

Cooperative output tracking of multi-agent systems under finite time

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Abstract: The cooperative output tracking problem of multi-agent systems in finite time is considered. In order to enable the agents to quickly track and converge to external system within a finite time, a novel distributed output feedback control strategy based on the finite-time state observer is designed. This distributed finite-time observer can not only solve cooperative output tracking problems when the agents can not get external system signal, but also make the systems have a faster convergence and a good robustness. The stability of the system in finite time is proved based on Lyapunov function. Numerical simulations results have been provided to demonstrate the effectiveness of the proposed protocol.

Key words: multi-agent systems; finite time observer; cooperative output tracking; distributed output feedback control

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0 Introduction

In recent years, the study of cooperative output control of multi-agent systems has attracted a lot of researchers' attention^[1-4]. Multi-agent systems are widely used in many fields such as multi unmanned aerial vehicle (UAV) cooperation control^[5], optimal formation control^[6] and target position estimate^[7]. The output regulation is mainly to design a distributed control law, which makes the output of multi-agent closed-loop systems asymptotically track a predetermined reference input generated by external system, or asymptotically reject the disturbance signals^[8]. Therefore, the output regulation problem can also be attributed to a class of output tracking problems^[9].

In many practical applications, usually there are two possibilities in the cooperative tracking problem setup. The first case is that all the followers can access the exogenous signal and the other is that the output to the exogenous is unknown to the followers. For the former, it was firstly studied in Ref. [10]. To overcome the limitation of the latter, a distributed observer is proposed so that each agent

can estimate the state of the exosystem, a dynamic state feedback controller can be based on the observer, and the heterogeneous multi-agent systems can realize output tracking problem^[11-13]. The output regulation problem of nonlinear system with distributed observer was considered by Tang, et al. [14], which enables the followers to track the state of the leader when the leader contains unknown input. In Ref. [15], a distributed fuzzy observer and controller were designed based on parallel distributed compensation scheme and internal reference models so that the heterogeneous nonlinear multi-agent systems can achieve output consensus. Ref. [16] further utilized the adaptive distributed observer approach to deal with the coordinated output adjustment problem of linear multi-agent systems.

From the above reviewed researches, we can see a common drawback that the designed observer can not realize the estimation of external system in finite time, thus the output of the agent can not be guaranteed to track external signals and suppress interference at a finite-time. However, in many cases, the system is required to converge within a limited time, especially when in state of external

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disturbance and uncertain factor, the shorter the convergence time, the better the performance. Sufficient conditions of finite time convergence and communication topological conditions are given in Refs. [17-20]. Motivated by Ref. [21], a finite-time state feedback controller is firstly proposed for a class of second-order nonlinear multi-agent systems with leader-follower architecture, the finite time convergence observer is constructed to estimate the velocity information, but with the disadvantage of long finite convergence time. By designing a new distributed fixed-time observer, it is possible to estimate the velocity information and achieve consensus problem in a bounded finite time fully independent of initial condition^[22]. In Ref. [23], to estimate the matched/mismatched disturbances and state the agents of the unknown elements for the finite-time output consensus problem, due to the fact that disturbances are fast time-varying and it is difficult to operate in reality, a finite-time generalized state observer is constructed for each agent. However, all the reviewed studies mentioned above on the finite-time output adjustment are mostly limited to the estimation of a certain reference quantity of the internal system, and the problem that some agents can not obtain the external signal in finite time is not considered. Moreover, the design of the existing finite time algorithm mainly focuses on the controller. In this paper, a new finite time distributed observer is proposed, which makes the state of the exosystem estimated in a limited time when a part of the agents can not obtain the exosystem information, so that every follower can quickly track the exosystem in a limited time and solve the coordinated output tracking in finite time. This is the main purpose of the research.

The key results of this paper involve three contributions. Firstly, a novel distributed finite time observer algorithm and a control law are designed. Secondly, the efficiency of heterogeneous multi-agent systems under the limited time distributed observer is proved. Finally, the stability of coordination output in finite time is proved.

The present paper is organized as follows. In Section 1, the problem formulation and some necessary preliminaries results from graph theory and the properties of finite time are given. The finite-time distributed observer and the corresponding control law are discussed in Section 2, which is illustrated by a simulation in Section 3. Finally, a

brief conclusion is presented in Section 4.

Throughout this paper, some notations to be used in the sequel are introduced. \mathbf{N} is a set of positive integers. $\mathbf{R}^{n \times n}$ is the set of $n \times n$ real matrices. $\text{sgn} \mathbf{x} = [\text{sgn} x_1, \dots, \text{sgn} x_n]^T$, $\text{sgn}(\cdot)$ denotes a sign function, $\text{sig}(x)^a = |x|^a \text{sgn}(x)$, $\mathbf{x}^a = [x_1^a, x_2^a, \dots, x_n^a]$, $|\mathbf{x}|^a = [|x_1|^a, |x_2|^a, \dots, |x_n|^a]$. \mathbf{A}^T denotes the transpose of matrix \mathbf{A} .

1 Preliminaries and problem formulation

1.1 Preliminaries

1.1.1 Graph

A graph $G = \{V, E, A\}$ is used to describe the topology of connections between agents and leader, where $V = \{1, 2, \dots, N\}$ is the set of vertices representing N agents, $E = V * V$ is the set of edges of the graph, i. e., the set of edges formed between nodes. The set of all neighbors of node i can be represented by $N_i = \{j \in V : (i, j) \in E, i \neq j\}$. The weighted adjacency matrix of a diagraph G is a nonnegative matrix $\mathbf{A} = (a_{ij}) \in \mathbf{R}^{n \times n}$. Suppose that there is no self-loop in the graph G , i. e., $(i, i) \notin E$. If $(i, j) \in E$, then $a_{ij} > 0$, and the node i is called a neighbor of the j node; Otherwise, $a_{ij} = 0$. If $(i, j) \in E$ and $(j, i) \in E$, then the graph G is called an undirected graph; otherwise G is called a directed graph.

The communication topology considered in this paper is undirected, define the degree of node i as $d_i(v_i) = \sum_{j=1}^n a_{ij}$ and the degree matrix of graph G as $\mathbf{D}_i = \text{diag}\{d_i(v_1), \dots, d_i(v_n)\}$. The Laplacian of a graph G is denoted by $\mathbf{L} = [l_{ij}] \in \mathbf{R}^{(N+1) \times (N+1)}$, which can be expressed as

$$l_{ij} = \begin{cases} -a_{ij}, & i \neq j, \\ \sum_{i=1}^n a_{ij}, & i = j. \end{cases} \quad (1)$$

1.1.2 Finite time stability

First, some lemmas and theorems which are stable for a finite time are given.

