Correction of Dynamic Error Result from Measurement System Limitations

Zhi-jie ZHANG(张志杰), Dai-hua WANG(王代华), Wen-lian WANG(王文廉), Wei WANG(王 巍)
(*Key Lab of Instrumentation Science and Dynamic Testing* (*Ministry of Education*), *North University of China*, *Taiyuan* 030051, *China*)

Abstract – The main cause of dynamic errors is due to frequency response limitation of measurement system. One way of solving this problem is designing an effective inverse filter. Since the problem is ill-conditioned, a small uncertainty in the measurement will cause large deviation in reconstructed signals. The amplified noise has to be suppressed at the sacrifice of biasing in estimation. The paper presents a kind of designing method of inverse filter in frequency domain based on stabilized solutions of Fredholm integral equations of the first kind in order to reduce dynamic errors. Compared with previous several work, the method has advantage of generalization. Simulations with different Signal-to-Noise ratio (SNR) are investigated. Flexibility of the method is verified. Application of correcting dynamic error is given.

Key words – dynamic error; *inverse filters*; *correction of dynamic characteristic*; *measurement system*

Manuscript Number: 1674-8042(2010)04-0307-05

dio: 10.3969/j.issn.1674-8042.2010.04.01

1 Introduction

The problem of correcting dynamic characteristic of measurement system can be traced back to some researches like recovery of ideal instrumentation, correcting frequency characteristic of instrumentation, reconstruction of input signals, correction of signals, compensation of dynamic error, method of deconvolution and technique of inverse filter[1]. The application of these methods can get a result more accurately reflecting the true value. Unfortunately, the problems is often ill-conditioned, which means the small uncertainties in the output signal, caused by the noise, lead to big differences in the estimated input. The amplified noise has to be suppressed at the sacrifice of biasing in the estimation. Several algorithms were proposed to compensate the effect of the measurement system with simultaneous noise suppression[2-11]. Most of deconvolution in the time domain is based on Van Cittert's method[2], and several methods have been extensively studied and used. However Van Cittert's method is only a particular case of Bialy's iterative algorithm[3]. Recent papers[12-14] present their solutions to deconvolution in the time domain, but they can not be applied in the measurement of transient signal or non-stationary process. Most of the deconvolution in the frequency domain involves the design of an inverse for the transfer function. These inverse filters are based on an optimization criterion for the selection of the iteration parameters in the optimal compensation deconvolution technique[4-6]. They are often constrained by fixed form of inverse filters. The paper deduces a group of inverse filters based on stabilized solutions of Fredholm integral equations of the first kind in order to correct dynamic characteristics of measurement systems or reduce dynamic errors. Compared with previous work, it is more flexible in the process of designing inverse filters besides these advantages like simplicity, a priori knowledge of the data's statistics is not required and relatively less knowledge of the signal's characteristics is required. At the same time, the paper gives several simulations with different SNR and an application of correcting dynamic error.

2 The principle of quasi-static measurement and dynamic measurement

2.1 The principle of quasi-static measurement

Assuming a measurement system is linear system. The principle of quasi-static measurement is to establish such a measurement system which is approximately satisfied with non-distortion transfer to measured signal according to priori knowledge, i.e. working frequency band width of measurement system is more than maximum frequency band width of measured signal or physical value variation and its phase characteristic is satisfied with linear

* **Received**: 2010-09-09
Project supported: The paper is sponsored by National Natural Science Foundation of China (No. 50675211); Natural Science Foundation (No. 2009011023); Returned Overseas Graduates Foundation (No. 2008067) of Shanxi Province in China
Corresponding author: Zhi-jie ZHANG(zhangzhijie@nuc.edu.cn)

phase. Its principle of quasi-static measurement is demonstrated in Fig.1.

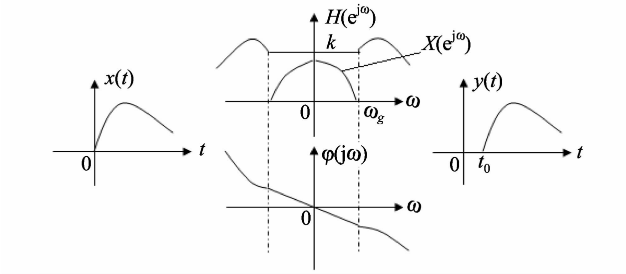

Fig.1 Principle of quasi-static measurement

For a quasi-static measurement system, the relationship between input and output in time domain is

$$y(t) = kx(t - t_0),$$

So, the relationship between input and output in frequency domain is

$$Y(e^{j\omega}) = kX(e^{j\omega})e^{j\omega t_0}.$$

2.2 The principle of dynamic measurement

Assuming a measurement system is linear system. The principle of dynamic measurement is the process of measurement under the situation that the measurement system is not satisfied with the principle of quasi-static measurement mentioned above. The output of the measurement system should be corrected because the effect of the measurement system on the input is to change the complex amplitude of each of the frequency components of the signal measured. Its principle of dynamic measurement is demonstrated in Fig.2.

