Cognitive amplify-and-forward dual-hop relay networks over α-μ fading channels

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Abstract: This paper provides an analytic performance evaluation of dual-hop cognitive amplify-and-forward (AF) relaying networks over independent nonidentically distributed (i. n. i. d.) fading channels. Two different transmit power constraint strategies at the secondary network are proposed to investigate the performance of the secondary network. In the case of combined power constraint, the maximum tolerable interference power on the primary network and the maximum transmit power at the secondary network are considered. Closed-form lower bound and its asymptotic expression for the outage probability (OP) are achieved. Utilizing the above results, average symbol error probability (ABEP) at high signal-to-noise ratios (SNRs) are also derived. In order to further study the performance of dual-hop cognitive AF relaying networks, the Closed-form lower bounds and asymptotic expressions for OP with single power constraint of the tolerable interference on the primary network is also obtained. Both analytical and simulation are employed to validate the accuracy of the theoretical analysis. The results show that the secondary network obtains a better performance when higher power constraint is employed.

Key words: cognitive relay network; amplify-and-forward relay; outage probability; α-μ fading channels

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0 Introduction

Cognitive radio (CR) is a promising paradigm that can improve spectral efficiency. In CR networks, the secondary users (unlicensed or cognitive users) are allowed to share the spectrum of primary users if the interference on the primary users is below a predefined interference threshold. Recently, relay technologies are introduced into CR networks to improve the total throughput and coverage area of networks. Numerous relaying protocols have been incorporated into cognitive networks, e.g., amplify-and-forward (AF) and decode-and-forward (DF). For the DF cognitive relay networks (CRNs)[1], the outage probability of CRNs over Raleigh fading was evaluated, where interference constraint on primary users and maximum allowable transmission power constraints of secondary users are considered. In Ref. [2], an exact outage probability (OP) expression of DF CRNs over Nakagami-m fading was derived. In Ref. [3], the authors derived the exact expression for the outage probability of a dual-hop DF CRNs over Rayleigh fading with interference from primary users. A DF relay based spectrum-sharing network with interference constraints over general α-μ fading channel was studied by Arzykulov, et al.[4]. Recently, because of its simplicity, AF relaying protocols have attracted researchers' interest. Duong, et al. investigated the outage performance for AF cognitive relay networks over Rayleigh fading channels and gave the tight lower bounds and asymptotic expressions of OP for the AF CRNs over Nakagami-m fading channels[5-6]. Sharma, et al. Considered the AF CRNs with direct link and gave an approximate outage probability for the relay selection[7]. Yang, et al. gave a thorough performance analysis of a dual-hop AF based CRNs over independent nonidentically distributed (i. n. i. d) η-μ fading channels[8].

In all the aforementioned studies, small-scale fading is modeled by the Rayleigh [1, 3, 5] or the Nakagami-m [2, 6, 7] fading distributions. Although these models are useful, they have limitations to accurately fit all existing wireless environment[9]. Additionally, the adequacy of the Nakagami-m and the Rayleigh distributions has been questioned[10].

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Motivated by this, We analyze the performance of a dual-hop cognitive AF relaying network over α-μ fading channels under interference power constraint, where primary users coexist with secondary ones.

1 System model

A dual-hop cognitive AF relaying network is considered as shown in Fig. 1, which includes one secondary user (SU) source (S), one AF SU relay (R), one S destination (D), and one primary user (PU) destination (P).

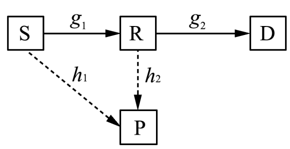

Fig. 1 Model for a dual-hop AF cognitive relay network

We assume that SU nodes are far away from the PU source, thus they do not impose any real interference, and all nodes are equipped with a single antenna and operate in half-duplex mode. The transmission from S to D is divided into two frames. In the first frame, S transmits information to R with transmission power P_S, then, the relay node R amplifies the message and forwards it to D with transmission power P_R. To ensure that the SUs and PU can coexist, the transmission powers at S and R are limited, so that S and R are allowed to transmit up to the maximum power P. Furthermore, the interference impinged on the PU receiver remains below the maximum tolerable interference power Q. Therefore, the transmission powers at S and R can be mathematically written as $P_S=\min\left(\frac{Q}{|h_1|^2},P\right)$ and $P_R=\min\left(\frac{Q}{|h_2|^2},P\right)$, where h_1 and h_2 represent the channel coefficients of the interference links S → P and R → P, respectively.

Considering these two transmission power constraints, the exact end-to-end instantaneous signal-to-noise ratio (SNR) at D is given by[5]

$$\gamma_D=\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2+1}, \tag{1}$$

$$\gamma_1=\min\left(\frac{\bar{\gamma}_Q}{|h_1|^2},\bar{\gamma}_P\right)|g_1|^2,$$

$$\gamma_2=\min\left(\frac{\bar{\gamma}_Q}{|h_2|^2},\bar{\gamma}_P\right)|g_2|^2, \tag{2}$$

where $\bar{\gamma}_Q=\frac{Q}{N_0}$, $\bar{\gamma}_P=\frac{P}{N_0}$, and N_0 is the noise variance.