Lemma 1^[17] Suppose there exists a continuous function $V(x)$ defined on a neighborhood of the origin, and real numbers $c > 0$ and $0 < \alpha < 1$, so that the following conditions hold:

- 1) $V(x)$ is positive definite;
- 2) $\dot{V}(x) + c(V(x))^\alpha \leq 0$, then the system is said to be stable for a finite time, and the stable time satisfies

$$T(x_0) \leq \frac{V(x_0)^{1-\alpha}}{c(1-\alpha)}.$$

1.2 Problem formulation

In this paper, the cooperative output system composed of external system and multi-agent systems is considered. The dynamics of each heterogeneous agent is

$$\begin{aligned}\dot{\mathbf{x}}_i(t) &= \mathbf{A}_i \mathbf{x}_i(t) + \mathbf{B}_i \mathbf{u}_i(t) + \mathbf{G}_i \mathbf{v}(t), \\ \mathbf{y}_i(t) &= \mathbf{C}_i \mathbf{x}_i(t) + \mathbf{D}_i \mathbf{u}_i(t) + \mathbf{E}_i \mathbf{v}(t),\end{aligned}\quad (2)$$

where $\mathbf{x}_i(t) \in \mathbf{R}^{n_i}$, $\mathbf{u}_i(t) \in \mathbf{R}^{m_i}$ and $\mathbf{y}_i(t) \in \mathbf{R}^{p_i}$ are the state, tracking error and input of the i -th subsystem, $\mathbf{A}_i \in \mathbf{R}^{n_i \times n_i}$, $\mathbf{B}_i \in \mathbf{R}^{n_i \times m_i}$, $\mathbf{C}_i \in \mathbf{R}^{p_i \times n_i}$, $\mathbf{D}_i \in \mathbf{R}^{p_i \times m_i}$, \mathbf{G}_i and \mathbf{E}_i are constant matrices, $\mathbf{v} \in \mathbf{R}^q$ is the exogenous signal composed of the reference signal $\mathbf{f}(t)$ and the disturbance signal $\mathbf{w}(t)$ as

$$\begin{aligned}\dot{\mathbf{f}}(t) &= \mathbf{A}_f \mathbf{f}(t), \\ \dot{\mathbf{w}}(t) &= \mathbf{A}_w \mathbf{w}(t).\end{aligned}\quad (3)$$

Let $\mathbf{A}_v = \text{diag}\{\mathbf{A}_f, \mathbf{A}_w\}$, then the external system can be written as

$$\dot{\mathbf{v}}(t) = \begin{bmatrix} \dot{\mathbf{f}}(t) \\ \dot{\mathbf{w}}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A}_f & 0 \\ 0 & \mathbf{A}_w \end{bmatrix} \begin{bmatrix} \mathbf{f}(t) \\ \mathbf{w}(t) \end{bmatrix} = \mathbf{A}_v \mathbf{v}(t). \quad (4)$$

Take the tracking error $\mathbf{e}_i(t) = \mathbf{y}_i(t) - \mathbf{v}(t)$, then multi-agent systems of Eq. (2) can be expressed in the following form

$$\begin{aligned}\dot{\mathbf{x}}_i(t) &= \mathbf{A}_i \mathbf{x}_i(t) + \mathbf{B}_i \mathbf{u}_i(t) + \mathbf{G}_i \mathbf{v}(t), \\ \mathbf{e}_i(t) &= \mathbf{C}_i \mathbf{x}_i(t) + \mathbf{D}_i \mathbf{u}_i(t) + \mathbf{E}_i \mathbf{v}(t), \\ \dot{\mathbf{v}}(t) &= \mathbf{A}_v \mathbf{v}(t).\end{aligned}\quad (5)$$

For convenience, some standard assumptions that are needed for the cooperative output tracing problem are discussed before deriving the main conclusions of this paper.

Assumption 1 \mathbf{A}_v has no eigenvalues with negative real parts.

Assumption 2 The pairs $(\mathbf{A}_i, \mathbf{B}_i)$ are stabilizable, the pairs $(\mathbf{A}_i, \mathbf{C}_i)$ and $(\mathbf{G}_i, \mathbf{E}_i)$ are observable, $i=1, 2, \dots, N$.

Assumption 3 Only part of the agents can get the signals from the external system, and the other part can not directly get the state of the external system, so it is necessary to estimate the state of the external system.

Definition 1 For the multi-agent systems of Eq. (5), design a controller and a distributed observer that satisfy the following conditions:

1) When $\mathbf{v}(t) = 0$, the closed-loop system of Eq. (5) has asymptotic stability.

2) For any initial condition $\mathbf{x}_i(0)$, $\boldsymbol{\eta}_i(0)$, $\mathbf{v}(0)$, $i=1, 2, \dots, N$, there is a $T \in [0, \infty]$ that makes the tracking error satisfy the following relationship

$$\lim_{t \rightarrow T} \mathbf{e}_i = 0.$$

2 Design of distributed finite time output feedback control protocol

In this section, a dynamic compensator^[11] is introduced as

$$\begin{aligned}\dot{\boldsymbol{\eta}}_i(t) &= \mathbf{A}_v \boldsymbol{\eta}_i(t) + \\ &\mu \left(\sum_{j \in N_i} a_{ij} (\boldsymbol{\eta}_j(t) - \boldsymbol{\eta}_i(t)) + a_{i0} (\mathbf{v}(t) - \boldsymbol{\eta}_i(t)) \right), \\ i &= 1, 2, \dots, n.\end{aligned}\quad (6)$$

