Influence of heat loss through probe electrical leads on thermal conductivity measurement with TPS method

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Abstract: The transient plane source (TPS) method is developed recently to measure the thermal conductivity of materials. In the measurement, the heating power is influenced by the heat which is transferred via the probe electrical leads. This fact further influences the measurement accuracy of thermal conductivity. To solve this problem, the influence of heat loss through the electrical leads on the heating power is studied theoretically. The mathematical formula of heat loss is deduced, and the corresponding correction model is presented. A series of measurement experiments on different materials have been conducted by using the hot disk thermal constant analyzer. The results show that the influence of the heat loss on the measurement is sensitive to different test materials and probes with different sizes. When the thermal conductivity of the material is greater than 0.2 W/(m·K), the influence of the heat loss is less than 0.16%, which can be ignored. As to the lower thermal conductivity materials, it is necessary to compensate the heat loss through the electrical leads, and the accuracy of thermal conductivity measurement can be effectively improved.

Key words: transient plane source (TPS) method; thermal conductivity; heat loss through electrical leads; heating power

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0 Introduction

Insulation materials play an important role in building energy conservation, aerospace insulation and automotive industry[1-3]. Thermal conductivity is a key parameter to evaluate the thermal insulation properties of insulation materials. Therefore, the accurate measurement of the thermal conductivity of insulation materials is of great significance.

The methods for measuring the thermal conductivity of materials are broadly classified as steady state methods and transient methods[4]. As to the steady state methods, the guarded hot plate (GHP)[5] method is regarded as the most commonly used method for measuring the thermal conductivity of insulation materials. The principle of this measurement is to establish a steady temperature gradient through a certain heat flux in the longitudinal one-dimensional direction of the sample. Then the thermal conductivity is calculated according to the Fourier law. Nevertheless, this technique requires a long time to establish a steady temperature gradient, and requires a relatively large sample size. The basic principle of the transient method is to heat the testing material in a short time. The thermal conductivity of the material is calculated by recording the surface temperature change of the probe with time. Compared with steady state methods, the transient methods have the advantages of short measurement time, a wide range of samples and high adaptability. At present, transient hot wire method[6] and transient plane source (TPS) method are the main applications of transient methods[7].

The TPS method is a transient method firstly developed by Gustafsson in 1991[8], which can obtain thermal conductivity, thermal diffusivity and volume specific heat capacity of the material simultaneously via a single transient measurement. Drawing on the ideal model of the TPS method, the heating power is assumed to be constant during the measurement. However, in fact, the heating power is influenced by the heat loss through the electrical leads. Therefore, the accuracy of thermal conductivity measurement will be reduced. Based on the ideal model of the TPS technique, this paper presents a model to correct the heat loss through the electrical leads so as to solve

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the foregoing problem. A series of experiments with different materials have been conducted within the proposed model. The results are presented and discussed in this paper.

1 Measurement principle of TPS method

The principle for measuring the thermal conductivity of material by the TPS method is based on the transient temperature response of the plane heat source in the infinite medium which is subjected to an abrupt electrical pulse[9]. The core element of this method is the TPS probe, which is composed of a bifilar spiral structure with an etching of metal foil, as seen in Fig. 1. The metal foil is covered on both sides by insulation film (Kapton or Mica) for protection and electrical insulation.

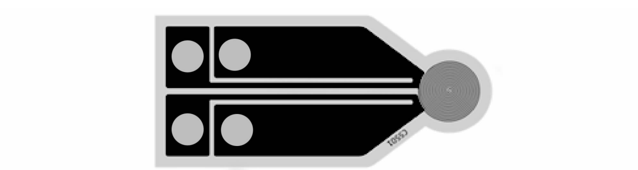

Fig. 1 TPS probe

During the experiment, the probe is tightly placed between two pieces of samples so that forming a sandwich structure as shown in Fig. 2. The probe not only acts as a heating element, but also as a temperature sensor for recording the temperature of the probe[10]. When a constant heating power is applied to the probe, due to the current heat effect, the probe temperature increases and the heat from the probe is transferred to the sample on both sides through conduction. By recording the changes of its voltage, the relations between the probe temperature and time can be obtained, which reflect the thermal properties of the testing material. In doing so, the thermal conductivity of the testing material can be calculated. In order to facilitate the theoretical analysis, the bifilar spiral structure is simplified into an equidistant concentric ring structure in the theoretical calculation.