Another situation is that when S and R are not power-limited terminals, the transmission power constraint only considers the PU tolerate level. In this case, $P_S=\frac{Q}{|h_1|^2}$ and $P_R=\frac{Q}{|h_2|^2}$, now the end-to-end instantaneous SNR at D can be rewritten as

$$\gamma_D^*=\frac{\gamma_3\gamma_4}{\gamma_3+\gamma_4+1}, \tag{3}$$

$$\gamma_3=\frac{\bar{\gamma}_Q|g_1|^2}{|h_1|^2},$$

$$\gamma_4=\frac{\bar{\gamma}_Q|g_2|^2}{|h_2|^2}. \tag{4}$$

Herein, we assume that all channel coefficients undergo i. n. i. d. α-μ fading. As a result, h_1, h_2, g_1 and g_2 are α-μ distributed with fading severity parameters α_{h_1}, α_{h_2}, α_{g_1}, α_{g_2} and μ_{h_1}, μ_{h_2}, μ_{g_1}, μ_{g_2}, respectively. Thus, the probability density function (PDF) and cumulative distribution function (CDF) of X, for $X\in\{|h_1|^2,|h_2|^2,|g_1|^2,|g_2|^2\}$, can be expressed respectively as

$$f_X(x)=\frac{\alpha\mu^{\mu}x^{\frac{\alpha\mu}{2}-1}}{2\hat{r}^{\alpha\mu}\Gamma(\mu)}\exp\left(-\frac{\mu x^{\frac{\alpha}{2}}}{\hat{r}^{\alpha}}\right), \tag{5}$$

$$F_X(x)=\frac{\gamma\left(\mu,\frac{\mu x^{\frac{\alpha}{2}}}{\hat{r}^{\alpha}}\right)}{\Gamma(\mu)}, \tag{6}$$

where $\Gamma(\cdot)$ and $\gamma(\cdot,\cdot)$ represent the Gamma function in Ref. [11, Eq. (8.310.1)] and the lower incomplete Gamma function in Ref. [11, Eq. (8.350.2)], respectively; $\alpha\in\{\alpha_{h_1},\alpha_{h_2},\alpha_{g_1},\alpha_{g_2}\}$; $\mu\in\{\mu_{h_1},\mu_{h_2},\mu_{g_1},\mu_{g_2}\}$; and $\hat{r}\in\{\hat{r}_{h_1},\hat{r}_{h_2},\hat{r}_{g_1},\hat{r}_{g_2}\}$.

2 Performance analysis and discussion

2.1 Combined power constraints: tolerable interference power at PU and maximum transmission power at SUs

2.1.1 Lower bound expression for OP

The OP is defined as the probability that the instantaneous SNR at D is below a predefined threshold γ_{th}, *i. e.*, $P_{out}=P(\gamma_D\leqslant\gamma_{th})$, where γ_D in Eq. (1) can be tightly upper bounded by $\gamma_D\leqslant\gamma_{up}=\min(\gamma_1,\gamma_2)$. Therefore, we get the lower bounds of OP, $P_{out}\geqslant F_{\gamma_{UP}}(\gamma_{th})=1-(1-F_{\gamma_1}(\gamma_{th}))(1-F_{\gamma_2}(\gamma_{th}))$. According to Eq. (2), since the derivations of the CDFs of γ_1 and γ_2 are similar, one can obtain one CDF from the other by replacing the corresponding parameters. The CDF of γ_1 is given as

$$F_{\gamma_1}(\gamma) = P\left\{\min\left(\frac{\bar{\gamma}_Q}{|h_1|^2}, \bar{\gamma}_P\right)|g_1|^2 \leqslant \gamma\right\} = P\left(|g_1|^2 \leqslant \frac{\gamma}{\bar{\gamma}_P}, \frac{\bar{\gamma}_Q}{|h_1|^2} \geqslant \bar{\gamma}_P\right) + P\left(\frac{|g_1|^2}{|h_1|^2} \leqslant \frac{\gamma}{\bar{\gamma}_P}, \frac{\bar{\gamma}_Q}{|h_1|^2} \leqslant \bar{\gamma}_P\right). \quad (7)$$