This distributed observer solves the problem that some agents can not obtain exosystem information. In order to make agents quickly track the exosystem in finite time, a distributed finite time observer and controller can be designed as

$$\begin{aligned}\mathbf{u}_i(t) &= \mathbf{K}_{1i} \mathbf{x}_i(t) + \mathbf{K}_{2i} \boldsymbol{\eta}_i(t), \\ \dot{\boldsymbol{\eta}}_i(t) &= \mathbf{A}_v \boldsymbol{\eta}_i(t) + \tau \left[\sum_{j \in N_i} a_{ij} \text{sig}(\boldsymbol{\eta}_j(t) - \boldsymbol{\eta}_i(t))^\sigma + \right. \\ &\quad \left. a_{i0} \text{sig}(\mathbf{v}(t) - \boldsymbol{\eta}_i(t))^\sigma + \beta \sum_{j \in N_i} (\boldsymbol{\eta}_j(t) - \boldsymbol{\eta}_i(t)) + \right. \\ &\quad \left. \beta a_{i0} (\mathbf{v}(t) - \boldsymbol{\eta}_i(t)) \right],\end{aligned}\quad (7)$$

where $\mathbf{K}_{1i} \in \mathbf{R}^{m_i \times n_i}$ and $\mathbf{K}_{2i} \in \mathbf{R}^{m_i \times q}$ are gain matrices to be determined later, for $i=1, \dots, N$; τ is some positive number; $0 < \sigma < 1$; a_{i0} is the weighted value between agent and leader; $\boldsymbol{\eta}_i \in \mathbf{R}^q$ as an estimate of \mathbf{v} can be seen from Eq. (6). Under the controller action of Eq. (7), the closed loop system Eq. (5) of multi-agent can be written as

$$\begin{cases} \dot{\mathbf{x}}_i(t) = \mathbf{A}_i \mathbf{x}_i(t) + \mathbf{B}_i \mathbf{K}_{1i} \mathbf{x}_i(t) + \mathbf{B}_i \mathbf{K}_{2i} \boldsymbol{\eta}_i(t) + \mathbf{G}_i \mathbf{v}(t), \\ \mathbf{e}_i(t) = \mathbf{C}_i \mathbf{x}_i(t) + \mathbf{D}_i \mathbf{K}_{1i} \mathbf{x}_i(t) + \mathbf{D}_i \mathbf{K}_{2i} \boldsymbol{\eta}_i(t) + \mathbf{E}_i \mathbf{v}(t), \\ \dot{\boldsymbol{\eta}}_i(t) = \mathbf{A}_v \boldsymbol{\eta}_i(t) + \tau \left[\sum_{j \in N_i} a_{ij} \text{sig}(\boldsymbol{\eta}_j(t) - \boldsymbol{\eta}_i(t))^\sigma + \right. \\ \quad \left. a_{i0} \text{sig}(\mathbf{v}(t) - \boldsymbol{\eta}_i(t))^\sigma + \beta \sum_{j \in N_i} (\boldsymbol{\eta}_j(t) - \boldsymbol{\eta}_i(t)) + \right. \\ \quad \left. \beta a_{i0} (\mathbf{v}(t) - \boldsymbol{\eta}_i(t)) \right], \\ \dot{\mathbf{v}}(t) = \mathbf{A}_v \mathbf{v}(t), \\ i = 1, 2, \dots, n. \end{cases}\quad (9)$$

To solve the performance of distributed observer, the following Lemma will be used.

Lemma 2^[17-18] For $y_1, y_2, \dots, y_n \geq 0$ and $0 < p < 1$,

there exists $\sum_{i=1}^n y_i^p \geq (\sum_{i=1}^n y_i)^p$.

Lemma 3^[19] For the undirected communication topology diagram G , $\mathbf{L}(\mathbf{A}) = [l_{ij}] \in \mathbf{R}^{n \times n}$ represents the Laplacian matrix of the undirected graph, which has the following properties:

$$1) \mathbf{x}^T \mathbf{L}(\mathbf{A}) \mathbf{x} = \frac{1}{2} \sum_{i,j=1}^n a_{ij} (x_j - x_i)^2 ;$$

2) If the topology G is connected, then $\mathbf{L}(\mathbf{A})$ is semi-definite. The algebraic connectivity of the graph is equal to $\min_{\mathbf{x} \neq 0, \mathbf{1}_n^T \mathbf{x} = 0} \frac{\mathbf{x}^T \mathbf{L}(\mathbf{A}) \mathbf{x}}{\mathbf{x}^T \mathbf{x}}$, that is, when $\mathbf{1}_n^T \mathbf{x} = 0$, there is $\mathbf{x}^T \mathbf{L}(\mathbf{A}) \mathbf{x} \geq \lambda_2(\mathbf{L}(\mathbf{A})) \mathbf{x}^T \mathbf{x}$, and the eigenvalues of $\mathbf{L}(\mathbf{A})$ satisfy $0 = \lambda_1(L) < \lambda_2(L) \leq \dots \leq \lambda_n(L)$;

3) If the undirected communication topology is connected, for vector $\mathbf{b}_i \geq 0, \forall i \in I, \mathbf{b} \neq 0$, then the matrix $\mathbf{L}(\mathbf{A}) + \text{Diag}(\mathbf{b}_1, \dots, \mathbf{b}_n)$ is positively definite.

Lemma 4^[20] For the undirected communication topology G , if there exists a function $\psi: \mathbf{R}^2 \rightarrow \mathbf{R}$, that satisfies $\psi(x_i, x_j) = -\psi(x_j, x_i)$, for any $i, j \in I$ and $i \neq j$, then there is a set of series y_1, y_2, \dots, y_n which satisfies

$$\sum_{i=1}^n \sum_{j \in N_i} a_{ij} y_j \psi(x_j, x_i) = -\frac{1}{2} \sum_{(v_i, v_j) \in E} a_{ij} (y_j - y_i) \psi(x_j, x_i).$$

Now consider the cooperative output tracking problem in finite time. By Lemma 2–4 and the above analysis, the following results can be obtained immediately.

Theorem 1 Suppose there exists a continuous function defined on a neighborhood of the origin. For any $t \geq T$, and real numbers $\rho > 0$ and $0 < \lambda < 1$, if the condition $T \leq \frac{V(0)^{1-\lambda}}{\rho(1-\lambda)}$ is satisfied, the observer of Eq. (8) can realize the fast tracking of the external signal $v(t)$, and the tracking error is reduced to 0. At the same time, each follower in the whole system can also quickly track the leader in a limited time, and the cooperative output tracking problem in finite time can be solved.

