Application of Wavelet Packet Energy Spectrum to Extract the Feature of the Pulse Signal

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Abstract **– The wavelet packet is presented as a new kind of multi-scale analysis technique followed by Wavelet analysis. The fundamental and realization arithmetic of the wavelet packet analysis method are described in this paper. A new application approach of the wavelet packed method to extract the feature of the pulse signal from energy distributing angle is expatiated. It is convenient for the microchip to process and judge by using the wavelet packet analysis method to make the pulse signals quantized and analyzed. Kinds of experiments are simulated in the lab, and the experiments prove that it is a convenient and accurate method to extract the feature of the pulse signal based on wavelet packed-energy spectrum analysis.**

Key words **–** ***wavelet packed*****;** ***energy spectrum*****;** ***pulse signal***

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1 Introduction

Traditional Chinese Medicine Pulse Diagnosis is an important means in the disease diagnosis. In its more than 2000 years-history, researchers have accumulated a wealth of experience. Pulse signals in the human body contains a wealth of physiological information, it has been shown from long-term clinical practice and experimental studies that pulse has great practical value and is an important clinical component of the clinical medicine foundation. It is an ongoing subject[1]. From the 50's since the 20th century, Chinese and overseas scholars have paid more attention and committed themselves to the objective of pulse signals, digital research and achieved some results.

The wavelet packet transform (WPT) method[2], which is a generalization of wavelet decomposition, offers a rich range of possibilities for signal analysis. The frequency bands of a hoist-motor signal as collected by the sensor system are wide. The useful information hides within the large amount of data. In general, some frequencies of the signal are amplified and some are depressed by the information. That is to say, these broadband signals contain a large amount of useful information. But the information can not be directly obtained from the data. The WPT is a fine signal analysis method that decomposes the signal into many layers and gives a better resolution in the time-frequency domain. The useful information within the different frequency bands will be expressed by different wavelet coefficients after the decomposition of the signal. The concept of "energy information" is presented to identify new information hidden the data. An energy eigenvector is then used to quickly mine information hiding within the large amount of data.

Wavelet packet energy spectrum has been successfully applied to a variety of fault diagnosis[3-6]. Pathological changes in some parts of the body, in a certain extent, can also be considered as some kind of a "failure", which would cause changes in the energy band of the pulse signal to a certainty. In this paper, we make a try to apply the wavelet packet-energy spectrum method to characteristics analysis of the pulse signal.

2 Theoretical background

2.1 Wavelet packet

The wavelet packet transform can provide a more accurate analysis. Since it has perfect performances in frequency localization, especially to higher-frequency signal, one can focalize himself on any details of a given signal and get good result. So this method has very important value in the real application.

Let the coefficients of wavelets base filters be h_n and g_n, respectively, meanwhile, scale function $\varphi(t)$ is noted $u_0(t)$ and wavelets function $\Psi(t)$ is $u_1(t)$, consequently, the double-scale equation becomes as the following

$$\begin{cases} u_0(t) = \sqrt{2}\sum_{k\in z} h_k u_0(2t-k), \\ u_1(t) = \sqrt{2}\sum_{k\in z} g_k u_1(2t-k). \end{cases} \tag{1}$$

Wavelets package is a function set including scale function $u_0(t)$ and wavelets base function $u_1(t)$; moreover, they have relations each other and its function set $\{u_n(t)\}_{n\in z}$ is defined by the following formula

$$\begin{cases} u_{2n}(t) = \sqrt{2}\sum_{k\in z} h(k) u_n(2t-k), \\ u_{2n+1}(t) = \sqrt{2}\sum_{k\in z} g(k) u_0(2t-k). \end{cases} \tag{2}$$

In the process of analysis using wavelets package, all "frequency window" is divided more minutely with increasing of scale, and data length becomes half after functioning of filter group once. Therefore, a set of data whose original length is 2^N and its sampling frequency is f_s applied on the wavelets package transform at the Lth scale; it will be divided as $n = 2^L$ series along frequency axis. Each of series has width of $f_s/2^L$, and the first frequency of the nth series is $f_n = (n-1)f_s/2^L$.

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2.2 Wavelet packet energy spectrum

When the result of wavelet packet decomposition is expressed by energy method, it can be called as wavelet packet energy spectrum[7].