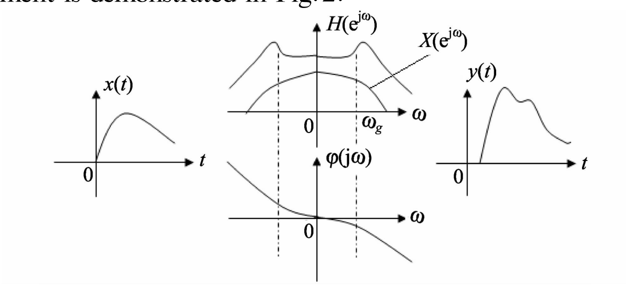

Fig.2 Principle of dynamic measurement

For a dynamic measurement system, the relationship between input and output in time domain is

$$y(t) = \int_{-\infty}^{\infty} h(t - \tau)x(\tau)d\tau,$$

So, the relationship between input and output in frequency domain is

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}).$$

3 Correction method of dynamic error result from system limitations

The problem of dynamic characteristics correction will be taken as solving stabilized solutions of Fredholm integral equations of the first kind

$$\int_{T_1}^{T_2} h(t,\tau)x(\tau)d\tau = A_h x = y(t),\quad t_1 < t < t_2. \tag{1}$$

Where $x(\tau)$ expresses continuous input signal in measurement system; $y(\tau)$ expresses continuous output signal in measurement system; $h(t,\tau)$ expresses pulse response of measurement system; A_h expresses arithmetic operators of pulse response of measurement system; t_1, T_1 expresses beginning time; t_2, T_2 expresses ending time.

According to the principle of solving inverse problems in mathematics, a general stabilized method of doing deconvolution is constructed. Select space $\boldsymbol{F}_1 = W_2^q[T_1, T_2]$, and take the square of integrable functional form with derivatives of q order in the region of $[T_1, T_2]$. Select stable functional

$$\Omega(x) = \int_{T_1}^{T_2} \sum_{r=0}^{q} qq_r(\tau)\left(\frac{d^r x}{d\tau^r}\right)^2 d\tau, \tag{2}$$

where $qq_r(\tau)$ is selected known constants or functions; When $r = 0,1,\cdots,q-1, qq_r(\tau)$, should be greater than or equal to zero; when $r = q, qq_r(\tau)$ should be greater than zero[15]. Then the stable solutions $\hat{x}$ should make the following smoothing functional a minimum

$$M^{\lambda}[\hat{x},y] = \| A_h\hat{x} - y \|^2 + \lambda\Omega(\hat{x}) = \int_{t_1}^{t_2} \left\{\int_{T_1}^{T_2} h(t,\tau)\hat{x}(\tau)d\tau - y(t)\right\}^2 dt + \lambda\int_{T_1}^{T_2} \sum_{r=0}^{q} qq_r(\tau)\left(\frac{d^r\hat{x}}{d\tau^r}\right)^2 d\tau. \tag{3}$$

Where $M^{\lambda}[\hat{x},y]$ expresses smoothing function.

For a linear time invariant measurement system, the model of correction principle of dynamic characteristics is shown in Fig.3. where: $x(n)$ expresses discrete sequence of input signal in measurement system; $y(n)$ expresses discrete sequence of output signal in measurement system; $n_y(x)$ expresses noise of output channel in measurement system; $\hat{x}(n)$ expresses estimated sequence of input signal in measurement system; $H(k)$ expresses discrete transfer function in measurement system; $K(k)$ expresses discrete inverse filter used for correcting dynamic characteristics in measurement system.