Proof: Let

$$\bar{\eta}_i(t) = \eta_i(t) - v(t). \quad (10)$$

According to Eqs. (8) and (10),

$$\begin{aligned} \dot{\bar{\eta}}_i(t) &= \mathbf{A}_v \bar{\eta}_i(t) + \tau \left[\sum_{j \in N_i} a_{ij} \text{sig}(\bar{\eta}_j(t) - \bar{\eta}_i(t))^\sigma + a_{i0} \phi_2 \text{sig}(v(t) - \eta_i(t))^\sigma + \beta a_{i0} (v(t) - \eta_i(t)) \right. \\ &\quad \left. + \beta \sum_{j \in N_i} a_{ij} (\bar{\eta}_j(t) - \bar{\eta}_i(t)) \right] = \mathbf{A}_v \bar{\eta}_i(t) + \tau \left[\sum_{j \in N_i} a_{ij} \phi_1 \text{sig}(\bar{\eta}_j(t) - \bar{\eta}_i(t))^\sigma - \right. \\ &\quad \left. a_{i0} \phi_2 \text{sig}(\bar{\eta}_i(t))^\sigma + \beta \sum_{j \in N_i} a_{ij} (\bar{\eta}_j(t) - \bar{\eta}_i(t)) - \beta a_{i0} (\bar{\eta}_i(t)) \right]. \end{aligned} \quad (11)$$

Consider the Lyapunov function candidate

$$V(t) = \sum_{i \in N_i} \bar{\eta}_i(t)^2.$$

Combined with Eqs. (10) – (11), the time derivative of this Lyapunov candidate along the trajectory of this system is

$$\begin{aligned} \dot{V}(t) &= 2 \sum_{i \in N_i} \bar{\eta}_i(t) \dot{\bar{\eta}}_i(t) = 2 \sum_{i \in N_i} \bar{\eta}_i(t) \left[\mathbf{A}_v \bar{\eta}_i(t) + \tau \sum_{j \in N_i} a_{ij} \text{sig}(\bar{\eta}_j(t) - \bar{\eta}_i(t))^\sigma - \tau a_{i0} \text{sig}(\bar{\eta}_i(t))^\sigma + \right. \\ &\quad \left. \tau \beta \sum_{j \in N_i} a_{ij} (\bar{\eta}_j(t) - \bar{\eta}_i(t)) - \tau \beta a_{i0} \bar{\eta}_i(t) \right] = \tau \left[2 \sum_{i \in N_i} \bar{\eta}_i(t) \sum_{j \in N_i} a_{ij} \text{sig}(\bar{\eta}_j(t) - \bar{\eta}_i(t))^\sigma - \right. \\ &\quad \left. 2 \sum_{i \in N_i} \bar{\eta}_i(t) a_{i0} \text{sig}(\bar{\eta}_i(t))^\sigma + 2 \beta \sum_{i \in N_i} \bar{\eta}_i(t) \sum_{j \in N_i} a_{ij} (\bar{\eta}_j(t) - \bar{\eta}_i(t)) - 2 \beta \sum_{i \in N_i} \bar{\eta}_i(t) a_{i0} \bar{\eta}_i(t) \right] + 2 \sum_{i \in N_i} \bar{\eta}_i(t) \mathbf{A}_v \bar{\eta}_i(t). \end{aligned}$$

According to Lemma 4, it can obtain that

$$\begin{aligned} \dot{V}(t) &= \tau \left[- \sum_{i \in N_i} \sum_{j \in N_i} (\bar{\eta}_j(t) - \bar{\eta}_i(t)) a_{ij} \text{sig}(\bar{\eta}_j(t) - \bar{\eta}_i(t))^\sigma - \beta \sum_{i \in N_i} \sum_{j \in N_i} a_{ij} (\bar{\eta}_j(t) - \bar{\eta}_i(t))^2 - \right. \\ &\quad \left. 2 \sum_{i \in N_i} \bar{\eta}_i(t) a_{i0} \text{sig}(\bar{\eta}_i(t))^\sigma - 2 \beta \sum_{i \in N_i} \bar{\eta}_i(t) a_{i0} \bar{\eta}_i(t) \right] + 2 \sum_{i \in N_i} \bar{\eta}_i(t) \mathbf{A}_v \bar{\eta}_i(t) = \\ &\quad \tau \left[- \sum_{i \in N_i} \sum_{j \in N_i} (\bar{\eta}_j(t) - \bar{\eta}_i(t)) a_{ij} \text{sig}(\bar{\eta}_j(t) - \bar{\eta}_i(t))^\sigma - \beta \sum_{i \in N_i} \sum_{j \in N_i} a_{ij} (\bar{\eta}_j(t) - \bar{\eta}_i(t))^2 \right] - \\ &\quad \tau 2 \sum_{i \in N_i} \bar{\eta}_i(t) a_{i0} \text{sig}(\bar{\eta}_i(t))^\sigma - \tau 2 \sum_{i \in N_i} \bar{\eta}_i(t) a_{i0} \beta \bar{\eta}_i(t) + 2 \sum_{i \in N_i} \bar{\eta}_i(t) \mathbf{A}_v \bar{\eta}_i(t) = \\ &\quad \tau \left[- \sum_{i \in N_i} \sum_{j \in N_i} (\bar{\eta}_j(t) - \bar{\eta}_i(t)) a_{ij} \text{sig}(\bar{\eta}_j(t) - \bar{\eta}_i(t))^\sigma - \beta \sum_{i \in N_i} \sum_{j \in N_i} a_{ij} (\bar{\eta}_j(t) - \bar{\eta}_i(t))^2 \right] - \\ &\quad 2 \sum_{i \in N_i} \bar{\eta}_i(t) [\tau a_{i0} \text{sig}(\bar{\eta}_i(t))^\sigma + \tau a_{i0} \beta \bar{\eta}_i(t) - \mathbf{A}_v \bar{\eta}_i(t)] < \tau \left[- \sum_{i \in N_i} \sum_{j \in N_i} (\bar{\eta}_j(t) - \bar{\eta}_i(t)) a_{ij} \text{sig}(\bar{\eta}_j(t) - \bar{\eta}_i(t))^\sigma - \right. \end{aligned}$$

$$\begin{aligned} \bar{\eta}_i(t)^\sigma - \beta \sum_{i \in N_i} \sum_{j \in N_i} a_{ij} (\bar{\eta}_j(t) - \bar{\eta}_i(t))^2 &= \tau \left[- \sum_{i \in N_i} \sum_{j \in N_i} a_{ij} |\bar{\eta}_j(t) - \bar{\eta}_i(t)|^{1+\sigma} - \beta \sum_{i \in N_i} \sum_{j \in N_i} a_{ij} (\bar{\eta}_j(t) - \bar{\eta}_i(t))^2 \right] < \\ &= -\tau \sum_{i \in N_i} \sum_{j \in N_i} a_{ij} |\bar{\eta}_j(t) - \bar{\eta}_i(t)|^{1+\sigma} = -\tau \sum_{i \in N_i} \sum_{j \in N_i} [a_{ij}^{\frac{2}{1+\sigma}} (\bar{\eta}_j(t) - \bar{\eta}_i(t))^2]^{\frac{1+\sigma}{2}}. \end{aligned}$$