The first term of Eq. (7) can be rewritten as

$$I_1 = P\left(|g_1|^2 \leqslant \frac{\gamma}{\bar{\gamma}_P}, \frac{\bar{\gamma}_Q}{|h_1|^2} \geqslant \bar{\gamma}_P\right) = F_{|g_1|^2}\left(\frac{\gamma}{\bar{\gamma}_P}\right) F_{|h_1|^2}\left(\frac{\bar{\gamma}_Q}{\bar{\gamma}_P}\right), \quad (8)$$

where $F_{|g_1|^2}(\cdot)$ and $F_{|h_1|^2}(\cdot)$ can be obtained from Eq. (6), then I_1 can be expressed as

$$I_1 = \frac{\psi_1 \gamma\left(\mu_{g_1}, \frac{\mu_{g_1}}{\hat{r}_{g_1}^{\alpha_{g_1}}}\left(\frac{\gamma}{\bar{\gamma}_P}\right)^{\frac{\alpha_{g_1}}{2}}\right)}{\Gamma(\mu_{g_1})}, \quad (9)$$

where $\psi_1 = \gamma\left(\mu_{h_1}, \mu_{h_1}\left(\frac{\bar{\gamma}_Q}{\bar{\gamma}_P}\right)^{\frac{\alpha_{h_1}}{2}} \Big/ \hat{r}_{h_1}^{\alpha_{h_1}}\right) \Big/ \Gamma(\mu_{h_1})$.

The second term of Eq. (7) can be rewritten as

$$I_2 = \int_{\frac{\bar{\gamma}_Q}{\bar{\gamma}_P}}^{\infty} f_{|h_1|^2}(y) \int_0^{\frac{\gamma y}{\bar{\gamma}_P}} f_{|g_1|^2}(x)\,\mathrm{d}x\,\mathrm{d}y = \int_{\frac{\bar{\gamma}_Q}{\bar{\gamma}_P}}^{\infty} f_{|h_1|^2}(y) F_{|g_1|^2}\left(\frac{\gamma y}{\bar{\gamma}_Q}\right)\mathrm{d}y. \quad (10)$$

Using power series representation in Ref. [11, Eq. (8.354.1)] instead of the lower incomplete Gamma function, Eq. (10) can be rewritten as

$$I_2 = \varphi_1\left[\sum_{n=0}^{\infty} \frac{(-1)^n \mu_{g_1}^{\mu_{g_1}+n} \gamma^{\frac{\alpha_{g_1}(\mu_{g_1}+n)}{2}}}{\bar{\gamma}_Q^{\frac{\alpha_{g_1}(\mu_{g_1}+n)}{2}} \hat{r}_{g_1}^{\alpha_{g_1}(\mu_{g_1}+n)} \Gamma(n+1)(\mu_{g_1}+n)} \times \int_{\frac{\bar{\gamma}_Q}{\bar{\gamma}_P}}^{\infty} y^{\frac{\alpha_{h_1}\mu_{h_1}}{2}+\frac{\alpha_{g_1}(\mu_{g_1}+n)}{2}-1} \exp\left(-\frac{\mu_{h_1} y^{\frac{\alpha_{g_1}}{2}}}{\hat{r}_{h_1}^{\alpha_{g_1}}}\right)\mathrm{d}y\right], \quad (11)$$

where $\varphi_1 = \frac{\alpha_{h_1}\mu_{h_1}^{\mu_{h_1}}}{2\Gamma(\mu_{g_1})\Gamma(\mu_{h_1})\hat{r}_{h_1}^{\alpha_{h_1}\mu_{h_1}}}$. With the help of Ref. [11, Eq. (3.351.2)], I_2 can be obtained as

$$I_2 = \varphi_1\left[\sum_{n=0}^{\infty} \frac{(-1)^n \mu_{g_1}^{\mu_{g_1}+n} \gamma^{\frac{\alpha_{g_1}(\mu_{g_1}+n)}{2}}}{\bar{\gamma}_Q^{\frac{\alpha_{g_1}(\mu_{g_1}+n)}{2}} \hat{r}_{g_1}^{\alpha_{g_1}(\mu_{g_1}+n)}} \times \frac{\Gamma\left(\mu_{h_1} + \frac{\alpha_{g_1}(\mu_{g_1}+n)}{\alpha_{h_1}} + 1, \frac{\mu_{h_1}}{\hat{r}_{h_1}^{\alpha_{h_1}}}\left(\frac{\bar{\gamma}_Q}{\bar{\gamma}_P}\right)^{\frac{\alpha_{h_1}}{2}}\right)}{\Gamma(n+1)(\mu_{g_1}+n)\left(\frac{\mu_{h_1}}{\hat{r}_{h_1}^{\alpha_{h_1}}}\right)^{\mu_{h_1}+\frac{\alpha_{g_1}(\mu_{g_1}+n)}{\alpha_{h_1}}+1}}\right]. \quad (12)$$

Substituting the expressions of I_1 and I_2 into Eq. (7), the closed form of $F_{\gamma_1}(\gamma)$ can be derived as

$$F_{\gamma_1}(\gamma) = I_1 + I_2. \quad (13)$$

The CDF of γ_2 can be directly achieved by substituting the parameters of $F_{\gamma_2}(\gamma)$ for the corresponding parameters of $F_{\gamma_1}(\gamma)$ (i. e., $\alpha_{h_1} \to \alpha_{h_2}$, $\alpha_{g_1} \to \alpha_{g_2}$, $\mu_{h_1} \to \mu_{h_2}$, $\mu_{g_1} \to \mu_{g_2}$, $\hat{r}_{h_1} \to \hat{r}_{h_2}$, $\hat{r}_{g_1} \to \hat{r}_{g_2}$). Finally, the OP can be lower bounded by