The original signal $f(x)$ would meet the following equation in $L^2(R)$

$$\int_R |f(x)|^2 \mathrm{d}x \, \| f \|_2^2 = \int_R \mathrm{d}a \int_R \left| \frac{W_f(a,b)}{a} \right|^2 \mathrm{d}b. \quad (3)$$

Wavelet packet energy spectrum can be selected within each band the signal squared, and as a symbol of energy. The energy in subspace W_{2^j} is

$$G = \sum_{k=1}^{2^j} | W(k) |^2. \quad (4)$$

After the wavelet packet decomposition, width of each frequency band would be the same. Thus we can use the wavelet packet decomposition results as the energy spectrum of the input, and according to the ratio between the energy in various frequency bands, a series of histograms can be made, and then normalized. Then the height of each histogram is 1, and the height of each histogram represents the proportion of the band energy to the total energy. Its can be applied to the human pulse wave energy distribution analysis, and would allow visual identification of pathological band characteristics.

3 The analysis about pulse signal based on wavelet packet energy spectrum

Selection of wavelet packet basis functions will affect the accuracy of signal analysis. Here we chose Daubechies 8 wavelets as the basic function, and achieved good results.

The number of wavelet packet decomposition levels directly related to the accuracy of the analysis of the time-frequency of pulse signal. Choice of low number, it would cause the rapid analysis, which is particularly evident for the high-frequency band signal, but at the same time with a low resolution; choosing the high number, then the analysis is slow, but the band can gain a high resolution. Taking into account the relationship between both above and the analysis of the signal in time domain and frequency domain features, we select the three layers decomposition, and eight bands characteristic curves can be gotten.

According to the theory discussed above, we will make a actual pulse signal analysis. And the steps are as follows[8]:

Step 1: First of all, the sampled signals are decomposed with three-layer based on wavelet packet. And the decomposition structure is shown in Fig. 1. Then the characteristics of the eight signals in the third layer are extracted from the low-frequency to high frequency.

As shown in Fig. 1, (i,j) indicates the number j nodes in the number i layer, where, $i=0,1,\cdots,7$. Each node represents a certain degree of signal characteristics. Among them, $(0,0)$ node represents the original signal S; $(1,0)$ represents the low-frequency coefficient X_{10} in first layer of wavelet packet decomposition; $(1,1)$ represents the high-frequency coefficient X_{11} in first layer of wavelet packet decomposition; $(3,0)$ represents the coefficient of first node, and so on.

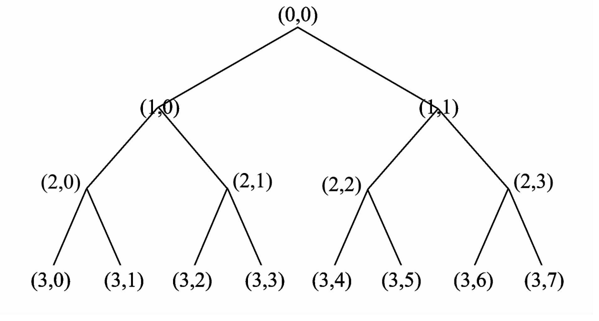

Fig. 1 The three-layer wavelet packet analysis tree

Step 2: The wavelet packet decomposition coefficients are reconstructed to extract the signals in every frequency range. We make S_{30} indicated the reconstruction of X_{30} signals, thus S_{31} indicated the reconstruction of X_{31} signals, and so on. Here, we would only make an analysis about all the nodes on the third layer, and the total signal S can be expressed as

$$S = S_{30} + S_{31} + S_{32} + S_3 + S_{34} + S_{35} + S_{36} + S_{37}.$$

$f_s = 128$, where f_s is the sampling frequency of original pulse sampled signal S. And thus the frequency of S covers 0~64 Hz. But in the process of feature extraction, only those features with the frequency between 0~40 Hz constitute a matter, the others which is with the frequency higher than 40 Hz would be believed to be of no practical help. Hence, we choose eight signals, $S_{3j}(j = 0,1,\cdots,7)$, and the frequency coverage of each signal is shown in Tab.1.

Step 3: The total energy of each band signal would be computed. As the pulse signal is a kind of random signal, we set the $S_{3j}(j = 0,1,\cdots,7)$ corresponding to the energy $E_{3j}(j = 0,1,\cdots,7)$, there is

$$E_{3j} = \int | S_{3j}(t) |^2 \mathrm{d}t = \sum_{k=1}^{n} | x_{jk} |^2. \quad (5)$$

Where $X_{jk}(j = 0,1,\cdots,7;\ k = 1,2,\cdots,n)$ means the amplitude of discrete node of the reconstructed signal S_{3j}.