It can be seen from Lemma 2 that the above equation can be expressed as

$$\begin{aligned} \dot{V}(t) &= -\tau \sum_{i \in N_i} \sum_{j \in N_i} [a_{ij}^{\frac{2}{1+\sigma}} (\bar{\eta}_j(t) - \bar{\eta}_i(t))^2]^{\frac{1+\sigma}{2}} \leq \\ &= -\tau \left[\sum_{i \in N_i} \left(\sum_{j \in N_i} a_{ij}^{\frac{2}{1+\sigma}} (\bar{\eta}_j(t) - \bar{\eta}_i(t))^2 \right)^{\frac{1+\sigma}{2}} \right]. \quad (12) \end{aligned}$$

Assuming that $\mathbf{P} = \mathbf{L}(\mathbf{A}) + \text{diag}(a_{10}, a_{20}, \dots, a_{n0})$, the eigenvalue conditions of $\mathbf{L}(\mathbf{A})$ in Lemma 3 can be obtained.

$$\frac{\sum_{i \in N_i} \left[\sum_{j \in N_i} a_{ij}^{\frac{2}{1+\sigma}} (\bar{\eta}_j - \bar{\eta}_i)^2 \right]}{V(t)} = \frac{2\bar{\eta}_i^T \mathbf{P}^T \mathbf{P} \bar{\eta}_i}{\bar{\eta}_i^T \mathbf{P}^T \bar{\eta}_i}. \quad (13)$$

Taking Eq. (12) into Eq. (13), the following can be obtained

$$\dot{V}(t) \leq -[2\lambda_2(L_P)]^{\frac{1+\sigma}{2}} V(t)^{\frac{1+\sigma}{2}}. \quad (14)$$

From Lemma 1, the designed observer satisfies the following conditions

$$T \leq \frac{2V(0)^{\frac{1+\sigma}{2}}}{(1-\sigma)[2\lambda_2(L_P)]^{\frac{1+\sigma}{2}}}. \quad (15)$$

Because $\tau > 0$, $0 < \sigma < 1$, through the above proof, the observation error can be gotten as $\lim_{t \rightarrow T} \bar{\eta}_i(t) = 0$, thus it can be seen that the designed observer of Eq. (8) can quickly track the external system signal in a limited time.

Theorem 2 Supposing that the system of Eq. (5) satisfies assumptions 1–5 and also satisfies the finite time conditions involved in Theorem 1 under the control law of Eq. (7), let \mathbf{K}_{1i} be a constant matrix so that $\mathbf{A}_i + \mathbf{B}_i \mathbf{K}_{1i}$ is Hurwitz, and $\mathbf{K}_{2i} = \mathbf{U}_i - \mathbf{K}_{1i} \mathbf{X}_i$, if and only if the following matrix Eq. (16) has a unique solution

$$\begin{aligned} \mathbf{A}_i \mathbf{X}_i + \mathbf{B}_i \mathbf{U}_i + \mathbf{G}_i &= \mathbf{X}_i \mathbf{A}_v, \\ \mathbf{C}_i \mathbf{X}_i + \mathbf{D}_i \mathbf{U}_i + \mathbf{E}_i &= \mathbf{0}. \end{aligned} \quad (16)$$

Then the multi-agent systems of Eq. (5) can solve the problem of cooperative output tracking stability in finite time so that the following statements hold:

- 1) If $\mathbf{v}(t) = 0$, $\lim_{t \rightarrow T} \bar{\eta}_i(t) = 0$;
- 2) For any initial condition $\mathbf{x}_i(0)$, $\boldsymbol{\eta}(0)$, $\mathbf{v}(0)$, $\lim_{t \rightarrow T} \bar{\mathbf{e}}_i(t) = 0$.

Proof: For compact expression, define the following variables

$$\mathbf{x}(t) = [\mathbf{x}_1^T, \dots, \mathbf{x}_n^T]^T, \mathbf{A} = \text{diag}[\mathbf{A}_1, \dots, \mathbf{A}_n],$$

$$\begin{aligned} \mathbf{B} &= \text{diag}[\mathbf{B}_1, \dots, \mathbf{B}_n], \mathbf{C} = \text{diag}[\mathbf{C}_1, \dots, \mathbf{C}_n], \\ \mathbf{D} &= \text{diag}[\mathbf{D}_1, \dots, \mathbf{D}_n], \mathbf{G} = \text{diag}[\mathbf{G}_1, \dots, \mathbf{G}_n], \\ \mathbf{E} &= \text{diag}[\mathbf{E}_1, \dots, \mathbf{E}_n], \mathbf{K}_1 = \text{diag}[\mathbf{K}_{11}, \dots, \mathbf{K}_{1n}], \\ \mathbf{K}_2 &= \text{diag}[\mathbf{K}_{21}, \dots, \mathbf{K}_{2n}], \mathbf{e}(t) = [\mathbf{e}_1^T, \dots, \mathbf{e}_n^T]^T, \end{aligned}$$

then systems of Eq. (9) can be written as

$$\begin{aligned} \dot{\mathbf{x}}(t) &= (\mathbf{A} + \mathbf{B} \mathbf{K}_1) \mathbf{x}(t) + \mathbf{B} \mathbf{K}_2 \boldsymbol{\eta} + \mathbf{G} \mathbf{v}, \\ \mathbf{e}(t) &= (\mathbf{C} + \mathbf{D} \mathbf{K}_1) \mathbf{x}(t) + \mathbf{B} \mathbf{K}_2 \boldsymbol{\eta} + \mathbf{E} \mathbf{v}, \\ \dot{\mathbf{v}}(t) &= \mathbf{A}_v \mathbf{v}(t). \end{aligned}$$

From the proof of Theorem 1, $\lim_{t \rightarrow T} \bar{\eta}_i(t) = 0$. When $\mathbf{v} = 0$, $\lim_{t \rightarrow T} \bar{\eta}_i(t) = 0$, thus the overall closed-loop system of Eq. (9) can be put in the following form

$$\dot{\mathbf{x}}(t) = (\mathbf{A} + \mathbf{B} \mathbf{K}_1) \mathbf{x}(t).$$

Let $\mathbf{A}_c = \mathbf{A} + \mathbf{B} \mathbf{K}_1$, then

$$\dot{\mathbf{x}}(t) = \mathbf{A}_c \mathbf{x}_c(t). \quad (17)$$

Since $\mathbf{A} + \mathbf{B} \mathbf{K}_1$ is Hurwitz, then \mathbf{A}_c is Hurwitz, so $\lim_{t \rightarrow T} \mathbf{x}_c(t) = 0$, i. e., $\lim_{t \rightarrow T} \mathbf{x}(t) = 0$. Through the above proof, the first statement of Theorem 2 takes place.