$$P_{\text{out}} \geqslant 1 - (1 - I_1 - I_2)(1 - I_3 - I_4),$$

where

$$I_3 = \frac{\psi_2 \gamma\left(\mu_{g_2}, \frac{\mu_{g_2}}{\hat{r}_{g_2}^{\alpha_{g_2}}}\left(\frac{\gamma}{\bar{\gamma}_P}\right)^{\frac{\alpha_{g_2}}{2}}\right)}{\Gamma(\mu_{g_1})},$$

$$I_4 = \varphi_2 \sum_{n=0}^{\infty}\left[\frac{(-1)^n \mu_{g_2}^{\mu_{g_2}+n} \gamma^{\frac{\alpha_{g_2}(\mu_{g_2}+n)}{2}}}{\bar{\gamma}_Q^{\frac{\alpha_{g_2}(\mu_{g_2}+n)}{2}} \hat{r}_{g_1}^{\alpha_{g_1}(\mu_{g_1}+n)}} \times \frac{\Gamma\left(\mu_{h_2} + \frac{\alpha_{g_2}(\mu_{g_2}+n)}{\alpha_{h_2}} + 1, \frac{\mu_{h_2}}{\hat{r}_{h_2}^{\alpha_{h_2}}}\left(\frac{\bar{\gamma}^Q}{\bar{\gamma}_P}\right)^{\frac{\alpha_{h_2}}{2}}\right)}{\Gamma(n+1)(\mu_{g_2}+n)\left(\frac{\mu_{h_2}}{\hat{r}_{h_2}^{\alpha_{h_2}}}\right)^{\mu_{h_2}+\frac{\alpha_{g_2}(\mu_{g_2}+n)}{\alpha_{h_2}}+1}}\right]. \quad (14)$$

2.1.2 Asymptotic analysis for OP

To further analyze the impact of fading parameters and interference power constraint on the network performance, we turn our attention to the high-SNR regimes. Without loss of generality, we suppose that $\bar{\gamma}_Q = \theta\bar{\gamma}_P$, where θ is a positive constant, and the average SNR is defined as $\bar{\gamma} = \bar{\gamma}_P$. Considering the case that $x \to 0$, the incomplete Gamma function is simplified as

$$\gamma(b, ax) \backsimeq \frac{(ax)^b}{b}, \text{ as } x \to 0. \quad (15)$$

By substituting Eq. (15) into Eq. (6), we get

$$F_X(x) = \frac{\gamma\left(\mu, \frac{\mu x^{\frac{\alpha}{2}}}{\hat{r}^\alpha}\right)}{\Gamma(\mu)} \backsimeq \frac{\mu^\mu x^{\frac{\alpha\mu}{2}}}{\hat{r}^{\alpha\mu}\Gamma(\mu+1)}, \text{ as } x \to 0. \quad (16)$$

Then by plugging Eq. (16) into Eq. (8) and Eq. (10), the CDF of γ_1, $F_{\gamma_1}(\gamma)$ can be approximated as

$$F\gamma_1(\gamma) = \Phi_1 \times \left(\frac{\gamma}{\bar{\gamma}}\right)^{\frac{\alpha_{g_1}\mu_{g_1}}{2}} \quad \text{as } \bar{\gamma} \to \infty, \quad (17)$$

Simiarly,

$$F\gamma_2(\gamma) = \Phi_2 \times \left(\frac{\gamma}{\bar{\gamma}}\right)^{\frac{\alpha_{g_2}\mu_{g_2}}{2}} \text{ as } \bar{\gamma} \to \infty, \quad (18)$$

where

$$\Phi_1 = \frac{\gamma\left(\mu_{h_1}, \frac{\mu_{h_1}\theta^{\frac{\alpha_{h_1}}{2}}}{\hat{r}_{h_1}^{\alpha_{h_1}}}\right)\mu_{g_1}^{\mu_{g_1}}}{\Gamma(\mu_{h_1})\hat{r}_{g_1}^{\alpha_{g_1}\mu_{g_1}}\Gamma(\mu_{g_1}+1)} + \frac{\theta^{-\frac{\alpha_{g_1}\mu_{g_1}}{2}}\mu_{\mu_{h_1}h_1}\mu_{g_1}^{\mu_{g_1}}\Gamma\left(\mu_{h_1}+\frac{\alpha_{g_1}\mu_{g_1}}{\alpha_{h_1}}, \frac{\mu_{h_1}}{\hat{r}_{h_1}^{\alpha_{h_1}}}\theta^{\frac{\alpha_{h_1}}{2}}\right)}{\Gamma(\mu_{h_1})\hat{r}_{h_1}^{\alpha_{h_1}\mu_{h_1}}\Gamma(\mu_{g_1}+1)\hat{r}_{g_1}^{\alpha_{g_1}\mu_{g_1}}\left(\frac{\mu_{h_1}}{\hat{r}_{h_1}^{\alpha_{h_1}}}\right)^{\left(\mu_{h_1}+\frac{\alpha_{g_1}\mu_{g_1}}{\alpha_{h_1}}\right)}}, \tag{19}$$