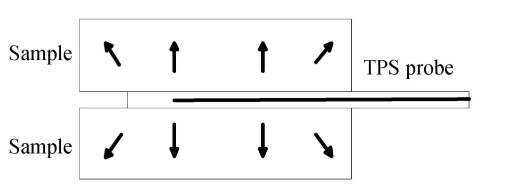

Fig. 2 Schematic diagram of measurement structure

The probe's temperature rises when electric current pass through the probe. The temperature increase at any point on the probe plane at time t is expressed as[11]

$$\Delta T(r,\tau) = \frac{P_0}{2\pi^{3/2}am(m+1)\lambda} \times \int_0^{\tau} \sum_{l=1}^{m} l e^{-\left[\frac{r^2/a^2+l^2/m^2}{4\sigma^2}\right]} I_0\left(\frac{rl}{2ma\sigma^2}\right)\frac{d\sigma}{\sigma^2}, \quad (1)$$

where P_0 is the heating power applied to the probe; λ is the thermal conductivity of the testing sample; a is the radius of the outermost ring of the probe; m is the number of concentric rings of the probe; $I_0(x)$ is the first class modified Bessel function of zero-order; τ is the non-dimensional time parameter defined as

$$\tau = (t/\theta)^{1/2} = \sqrt{\kappa t}/a, \quad (2)$$

$$\theta = a^2/\kappa, \quad (3)$$

where κ is the thermal diffusivity of the testing sample; θ is the characteristic time described by Eq. (3).

The total length of the metallic wire is

$$L = \sum_{l=1}^{m} 2\pi l \frac{a}{m} = (m+1)\pi a. \quad (4)$$

Then the average temperature increase of the probe surface can be obtained by averaging over the length of the concentric rings

$$\Delta T(\tau) = \frac{1}{L}\int_0^{2\pi} \Delta T(r,\tau) \sum_{k=1}^{m} \delta\left(r - \frac{k}{m}a\right) r d\theta = \frac{P_0}{\pi^{3/2}a\lambda} D(\tau), \quad (5)$$

where $D(\tau)$ is a dimensionless time function given by

$$D(\tau) = \frac{1}{m^2(m+1)^2} \times \int_0^{\tau} \sum_{k=1}^{m} k \sum_{l=1}^{m} l e^{-\left[\frac{k^2+l^2}{4m^2\sigma^2}\right]} I_0\left(\frac{kl}{2m^2\sigma^2}\right)\frac{d\sigma}{\sigma^2}. \quad (6)$$

Eq. (5) is the ideal model for the average temperature increase of the probe surface by TPS method.

During the measurement, the temperature increase of the probe surface is obtained by measuring the change of the probe resistance. The relationship between the resistance increase in probe and time is

$$R(t) = R_0[1 + \alpha \Delta \overline{T}(t)]. \quad (7)$$

From Eq. (7), the temperature increase of the probe surface can be obtained by

$$\Delta \overline{T}(t) = \frac{R(t) - R_0}{\alpha R_0}, \quad (8)$$

where $R(t)$ is the resistance of probe at time t; R_0 is the initial resistance of probe; α is the temperature coefficient of probe resistivity.

From Eq. (5), it is observed that when a constant heating power is applied to the probe, the average temperature increase $\Delta\overline{T}(\tau)$ of the probe surface is proportional to a function $D(\tau)$. Combining Eqs. (2) and (6), it can be seen that different κ corresponds to different $D(\tau)$. By an iteration procedure with thermal diffusivity κ, the obtained $D(\tau)$ is linearly fitted with the average temperature increase $\Delta\overline{T}(\tau)$ of the probe surface. The value of κ corresponding to the best-fit straight line is the thermal diffusivity of the sample. The thermal conductivity is determined from the slope of the fit straight line. Therefore, both thermal conductivity and thermal diffusivity of the sample can be obtained from a single transient measurement.

2 Correction for heat loss through electrical leads

In light of the ideal model of the TPS method, the heating power is considered to be constant. However, there is a heat transfer via the electrical leads during the measurement that will cause power loss. Consequently, the actual heating power of the sample is less than the rated input power of the probe, which reduces the accuracy of the thermal conductivity measurement. In order to improve the accuracy of this measurement, the heat loss caused by the heat transfer via the probe leads is analyzed and corrected. A corrected model of the average temperature increase of the probe surface is proposed.

The etched-out leads of the probe are of the shape depicted in Fig. 3.

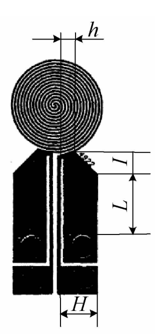

Fig. 3 Probe dimension diagram

With the assumptions that the lead pattern is wide enough to avoid self-heating and the ambient temperature remains constant during the measurement, the loss of power through the electrical leads ΔP_l is given by

$$\Delta P_l = 2\,\frac{\Delta\overline{T}_l}{R_l}, \tag{9}$$

where ΔT_l is the average temperature difference between the edge of the sensor and the points of contact with the heavy electrical leads in heating time; R_l is the heat transfer thermal resistance of the electrical leads.