Denote $\bar{\mathbf{x}}_i(t) = \mathbf{x}_i(t) - \mathbf{X}_i \mathbf{v}$, $\bar{\boldsymbol{\eta}}_i(t) = \boldsymbol{\eta}_i(t) - \mathbf{v}(t)$, $\mathbf{U}_i(t) = \mathbf{K}_{1i} \mathbf{X}_i + \mathbf{K}_{2i}$, the time derivative of this $\bar{\mathbf{x}}_i(t)$ is

$$\begin{aligned} \dot{\bar{\mathbf{x}}}_i(t) &= \mathbf{A}_i \mathbf{x}_i(t) + \mathbf{B}_i \mathbf{u}_i + \mathbf{G}_i \mathbf{v}(t) - \mathbf{X}_i \mathbf{A}_v \mathbf{v}(t) = \\ &= \mathbf{A}_i (\bar{\mathbf{x}}_i(t) + \mathbf{X}_i \mathbf{v}(t)) + \mathbf{B}_i \mathbf{K}_{1i} (\bar{\mathbf{x}}_i(t) + \mathbf{X}_i \mathbf{v}(t)) + \\ &= \mathbf{B}_i \mathbf{K}_{2i} (\bar{\boldsymbol{\eta}}_i(t) + \mathbf{v}(t)) + \mathbf{G}_i \mathbf{v}(t) - \mathbf{X}_i \mathbf{A}_v \mathbf{v}(t) = \\ &= \mathbf{A}_i \bar{\mathbf{x}}_i(t) + \mathbf{A}_i \mathbf{X}_i \mathbf{v}(t) + \mathbf{B}_i \mathbf{K}_{1i} \bar{\mathbf{x}}_i(t) + \mathbf{B}_i \mathbf{K}_{1i} \mathbf{X}_i \mathbf{v}(t) + \\ &= \mathbf{B}_i \mathbf{K}_{2i} \bar{\boldsymbol{\eta}}_i(t) + \mathbf{B}_i \mathbf{K}_{2i} \mathbf{v}(t) + \mathbf{G}_i \mathbf{v}(t) - \mathbf{X}_i \mathbf{A}_v \mathbf{v}(t). \end{aligned}$$

When $\mathbf{v} = 0$, $\dot{\bar{\mathbf{x}}}_i = (\mathbf{A}_i + \mathbf{B}_i \mathbf{K}_{1i}) \bar{\mathbf{x}}_i(t) + \mathbf{B}_i \mathbf{K}_{2i} \bar{\boldsymbol{\eta}}_i(t)$, since $\lim_{t \rightarrow T} \bar{\boldsymbol{\eta}}_i(t) = 0$, then $\lim_{t \rightarrow T} \bar{\mathbf{x}}_i(t) = 0$.

Similarly, the output of a multi-agent is

$$\begin{aligned} \mathbf{e}_i(t) &= \mathbf{C}_i \mathbf{x}_i(t) + \mathbf{D}_i \mathbf{u}_i(t) + \mathbf{E}_i \mathbf{v}(t) = \\ &= \mathbf{C}_i (\bar{\mathbf{x}}_i(t) + \mathbf{X}_i \mathbf{v}(t)) + \mathbf{D}_i \mathbf{K}_{1i} (\bar{\mathbf{x}}_i(t) + \mathbf{X}_i \mathbf{v}(t)) + \\ &= \mathbf{D}_i \mathbf{K}_{2i} (\bar{\boldsymbol{\eta}}_i(t) + \mathbf{v}(t)) + \mathbf{E}_i \mathbf{v}(t) = \\ &= (\mathbf{C}_i + \mathbf{D}_i \mathbf{K}_{1i}) \bar{\mathbf{x}}_i(t) + \mathbf{D}_i \mathbf{K}_{2i} \bar{\boldsymbol{\eta}}_i(t) + (\mathbf{C}_i \mathbf{X}_i + \\ &= \mathbf{D}_i \mathbf{K}_{1i} \mathbf{X}_i + \mathbf{D}_i \mathbf{K}_{2i} + \mathbf{E}_i) \mathbf{v}(t). \end{aligned} \quad (18)$$

Since $\lim_{t \rightarrow T} \bar{\boldsymbol{\eta}}_i(t) = 0$ and $\lim_{t \rightarrow T} \bar{\mathbf{x}}_i(t) = 0$, according to Theorem 2, if and only if \mathbf{X}_i there is the solution of Eq. (16), and the solution is unique, then the above formula can be expressed as

$$\lim_{t \rightarrow T} \mathbf{e}_i(t) = (\mathbf{C}_i \mathbf{X}_i + \mathbf{D}_i \mathbf{U}_i + \mathbf{E}_i) \mathbf{v}(t). \quad (19)$$

If $\mathbf{C}_i \mathbf{X}_i + \mathbf{D}_i \mathbf{U}_i + \mathbf{E}_i = 0$ holds, get $\lim_{t \rightarrow T} \mathbf{e}_i(t) = 0$. If $\lim_{t \rightarrow T} \mathbf{e}_i(t) = 0$ holds, according to Assumption 2, one can find that $\lim_{t \rightarrow T} \mathbf{v}(t) \neq 0$, hence $\mathbf{C}_i \mathbf{X}_i + \mathbf{D}_i \mathbf{U}_i + \mathbf{E}_i = 0$. Thus the first and second statements of Theorem 2 can be proved.

Based on the above proofs, the distributed protocol and the finite time control algorithm are designed which can be used to solve the finite time output tracking problem of the multi-agent systems.