Step 4: The feature vectors are structured and the histogram is made. When people fall ill, there will be a greater influence on the signals on some bands. Therefore, the energy can be constructed as the elements of a feature vector. Feature vector T then can be constructed as follows: $T = [E_{30}, E_{31}, E_{32}, E_{33}, E_{34}, E_{35}, E_{36}, E_{37}]$. When the energy is large, $E_{3j}(j = 0,1,\cdots,7)$ is usually a large value, which would bring some problems in the data analysis. As a result, feature vector T can be normalized. Making $E = \sum_{j=0}^{7} E_{3j}$, the new vector then $T' = [E_{30}/E, E_{31}/E, E_{32}/E, E_{33}/E$, is E_{34}/E, E_{35}/E, E_{36}/E, $E_{37}/E]$. T' is the normalized vector and the histogram of the energy spectrum is made based on T'.

Tab. 1 Signals and their frequency coverage

signal	S_{30}	S_{31}	S_{32}	S_{33}	S_{34}	S_{35}	S_{36}	S_{37}
Frequency coverage	0~8	8~16	16~24	24~32	32~40	40~48	48~56	56~64

Tab. 2 The proportion of the band energy to the total energy

	$T'(1)$	$T'(2)$	$T'(3)$	$T'(4)$	$T'(5)$	$T'(6)$	$T'(7)$	$T'(8)$
Rapid pulse	0.9952	0.0027	0.0011	0.0002	0.0004	0.0001	0.0002	0.0001
Thready pulse	0.9916	0.0070	0.0007	0.0005	0.0001	0.0001	0.0000	00.000
Deep pulse	0.9972	0.0019	0.0003	0.0002	0.0002	0.0001	0.0000	0.0001
Floating pulse	0.9986	0.0008	0.0002	0.0001	0.0002	0.0000	0.0001	0.0000
Slippery pulse	0.9896	0.0064	0.0023	0.0010	0.0004	0.0002	0.0001	0.0001
Stringy pulse	0.9740	0.0200	0.0040	0.0013	0.0003	0.0002	0.0001	0.0001
Moderate pulse	0.9960	0.0028	0.0007	0.0003	0.0001	0.0001	0.0000	0.0000

4 Analysis and experiment results

In this paper, we make an assay on the rapid pulse, the thready pulse, the deep pulse, the floating pulse, the slippery pulse, the stringy pulse and the moderate pulse, according the algorithm we introduced above. T' means the proportion of the band energy to the total energy, as shown in Fig. 2. $T'(1)$ epresents the ratio of the energy between 0~8 Hz frequency bands to the total energy. It can be found that almost all of the energy of human pulse signal are distributed in 0~10 Hz, and only a very little in other frequency coverage, just as the energy spectrum problem shows we find before.

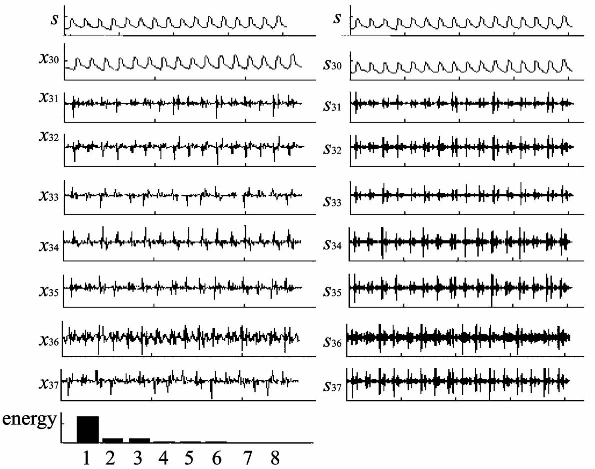

Fig. 2 The decomposition, reconstruction and histogram of multi-wavelet packets

Here is an example with the analysis of wavelet package in Fig. 2. S is the original pulse curve, X_{30}, X_{31}, X_{32}, X_{33}, X_{34}, X_{35}, X_{37}, X_{37} represent the wavelet packet decomposition coefficients, and S_{30}, S_{31}, S_{32}, S_{33}, S_{34}, S_{35}, S_{36}, S_{37} sepresent the wavelet packet reconstruction coefficients. S30 can basically recoostructs the pulse signal. This farther indicates that the feature of the pulse signal is between 0~10 Hz. It is accordant to the fact. In the limited space of this paper, only one example of the processing result is given here.

5 Conclusion

Wavelet packet-energy spectrum can get precise time and frequency distribution of the signal, especially in the analysis of high-frequency density signal which can be clearly analyzed the frequency distribution in different time slots. Wavelet packet energy spectrum has been successfully applied to a variety of fault diagnosis. Human disease also can be equivalent to some "fault" which will certainly cause some changes in the energy band. This article applies this method to pulse wave analysis. Comparing the experiments result and the previous power spectrum analysis, we can prove the effectiveness of the new method which express the full information responded by the power spectral. However, much more work should be done to find how a specific disease corresponding to the frequency information. And we believe that the method in this paper would have a new prospect if we can find the relationship of a band of energy and the disease.

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