$$\Phi_2 = \frac{\gamma\left(\mu_{h_2}, \frac{\mu_{h_2}\theta^{\frac{\alpha_{h_2}}{2}}}{\hat{r}_{h_2}^{\alpha_{h_2}}}\right)\mu_{g_2}^{\mu_{g_2}}}{\Gamma(\mu_{h_2})\hat{r}_{g_2}^{\alpha_{g_2}\mu_{g_2}}\Gamma(\mu_{g_2}+1)} + \frac{\theta^{-\frac{\alpha_{g_2}\mu_{g_2}}{2}}\mu_{\mu_{h_2}h_2}\mu_{g_1}^{\mu_{g_1}}\Gamma\left(\mu_{h_2}+\frac{\alpha_{g_2}\mu_{g_2}}{\alpha_{h_2}}, \frac{\mu_{h_2}}{\hat{r}_{h_1}^{\alpha_{h_1}}}\theta^{\frac{\alpha_{h_2}}{2}}\right)}{\Gamma(\mu_{h_2})\hat{r}_{h_1}^{\alpha_{h_2}\mu_{h_2}}\Gamma(\mu_{g_2}+1)\hat{r}_{g_2}^{\alpha_{g_2}\mu_{g_2}}\left(\frac{\mu_{h_2}}{\hat{r}_{h_2}^{\alpha_{h_2}}}\right)^{\left(\mu_{h_2}+\frac{\alpha_{g_2}\mu_{g_2}}{\alpha_{h_2}}\right)}}. \tag{20}$$

Further, if $\alpha_{h_1}=\alpha_{g_1}=2$, $\mu_{h_1}=m_{h_1}$, and $\mu_{g_1}=m_{h_1}$ in Eq. (17), after some mathematical simplifications, asymptotic expression for $F_{\gamma_1}(\gamma)$ under nakagami-m fading is obtained in Ref. [6, Eq. (16)]. The lower bound of OP is rewritten as $P_{\text{out}}=F_{\gamma_1}(\gamma_{\text{th}})+F_{\gamma_2}(\gamma_{\text{th}})-F_{\gamma_1}(\gamma_{\text{th}})F_{\gamma_2}(\gamma_{\text{th}})$. According to the fact that $F_{\gamma_1}(\gamma_{\text{th}})F_{\gamma_2}(\gamma_{\text{th}})\ll F_{\gamma_1}(\gamma_{\text{th}})+F_{\gamma_2}(\gamma_{\text{th}})$, after some simplifications and omission of the small terms, an appropriate expression of OP can be got as

$$P_{\text{out}} \backsimeq \phi_A\left(\frac{\gamma_{\text{th}}}{\bar{\gamma}}\right)^{\min\left(\frac{\alpha_{g_1}\mu_{g_1}}{2}, \frac{\alpha_{g_2}\mu_{g_2}}{2}\right)} \text{ as } \bar{\gamma}\to\infty, \tag{21}$$

where ϕ_A is given by

$$\phi_A = \begin{cases} \Phi_1, & \text{if } \frac{\alpha_{g_1}\mu_{g_1}}{2} < \frac{\alpha_{g_2}\mu_{g_2}}{2}, \\ \Phi_1+\Phi_2, & \text{if } \frac{\alpha_{g_1}\mu_{g_1}}{2} = \frac{\alpha_{g_2}\mu_{g_2}}{2}, \\ \Phi_2, & \text{if } \frac{\alpha_{g_1}\mu_{g_1}}{2} > \frac{\alpha_{g_2}\mu_{g_2}}{2}. \end{cases} \tag{22}$$

2.1.3 Average bit-error probability

The average bit-error probability $\bar{P}_{\text{be}}$ of a great variety of digital modulation schemes over fading channels can be calculated by Ref. [12, Eq. (12)] as

$$\bar{P}_{\text{be}} = \frac{1}{\sqrt{\pi}}\int_0^{\infty}\exp(-t^2)P_{\text{out}}\left(\frac{2t^2}{\beta}\right)\mathrm{d}t. \tag{23}$$

Substituting Eq. (18) into Eq. (23), using Ref. [11, Eq. (3.351.3)], we get the average bit-error probability at high SNRs as

$$\bar{P}_{\text{be}} = \frac{\phi_A\beta^{C-1}\Gamma\left(C+\frac{1}{2}\right)}{\sqrt{\pi}\bar{\gamma}^C\beta^C}, \tag{24}$$

where $C=\min\left(\frac{\alpha_{g_1}\mu_{g_1}}{2}, \frac{\alpha_{g_2}\mu_{g_2}}{2}\right)$; β depends on the specific modulation scheme in Ref. [13, Ch. 5], for example, by setting $\beta=1$ and $\beta=2$, the performance of quadrature phase-shift keying and binary phase-shift keying (BPSK) is obtained, respectively.