3 Experiments and results

3.1 Simulations example

In what follows, a simulation is provided to indicate the effectiveness of the finite time distributed control algorithm proposed in the previous section. Considering the cooperative output regulation of heterogeneous multi-agent for a kind of double-integrator systems with sinusoidal disturbances, including four agents and an external system, the dynamic equation of multi-agent is

$$\begin{aligned} \dot{\mathbf{x}}_{1i} &= \mathbf{x}_{2i}, \\ \dot{\mathbf{x}}_{2i} &= 0.5i\mathbf{v}_2 + \mathbf{u}_i, \quad i = 1, 2, 3, 4, \\ \mathbf{e}_i &= \mathbf{x}_{1i} - \mathbf{v}_1, \end{aligned}$$

and the exosystem is

$$\dot{\mathbf{v}}_1 = \mathbf{v}_2, \quad \dot{\mathbf{v}}_2 = -\mathbf{v}_1,$$

which is an unforced harmonic oscillator. The information exchange between agents is represented by a network topology as shown in Fig. 1, where node 0 is an external system and node i represents agent i .

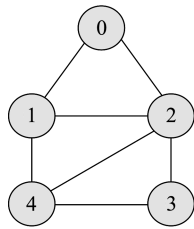


Fig. 1 Network topology for example

Taking the state variable $\mathbf{x}^T = (\mathbf{x}_{1i}, \mathbf{x}_{2i})$, the above system can be described with the same state space as Eq. (2),

$$\mathbf{A}_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{B}_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{C}_i = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

$$\begin{aligned} \mathbf{G}_i &= \begin{bmatrix} 0 & 0 \\ 0 & 0.5i \end{bmatrix}, \quad \mathbf{E}_i = \begin{bmatrix} -1 & 0 \end{bmatrix}, \\ \mathbf{A}_v &= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad \mathbf{D}_i = 0, \quad i = 1, 2, 3, 4. \end{aligned}$$

It can be seen that Assumptions 1–5 are satisfied. Solving the Eq. (16), the satisfied solution is $\mathbf{X}_i = \mathbf{I}_n$ and $\mathbf{U}_i = [-1, -0.5i]$. Since the controller gain matrix \mathbf{K}_{1i} satisfies $\mathbf{A} + \mathbf{B}\mathbf{K}_{1i}$ is Hurwitz, to be more specific, let $\mathbf{K}_{1i} = [-8, -4]$, then $\mathbf{K}_{2i} = \mathbf{U}_i - \mathbf{K}_{1i} = [7, 4 - 0.5i]$. Besides, let $\tau = 1$, $\sigma = 0.3$ and $\beta = 3$.

3.2 Results

The simulation results are shown in Figs. 2–11. By contrasting Figs. 2–5, the regulated output component $\mathbf{e}_{1i}(t)$ of the agent converges to 0 for about 6 s under the observer of Eq. (6), and the component $\mathbf{e}_{2i}(t)$ converges to 0 at approximately 6.5 s.

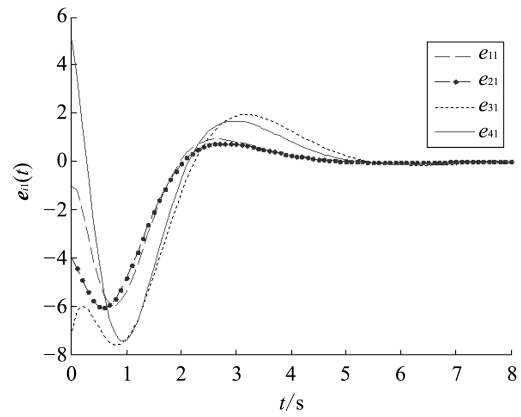


Fig. 2 $\mathbf{e}_{1i}(t)$ with adaptive distributed observer

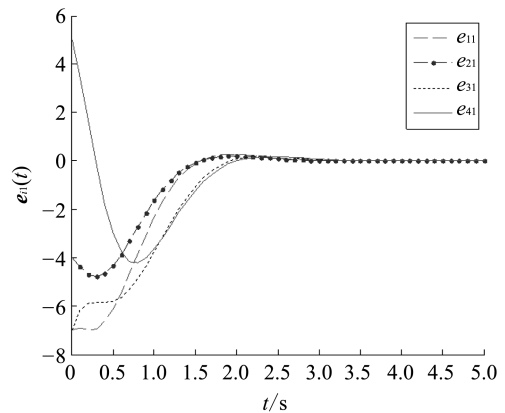


Fig. 3 $\mathbf{e}_{1i}(t)$ with finite-time distributed observer

Obviously, the regulation errors of all the agents with the finite time distributed observer of Eq. (8) can converge to 0 before $t = 2.4$ s. One can find that the convergence speed with this finite-time distributed observer increases nearly 2.6 times than

that using the adaptive distributed observer, and the overshoot reduces greatly.

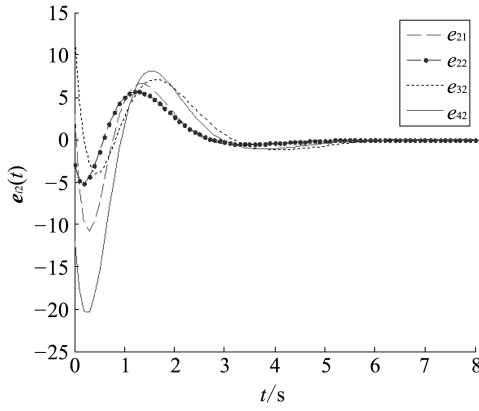


Fig. 4 $e_{i2}(t)$ with adaptive distributed observer

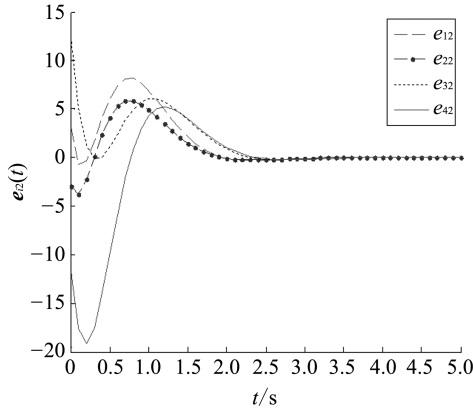


Fig. 5 $e_{i2}(t)$ with finite-time distributed observer

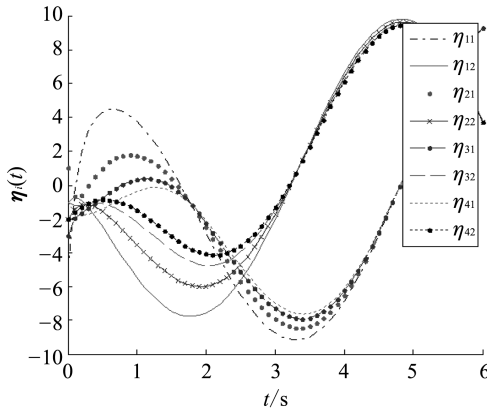


Fig. 6 $\eta_i(t) = [\eta_{i1}(t), \eta_{i2}(t)]^T$, with the adaptive distributed observer