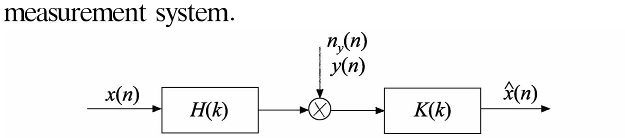

Fig.3 A general model of correction principle of dynamic characteristics

Using stable functional $\Omega(x) = \int_{T_1}^{T_2} \sum_{r=0}^{q} \lambda_r(d^r\hat{x})^2 d\tau$, $\lambda = 1$, then

$$M^{\lambda}[\hat{x},y] = \| A_h\hat{x} - y \|^2 + \Omega(\hat{x}) = \| A_h\hat{x} - y \|^2 + \int_{T_1}^{T_2} \sum_{r=0}^{q} \lambda_r(d^r\hat{x})^2 d\tau. \tag{4}$$

Where $d^r\hat{x}$ is expression of r-order difference of $\hat{x}$. It can be expressed as

$$LM^{\lambda}[\hat{x}(n),y(n)] = T_s\sum_{n=0}^{N-1}[y(n)-\sum_m h(m)\hat{x}(n-m)]^2 + T_s\sum_{r=0}^{q}\sum_{n=0}^{N-1}\lambda_r[\sum_m c_r(m)\hat{x}(n-m)]^2. \quad (5)$$

Where L is an operator; $c_r(n)$ is a r-order difference alignment with a length of N.

The same expression in the frequency domain is

$$LM^{\lambda}[\hat{X}(k),Y(k)] = \frac{T_s}{N}\sum_{k=0}^{N-1}|Y(k)-H(k)\hat{X}(k)|^2 + \frac{T_s}{N}\sum_{r=0}^{q}\sum_{k=0}^{N-1}\lambda_r|C_r(k)\hat{X}(k)|^2. \quad (6)$$

Thus,

$$\frac{\partial LM^{\lambda}[\hat{X}(k),Y(k)]}{\partial\hat{X}(k)} = \frac{T_s}{N}\sum_{k=0}^{N-1}\{2[Y(k)-H(k)\hat{X}(k)][-H'(k)] + \sum_{r=0}^{q}2\lambda_r|C_r(k)|^2\hat{X}(k)\}. \quad (7)$$

If

$$\frac{\partial LM^{\lambda}[\hat{X}(k),Y(k)]}{\partial\hat{X}(k)} = 0,$$

then,

$$-2Y(k)H'(k)+2|H(k)|^2\hat{X}(k) + 2\sum_{r=0}^{q}\lambda_r|C_r(k)|^2\hat{X}(k) = 0. \quad (8)$$

Thus,

$$\hat{X}(k) = Y(k)\frac{H'(k)}{|H(k)|^2+\sum_{r=0}^{q}\lambda_r|C_r(k)|^2} = Y(k)K(k). \quad (9)$$

Where $H'(k)$ is conjugate expression of $H(k)$ and $c_r(n)$ is a r-order difference series with a length of N

$$\begin{aligned} c_0(n) &= \{1,0,0,0,\cdots,0_{N-1}\},\\ c_1(n) &= c_0(n)-c_0(n-1) = \{1,-1,0,0,\cdots,0_{N-1}\},\\ c_2(n) &= c_1(n)-c_1(n-1) = \{1,-2,1,0,\cdots,0_{N-1}\},\\ c_3(n) &= c_2(n)-c_2(n-1) = \{1,-3,3,-1,\cdots,0_{N-1}\}. \end{aligned} \quad (10)$$

The Discrete Fourier Transform (DFT) of $c_0(n)$ is

$$C_0(k) = DFT[c_0(n)] = 1, \quad \text{for } k=0,1,\cdots,N-1. \quad (11)$$

Thus,

$$|C_0(k)|^2 = 1.$$

Next,

$$C_1(k) = DFT[c_0(n)-c_0(n-1)] = 1-\exp(-j\frac{2\pi}{N}k) = 2\sin\frac{\pi k}{N}\exp(j\frac{\pi k}{N}). \quad (12)$$

Thus, $$|C_1(k)|^2 = 4\sin^2\frac{\pi k}{N}. \quad (13)$$

Next,

$$C_2(k) = DFT[c_1(n)-c_1(n-1)] = 2\sin\frac{\pi k}{N}\exp(j\frac{\pi k}{N})\{1-\exp(-j\frac{2\pi}{N}k)\} = 4\sin^2\frac{\pi k}{N}\exp^2(j\frac{\pi k}{N}). \quad (14)$$

Thus, $$|C_2(k)|^2 = 16\sin^4\frac{\pi k}{N}. \quad (15)$$

Next, $$C_3(k) = DFT[c_2(n)-c_2(n-1)] = 4\sin^2\frac{\pi k}{N}\exp^2(j\frac{\pi k}{N})\{1-\exp(-j\frac{2\pi}{N}k)\} = 4\sin^3\frac{\pi k}{N}\exp^3(j\frac{\pi k}{N}). \quad (16)$$