2.2 Single power constraint: tolerable interference power at PU

2.2.1 Tight lower bound for OP

In this section, as analysis in section 1.1, we are committed to investigate OP with the single power constraint of the interference on the primary network. Once again, the CDFs of γ_3 and γ_4 are very similar, and herein, we concentrate on the derivation of the CDF of γ_3, namely

$$F_{\gamma_3}(\gamma) = \int_0^{\infty}F_{|g_1|^2}\left(\frac{\gamma y}{\gamma_Q}\right)f_{|h_1|^2}(y)\mathrm{d}y. \tag{25}$$

By expanding $F_{|g_1|^2}\left(\frac{\gamma y}{\gamma_Q}\right)$ in the power series, with the aid of Ref. [11, Eq. (3.351.3)], Eq. (25) can be solved as

$$F_{\gamma_3}(\gamma) = \frac{\mu_{h_1}^{\mu_{h_1}}}{\Gamma(\mu_{g_1})\Gamma(\mu_{h_1})\hat{r}^{\alpha_{h_1}\mu_{h_1}}}\sum_{n=0}^{\infty}\left[\frac{(-1)^n\mu_{g_1}^{\mu_{g_1}+n}}{\bar{\gamma}_Q^{\frac{\alpha_{g_1}(\mu_{g_1}+n)}{2}}}\times \frac{\gamma^{\frac{\alpha_{g_1}(\mu_{g_1}+n)}{2}}\Gamma\left(\mu_{h_1}+\frac{\alpha_{g_1}(\mu_{g_1}+n)}{\alpha_{h_1}}\right)}{\hat{r}_{g_1}^{\alpha_{g_1}(\mu_{g_1}+n)}\Gamma(n+1)(\mu_{g_1}+n)\left(\frac{\mu_{h_1}}{\hat{r}_{h_1}^{\alpha_{h_1}}}\right)^{\mu_{h_1}+\frac{\alpha_{g_1}(\mu_{g_1}+n)}{\alpha_{h_1}}}}\right]. \tag{26}$$

Using the similar approach, the CDF of γ_r can be directly derived from the CDF of γ_3 after substituting the respective parameters by their counterparts (i.e., $\alpha_{h_1}\to\alpha_{h_2}$, $\alpha_{g_1}\to\alpha_{g_2}$ and $\mu_{h_1}\to\mu_{h_2}$, $\mu_{g_1}\to\mu_{g_2}$) as

$$F_{\gamma_4} = 1 - \frac{\mu_{h_2}^{\mu_{h_2}}}{\Gamma(\mu_{g_2})\Gamma(\mu_{h_2})\hat{r}^{\alpha_{h_2}\mu_{h_2}}} \times \sum_{n=0}^{\infty} \frac{(-1)^n \mu_{g_2}^{\mu_{g_2}+n} \gamma^{\frac{\alpha_{g_2}(\mu_{g_2}+n)}{2}} \Gamma\left(\mu_{h_2} + \frac{\alpha_{g_2}(\mu_{g_2}+n)}{\alpha_{h_2}}\right)}{\bar{\gamma}_Q^{\frac{\alpha_{g_2}(\mu_{g_2}+n)}{2}} \hat{r}^{\alpha_{g_2}(\mu_{g_2}+n)} \left(\frac{\mu_{h_2}}{\hat{r}_{h_2}^{\alpha_{h_2}}}\right)^{\mu_{h_1}+\frac{\alpha_{g_2}(\mu_{g_2}+n)}{\alpha_{h_2}}}}. \tag{27}$$

Thus, the lower bounds of OP with single power constraint is given by

$$P_{\text{out}} \geqslant 1 - [1 - F_{\gamma_3}(\gamma)][1 - F_{\gamma_4}(\gamma)].$$

2.2.2 Asymptotic OP analysis

For high values of SNRs, $F_{\gamma_3}(\gamma)$ can be approximated by

$$F_{\gamma_3}(\gamma) = \frac{\mu_{h_1}^{\mu_{h_1}} \mu_{g_1}^{\mu_{g_1}} \left(\frac{\gamma}{\bar{\gamma}\theta}\right)^{\frac{\alpha_{g_1}\mu_{g_1}}{2}}}{\Gamma(\mu_{h_1})\hat{r}_{h_1}^{\alpha_{h_1}\mu_{h_1}} \Gamma(\mu_{g_1}+1)\hat{r}_{g_1}^{\alpha_{g_1}\mu_{g_1}}} \times \frac{\Gamma\left(\mu_{h_1} + \frac{\alpha_{g_1}\mu_{g_1}}{\alpha_{h_1}}\right)}{\left(\frac{\mu_{h_1}}{\hat{r}_{h_1}^{\alpha_{h_1}}}\right)^{\left(\mu_{h_1}+\frac{\alpha_{g_1}\mu_{g_1}}{\alpha_{h_1}}\right)}} \quad \text{as } \bar{\gamma} \to \infty. \tag{28}$$