As shown in Fig. 6, the component η_{i1} under the observer of Eq. (6) is tracked to the external system for about 3.5 s, and the component η_{i2} is tracked to the external system for about 5 s. It can be seen from Fig. 7 that the component η_{i1} under the observer of Eq. (8) is traced to the external system for about

0.6 s, and the component η_{i2} is traced to the external system for about 0.9 s, which is a good solution for the problem of fast tracking when a part of the agent is unable to obtain the external system signal. Similarly, as shown in Figs. 8–11, the states of all agents are quickly consistent with the states of the leader, i. e. , the external system in a limited time.

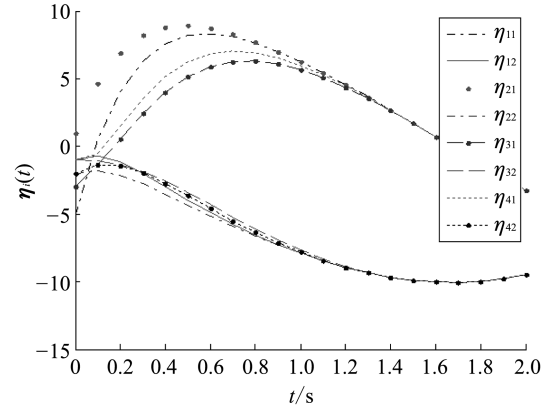


Fig. 7 $\eta_i(t) = [\eta_{i1}(t), \eta_{i2}(t)]^T$ with finite-time distributed observer

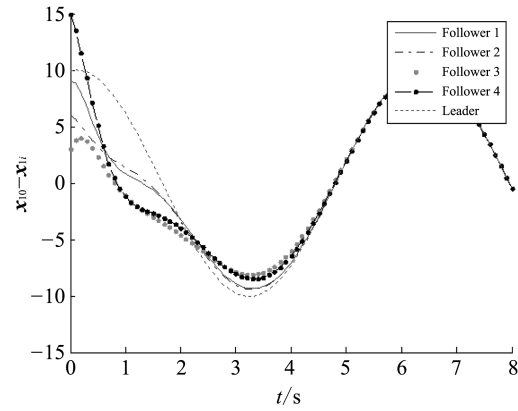


Fig. 8 $x_{i0} - x_{i1}$ with adaptive distributed observer

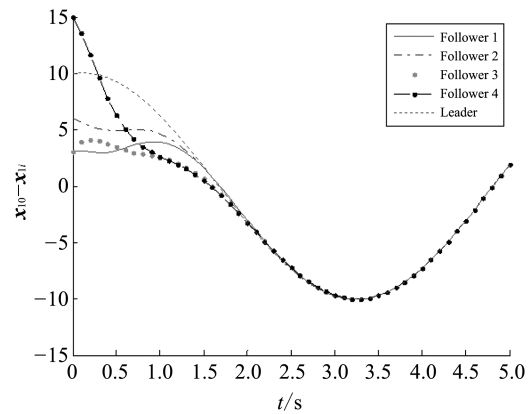


Fig. 9 $x_{i0} - x_{i1}$ with finite-time distributed observer

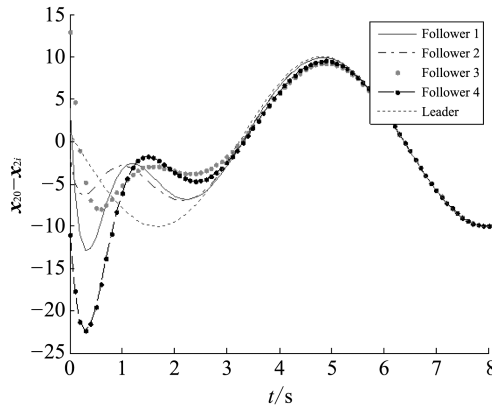


Fig. 10 $x_{20} - x_{2i}$ with adaptive distributed observer

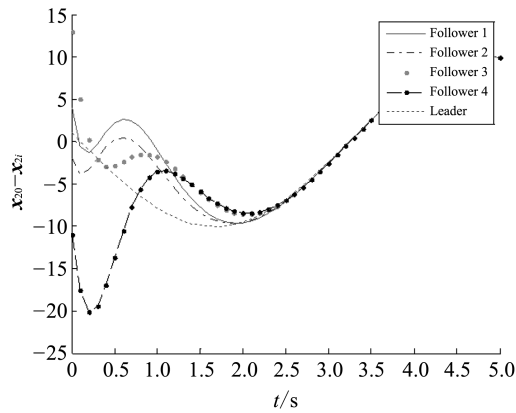


Fig. 11 $x_{20} - x_{2i}$ with finite-time distributed observer

4 Conclusion

This paper proposes a finite-time observer control algorithm to solve the problem of the heterogeneous multi-agent systems which can not converge in a limited time by using graph theory and Lyapunov theorem. The finite time stability of the system under this control algorithm is proved. The most prominent feature of this algorithm is that the designed observer can estimate the state of the external system in a limited time, so that the followers of the whole system can quickly track the external system and obtain the convergent characteristics of output regulation for a limited time.

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多智能体系统在有限时间内的协同输出跟踪

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摘 要: 研究了有限时间内多智能体系统的输出跟踪问题。为了使智能体能够在有限时间内快速跟踪并收敛到外部系统, 设计了一种基于有限时间状态观测器的新型分布式输出反馈控制策略。该分布式有限时间观测器不仅可以在智能体无法获得外部系统信号的情况下解决协同输出跟踪问题, 而且可以使系统获得较快的收敛性和良好的鲁棒性。最后, 基于 Lyapunov 函数证明了该系统在有限时间内的稳定性且提供了数值仿真实验, 证明了该算法和协议的有效性。

关键词: 多智能体系统; 有限时间观测器; 协同输出跟踪; 分布式输出反馈控制

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