Thus, $$|C_3(k)|^2 = 64\sin^6\frac{\pi k}{N}. \quad (17)$$

In general, then

$$|C_q(k)|^2 = (4\sin^2\frac{\pi k}{N})^q, \quad \text{for } k=0,1,\cdots,N-1. \quad (18)$$

So, a group of inverse filters in frequency domain $K(k)$, can be expressed by

$$K(k) = \frac{H'(k)}{|H(k)|^2+\sum_{r=0}^{q}\lambda_r|C_r(k)|^2} = \frac{H(k)}{|H(k)|^2+\{\lambda_0+\lambda_1 4\sin^2\frac{\pi k}{N}+\lambda_2(4\sin^2\frac{\pi k}{N})^2+\cdots+\lambda_q(4\sin^2\frac{\pi k}{N})^q\}}. \quad (19)$$

Where $\lambda_0,\lambda_1,\cdots,\lambda_q$ and q can be selected or optimized parameters[17].

4 Comparison with other inverse filters

The following results are special cases of formula (19).

1) Using stable functional

$$\Omega(\hat{x}) = \int_{T_1}^{T_2}\hat{x}^2 d\tau,$$

then,

$$K(k) = \frac{H'(k)}{|H(k)|^2+\lambda}, \quad (20)$$

where λ is adjustable parameter. This is a compensating method of inverse filter[4].

2) Using stable functional

$$\Omega(\hat{x}) = \int_{T_1}^{T_2}(d^2\hat{x})^2 d\tau,$$

then,

$$K(k) = \frac{H'(k)}{|H(k)|^2+16\lambda\sin^4\frac{\pi k}{N}}, \quad (21)$$

where λ is adjustable parameter and N is a length of discrete points.

This is an inverse filtering method with one parameter[5].

3) Using stable functional

$$\Omega(\hat{x}) = \int_{T_1}^{T_2}\sum_{r=0}^{2}\lambda_r(d^r\hat{x})^2 d\tau,$$

then,

$$K(k) = \frac{H'(k)}{|H(k)|^2 + \delta |L(k)|^4 + \gamma |L(k)|^2 + \lambda}. \quad (22)$$

Where δ, γ, λ are adjustable parameters; $L(k)$ is the Fourier transform of the second-order backward difference sequence.

This is an inverse filtering method with three parameters[6].

4) Using stable functional

$$\Omega(\hat{x}) = \lambda \int_{T_1}^{T_2} \sum_{r=0}^{2} (d^r \hat{x})^2 \mathrm{d}\tau,$$

then,

$$K(k) = \frac{H'(k)}{|H(k)|^2 + \lambda\{1 + 4\sin\frac{\pi k}{N} + (4\sin^2\frac{\pi k}{N})^2 + \cdots + (4\sin^2\frac{\pi k}{N})^q\}}. \quad (23)$$

Where λ and q are adjustable parameters.

This is an inverse filtering method presented by author[16].

5 Simulations

In order to illustrate the effectiveness of the method mentioned above, a low pass measurement system with a high-bandwidth input signal is studied. The system function of simulated low pass measurement system is

$$H(S) = \frac{(10^5\pi)^2}{S^2 + 6\pi \times 10^3 S + (10^5\pi)^2}.$$

If the sampling period is T_s, the number of sampling point is N, then the DFT of $H(S)$ is given by

$$H(k) = \frac{(10^5\pi)^2}{(\mathrm{j}\frac{k}{NT_s})^2 + 6\pi \times 10^4 \times (\mathrm{j}\frac{k}{NT_s}) + (10^5\pi)^2},$$

$$\text{for } k = 0, 1, \cdots, N-1.$$

The simulation results are shown in Fig.4～Fig.9.

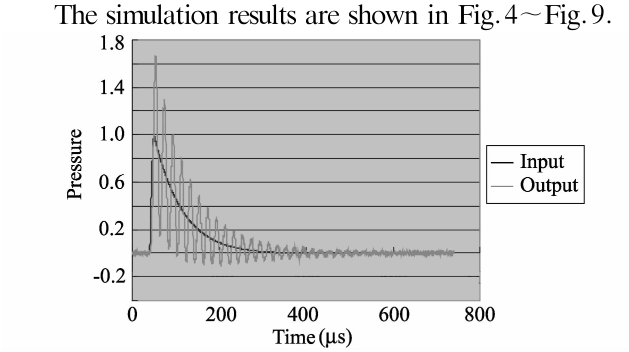

Fig.4 Input and output of the system. Normally distributed noise (its SNR is 40 dB) is added to the output

6 Applications of correcting dynamic error

The method presented by the paper is successful in correcting dynamic characteristic of sensors and measurement system. Figure 10 is a real example in dynamic calibration experiment by use of step pressure generator.

