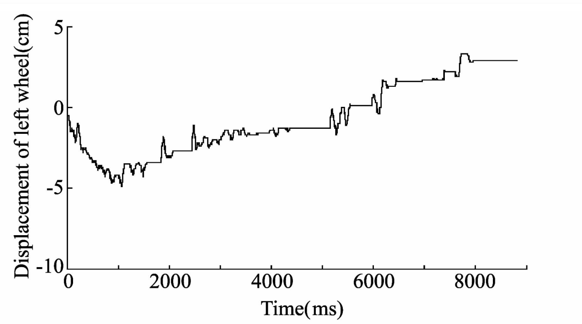
$$0.259 \times 180/\pi = 14.84°. \tag{37}$$

When the manipulator only moves its and fourth joints of one arm, the compensatory angle are from $-3.03°$ to $10.52°$.

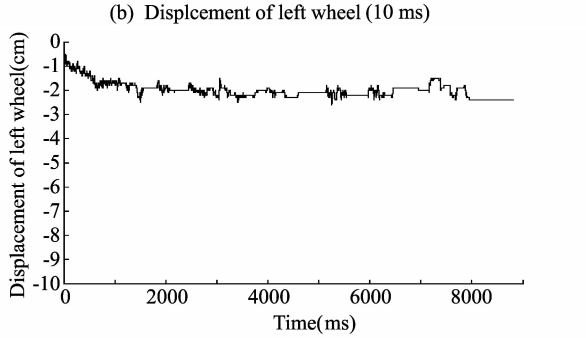
In fact, in this experiment, all the joints only change from 0° to 90°, which is enough to check the method. Fig.8 shows the experiment results about the first joint of both arms. From 10 to 40 s, θ_{l1} transfers from 0 to 90°, and then from 50 to 80 s θ_{r1} transfers from 0 to 90°. Experiment results show that the platform can still keep balance by using compensatory angle. Still, it has the displacement of 1.5 cm and 2.7 cm for left and right side with a body rotation angle about 1.09°.



(a) Compensatory angle θ and tilt angle of platform θ_{tilt} (10 ms)

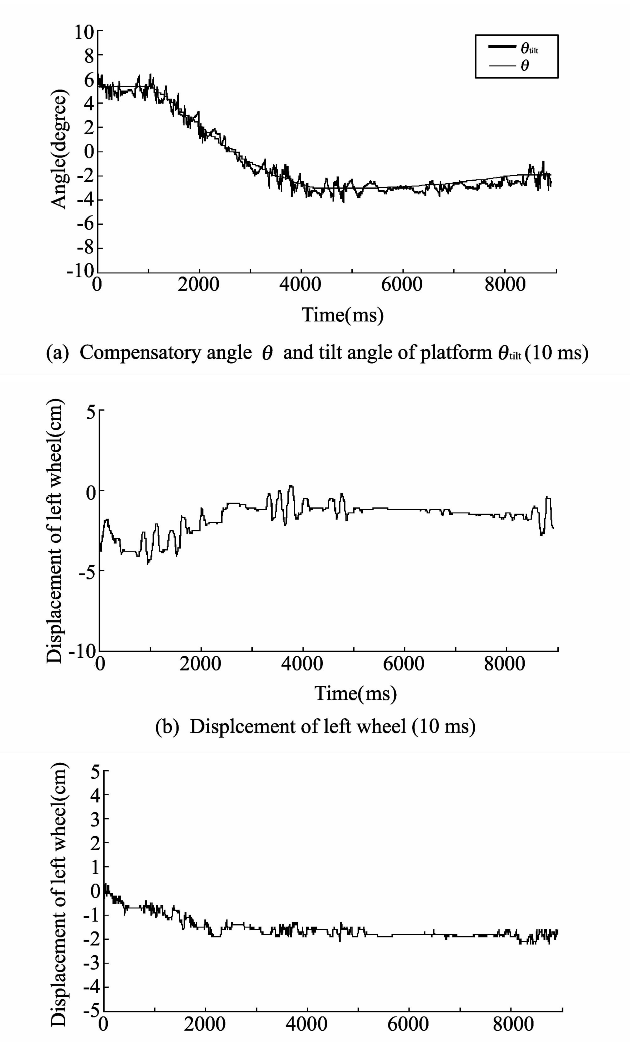


(b) Displcement of left wheel (10 ms)



(c) Difference of both wheel (10 ms)

Fig.8 Experiment about the first joint of both arms



(a) Compensatory angle θ and tilt angle of platform θ_{tilt} (10 ms)

(b) Displcement of left wheel (10 ms)

(c) Difference of both wheel (10 ms)

Fig. 9 Experiment about the first and the fourth joint of left arm

Fig. 9 shows the experiment results about the first and the fourth joint of left arm. From 10 to 40 s, θ_{l1} transfers from 0° to 90°, and then from 50 to 80 s, θ_{l4} transfers from 0° to 90°. Experiment result shows the similar data as shown in Fig. 7.

5 Conclusion

This paper presents a method to control the balance of a mobile manipulator based on two-wheel inverted pendulum robot. By using compensatory angle, the robot can keep balance. As coordinate transformation is widely used in robot engineering, this method is also useful for other similar manipulator which is based on two wheel inverted pendulum robot.

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