As discussed in the previous section, the CDF of γ_4 can be obtained easily. Then, similarly, the lower bounds for OP can be asymptotically expressed by

$$P_{\text{out}} \cong \phi_B \left(\frac{\gamma_{\text{th}}}{\bar{\gamma}}\right)^{\min\left(\frac{\alpha_{g_1}\mu_{g_1}}{2}, \frac{\alpha_{g_2}\mu_{g_2}}{2}\right)} \quad \text{as } \bar{\gamma} \to \infty, \tag{29}$$

where ϕ_B is given by

$$\phi_B = \begin{cases} \Phi_3, & \text{if } \frac{\alpha_{g_1}\mu_{g_1}}{2} < \frac{\alpha_{g_2}\mu_{g_2}}{2}, \\ \Phi_3 + \Phi_4, & \text{if } \frac{\alpha_{g_1}\mu_{g_1}}{2} = \frac{\alpha_{g_2}\mu_{g_2}}{2}, \\ \Phi_4, & \text{if } \frac{\alpha_{g_1}\mu_{g_1}}{2} > \frac{\alpha_{g_2}\mu_{g_2}}{2}, \end{cases} \tag{30}$$

$$\Phi_3 = \frac{\mu_{h_1}^{\mu_{h_1}} \mu_{g_1}^{\mu_{g_1}}}{\theta^{\frac{\alpha_{g_1}\mu_{g_1}}{2}} \Gamma(\mu_{h_1})\hat{r}_{h_1}^{\alpha_{h_1}\mu_{h_1}} \Gamma(\mu_{g_1}+1)\hat{r}_{g_1}^{\alpha_{g_1}\mu_{g_1}}} \times \frac{\Gamma\left(\mu_{h_1} + \frac{\alpha_{g_1}\mu_{g_1}}{\alpha_{h_1}}\right)}{\left(\frac{\mu_{h_1}}{\hat{r}_{h_1}^{\alpha_{h_1}}}\right)^{\left(\mu_{h_1}+\frac{\alpha_{g_1}\mu_{g_1}}{\alpha_{h_1}}\right)}}, \tag{31}$$

$$\Phi_4 = \frac{\mu_{h_2}^{\mu_{h_2}} \mu_{g_2}^{\mu_{g_2}}}{\theta^{\frac{\alpha_{g_2}\mu_{g_2}}{2}} \Gamma(\mu_{h_1})\hat{r}_{h_2}^{\alpha_{h_2}\mu_{h_2}} \Gamma(\mu_{g_2}+1)\hat{r}_{g_2}^{\alpha_{g_2}\mu_{g_2}}} \times \frac{\Gamma\left(\mu_{h_2} + \frac{\alpha_{g_2}\mu_{g_2}}{\alpha_{h_2}}\right)}{\left(\frac{\mu_{h_2}}{\hat{r}_{h_2}^{\alpha_{h_2}}}\right)^{\left(\mu_{h_2}+\frac{\alpha_{g_2}\mu_{g_2}}{\alpha_{h_2}}\right)}}. \tag{32}$$

2.2.3 Average bit-error probability analysis

By the way, one can get the asymptotical expression of average bit-error probability under single power constraint as

$$\bar{P}_{\text{be}} = \frac{\phi_B \beta^{C-1} \Gamma\left(C + \frac{1}{2}\right)}{\sqrt{\pi}\,\bar{\gamma}^C \beta^C}. \tag{33}$$

3 Numerical results and simulation

Using the aforementioned numerical analysis, the performance of cognitive AF relay networks is discussed here. The outage threshold γ_{th} is set at -3 dB, and the root mean value of the secondary network is given by $\hat{r}_{h_1} = \hat{r}_{h_2} = \hat{r}_{g_1} = \hat{r}_{g_2} = 2$. In order to confirm the tightness of the bounds, the curves obtained by Monte Carlo simulations are included. It can be seen that the asymptotic curves tightly converge to the lower bounds in the high SNR regimes, which meets our analysis.

Fig. 2 depicts the OP for the considered network with combined power constraint under α-μ fading channels.

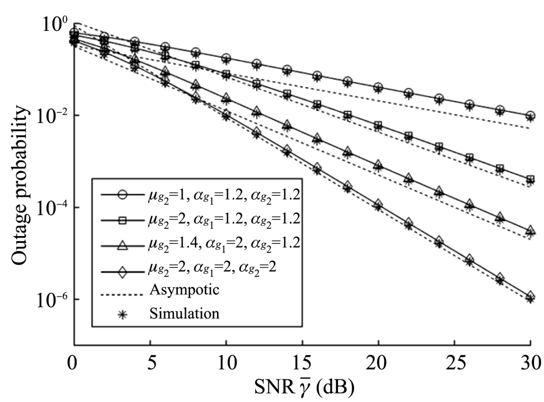

Fig. 2 OP for combined power constraint over α-μ fading channels with parameters $\alpha_{h_1} = \alpha_{h_2} = 3$, $\mu_{h_1} = 1$, $\mu_{h_2} = 2$ and $\mu_{g_1} = 2$

For $\alpha_{h_1} = \alpha_{h_2} = 3$, $\mu_{h_1} = 1$, $\mu_{g_1} = 2$, we investigate the impact of parameters μ_{g_2}, α_{g_1} and α_{g_2} on the performance of OP. As it shows, the outage performance improves as μ_{g_2} increases and/or α_{g_1} and α_{g_2} increase.