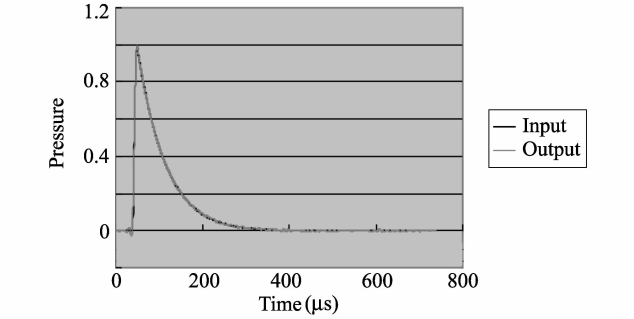

Fig.5 Input and its estimation of the output in the Fig.4

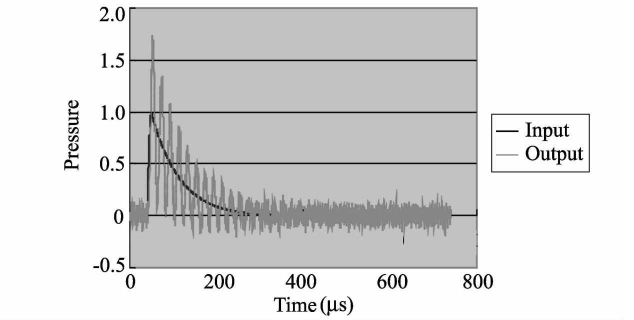

Fig.6 Input and output of the system. Normally distributed noise (its SNR is 26 dB) is added to the output

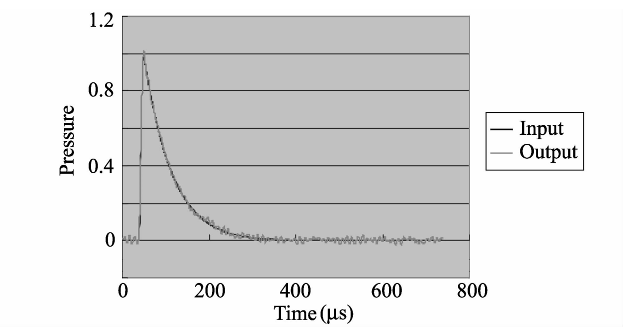

Fig.7 Input and its estimation of the output in the Fig.6

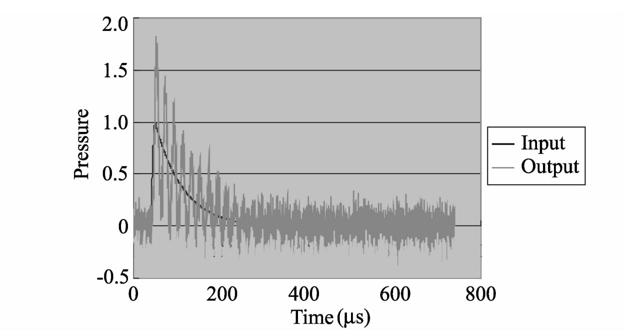

Fig.8 Input and output of the system. Normally distributed noise (its SNR is 20 dB) is added to the output

7 Conclusions

The paper presents a group of inverse filters in frequency domain based on the principium of solving inverse problems in Mathematics in order to correct dynamic error

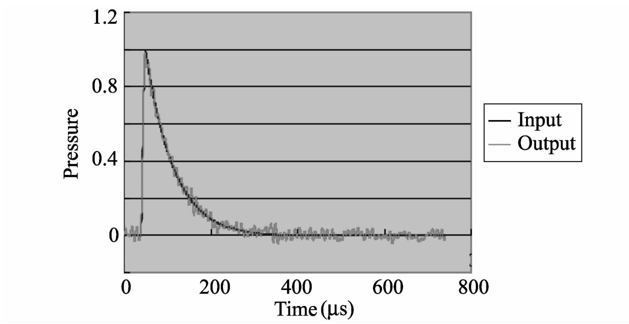

Fig. 9 Input and its estimation of the output in the Fig. 8

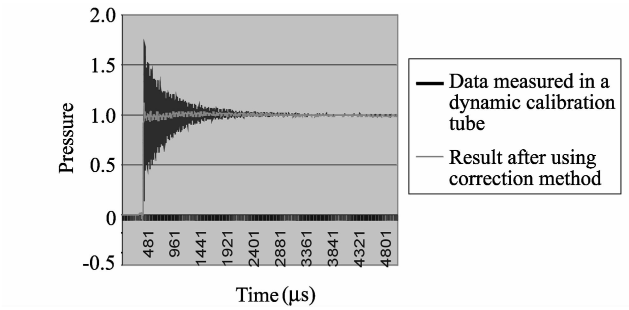

Fig. 10 Comparison between the data from pressure sensor calibrated and result by use of the method

result from measurement system limitations. Most of previous deconvolution in frequency domain is only special cases of method presented in the paper. It provides a kind of easier applied and more flexible method of correcting dynamic error of sensors or measurement systems in the process of selecting or designing inverse filters.

8 Acknowledge

The author wishes to thank Professor Junqin Huang of Beijing University of Aeronautics & Astronautics and all other friends who previously discussed each other and provided technical support.

References

[1] Zhi-jie Zhang, 1998. The Confidence Level of Measured Data of Single Process in Dynamic Measurement: Study on Environmental Factor and Inverse Filtering Method, Doctoral Dissertation, Beijing Institute of Technology, Beijing.

[2] P. B. Crilly, 1991. A quantitative evaluation of various iterative deconvolution algorithms. *IEEE Trans. on Instrum. and Meas.*, 40(3): 558-562.

[3] Yan-da Li, Jun Chang, 1991. Signal Reconstruction Theory and its Applications, Tsinghua University Press, p. 135-138.

[4] S. M. Riad, R. B. Stafford, 1980. Impulse Response Evaluation using Frequency Domain Optimal Compensation Deconvolution. Proc. 23rd Midwest Symp. on Circuits and Systems, Toledo, OH, p. 521-525.

[5] N. S. Nahman, M. E. Guillaume, 1981. Deconvolution of Time Domain Waveforms in the Presence of Noise. NBS Tech. Note 1047, NBS Boulder, CO.

[6] Tamas Daboczi, Istran Kollar, 1996. Multiparameter optimization of inverse filtering algorithms. *IEEE Trans. on Instrum. and Meas.*, 45(2): 417-421.

[7] G. N. Solopchenko, 1987. Inverse problems in measurement. *Measurement*, 5(1): 10-19.

[8] B. Parruck, S. M. Riad, 1983. An optimization criterion for iterative deconvolution. *IEEE Trans. on Instrum. and Meas.*, IM-32(1): 137-140.

[9] Sun Shenghe, 1989. Modern Measurement in the Time Domain. Harbin Industry University Press, p. 84-93.

[10] A. N. Tikhonov, V. Y. Arsenin, 1977. Solution of Ill-Posed Problems, Wiley, New York.

[11] K. Miller, 1970. Least squares methods for ill-posed problems with a prescribed bound. *SIAM J. Math Anal.*, 1: 52-74,

[12] Ke-jun Xu, Cheng Li, Zhi-neng Zhu, 2007. Dynamic modeling and compensation of robot six-axis wrist force/torque sensor. *IEEE Trans. on Instrum. and Meas.*, 56(5): 2094-2100.

[13] Marco P. Schoen, 2007. Dynamic compensation of intelligent sensors. *IEEE Trans. on Instrum. and Meas.*, 56(5): 1992-2001.

[14] Claudio Narduzzi, 2005. Inverse filtering with signal-adaptive constraints. *IEEE Trans. on Instrum. and Meas.*, 54(4): 1992-2001.

[15] Shi-xiong Li, Jia-qi Liu, 1994. Wavelet Transform and Mathematic Groundwork to Inverse Problem, Geological Publishing Company.

[16] Zhi-jie Zhang, Jing Zu, Hong-mian Du, An-shi Chen, 2007. Correction of signal distortion resulting from measurement systems limitation. *The Journal of Measurement Science*, 2(2): 65-71.

[17] Zhi-jie Zhang, et al, 2008. A Group of Inverse Filters Based on Stabilized Solutions of Fredholm Integral Equations of the First Kind. IEEE International Instrumentation and Measurement Technology Conference, Victoria, Vancouver Island, Canada, 2008.