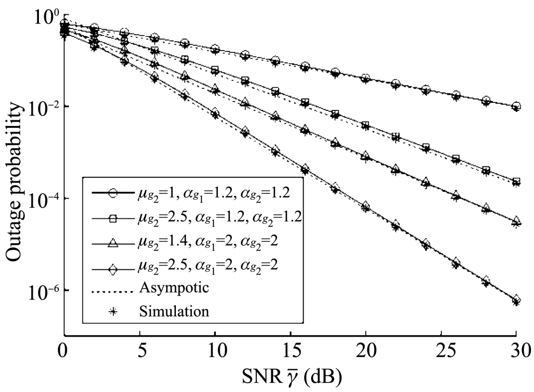

Fig. 3 OP of single power constraint over α-μ fading channels with parameters $\alpha_{h_1}=\alpha_{h_2}=3$, $\mu_{h_1}=1$, $\mu_{h_2}=2$ and $\mu_{g_1}=2$

Moreover, the outage probability decreases as the average SNR increases, which means the performance of the secondary network gets better as the values of tolerable power or/and the maximum allowable transmit power increase. Fig. 3 shows the OP performance of single power constraint of the interference on the primary network. In Fig. 3, we assume $\alpha_{h_1}=\alpha_{h_2}=3$, $\mu_{h_1}=1$, $\mu_{h_2}=2$, $\mu_{g_1}=2$ but the values of μ_{g_2}, α_{g_1} and α_{g_2} vary. One can see that the OP decreases as μ_{g_2} increases and/or α_{g_1} and α_{g_2} increase.

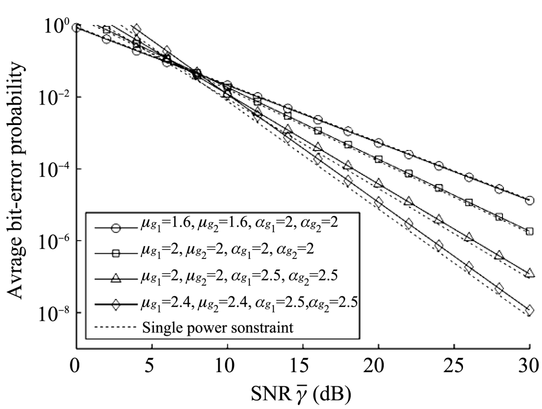

Fig. 4 Asymptotical $\bar{P}_{\rm be}$ of BPSK for considered networks over α-μ fading channels as a function of average SNR and for different values of μ_{g_1}, μ_{g_2}, α_{g_1} and α_{g_2} ($\alpha_{h_1}=\alpha_{h_2}=3$, $\mu_{h_1}=1$, $\mu_{h_2}=2$, $\beta=2$)

In Fig. 4, for $\alpha_{h_1}=\alpha_{h_2}=3$, $\mu_{h_1}=1$, $\mu_{h_2}=2$ and several values of μ_{g_1}, μ_{g_2}, α_{g_1} and α_{g_2}, the performance evaluation results show that $\bar{P}_{\rm be}$ improves as μ_{g_1}, μ_{g_2} and/or α_{g_1}, α_{g_2} inecrease. It also can be seen that the performance of single power constraint of CRNs is better than the combined power constraint.

4 Conclusion

In this paper, we have investigated the outage probability and average bit-error probability of a cognitive AF dual-hop relaying system in α-μ fading channels under two different transmit power constraint strategies. The closed lower bounds and its asymptotical expressions for OP with the combined power constraint or the single power constraint are attained. The results show that the performance of secondary network get better as the values of tolerable power or/and the maximum allowable transmit power increase.

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认知两跳 AF 中继网络在 α-μ 衰落信道下的性能分析

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摘　要： 基于独立非同分布 α-μ 衰落信道研究认知两跳放大转发(amplify-and forward, AF)中继网络的性能，分别在两种认知用户发射功率控制策略下分析二级用户网络性能。首先，综合考虑了主用户网络最大可容忍干扰功率和认知网络最大允许发射功率，分别给出了中断概率下界的封闭表达和渐近表达。利用以上结果，得出高信噪比下的平均误码率。为了进一步分析认知两跳 AF 中继网络的性能，在只考虑主用户网络最大可容忍干扰功率条件下，给出了中断概率下界闭式表达和渐近表达。最后，通过数值仿真及蒙特卡洛仿真验证了理论分析的准确性。结果表明，当主用户网络最大可容忍干扰功率或认知网络最大允许发射功率变大，二级用户网络的中断概率和误码率均降低。

关键词： 认知中继网络；放大转发中继；中断概率；α-μ 衰落信道

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