To simplify the work with correction terms, it is assumed in Eq. (9) that the heat flow from the probe through the leads is controlled by the average temperature increase of the probe. However, the heat flow is in fact controlled by the temperature increase of the outer-most ring source of the probe. Hence, γ is introduced as a correction factor, so $\Delta\overline{T}_l$ can be expressed as

$$\Delta\overline{T}_l = \gamma\Delta\overline{T}, \tag{10}$$

$$\Delta\overline{T} = \frac{1}{t}\int_0^t \Delta\overline{T}(t)\,\mathrm{d}t, \tag{11}$$

where t is the measurement time; $\Delta\overline{T}$ is the average temperature increase of the probe in time t; $\Delta\overline{T}$ is the temperature increase of the probe at time t.

When heat transfer occurs on the leads, the heat transfer thermal resistance of the leads is

$$R_l = \frac{1}{d_m\lambda_m}\left[\frac{l}{H-h}\ln\left(\frac{H}{h}\right)+\frac{L}{H}\right]. \tag{12}$$

According to Eq. (9), the heat loss through the leads can be written as

$$\Delta P_l = \frac{2\gamma d_m\lambda_m\Delta T}{\left[\frac{L}{H}+\frac{l}{H-h}\ln\left(\frac{H}{h}\right)\right]}, \tag{13}$$

where d_m is the thickness of the metal used as sensing material of the probe. The influence of the insulation layers is ignored because the heat loss via the contacts to the metal pattern is normally higher than the conduction via the insulation layers; λ_m is the thermal conductivity of the metal material; H, h, L, l are given in Fig. 3; γ is a correction factor, which is determined by using a standard reference material with low thermal conductivity. The actual heating power can be written as

$$P = P_0 - \Delta P_l = P_0 - \frac{2\gamma d_m\lambda_m\Delta\overline{T}}{\left[\frac{L}{H}+\frac{l}{H-h}\ln\left(\frac{H}{h}\right)\right]}. \tag{14}$$

By replacing P_0 in Eq. (5) with P in Eq. (14), the average temperature increase of the TPS probe can be rewritten as

$$\Delta\overline{T}(\tau) = P_0 \frac{D(\tau)}{\pi^{3/2} a\lambda}\left(1 - \frac{\Delta P_l}{P_0}\right) =$$

$$\left[P_0 - \frac{2\gamma d_m \lambda_m \Delta T}{\left[\frac{L}{H} + \frac{l}{H-h}\ln\left(\frac{H}{h}\right)\right]}\right]\frac{D(\tau)}{\pi^{3/2} a\lambda}. \quad (15)$$

The above formula is the corrected probe temperature increase model after considering the leads heat loss. In the following, the influence of the heat loss through the electrical leads on the measurement of the thermal conductivity is studied experimentally based respectively on Eqs. (5), (13) and (15).

3 Experiment

3.1 Experimental materials

In order to evaluate the influence of the heat loss through the electrical leads on the thermal conductivity of different materials, the following eight materials were examined in this study, namely black rubber sheet, polymethyl methacrylate (PMMA), wood, marble, stainless steel, lead, industrial pure iron and brass. Three kinds of insulation materials, namely black rubber sheet, extruded polystyrene (XPS) and polyurethane, were selected for comparison experiments to compare the heat loss before and after correction. The standard values of thermal conductivity of these materials are polyurethane of 0.022 W/(m·K), XPS of 0.032 W/(m·K), black rubber sheet of 0.037 W/(m·K), PMMA of 0.2 W/(m·K), wood of 0.445 W/(m·K), marble of 1.822 W/(m·K), stainless steel of 14.5 W/(m·K), lead of 34.8 W/(m·K), industrial pure iron of 74.4 W/(m·K) and brass of 119.1 W/(m·K).

3.2 Experimental parameters

When performing the measurements, the TPS probe was sandwiched between two identical samples, as shown in Fig. 2. The probe simultaneously acted as the heat source and the temperature sensor. In order to ensure that the thermal effect of the probe was the only factor that caused the temperature increase of the sample, before the experiment, the experimental device was placed in a room with constant temperature so as to keep the experimental device consistent with ambient temperature.

In the TPS method, it is assumed that the probe is placed in a sample that is infinitely large. Therefore, the heat flow can't reach the sample's boundary. The distance travelled by the heat flow during the measurement is defined as the probing depth D by

$$D = \sqrt{\kappa t}, \quad (16)$$

which depends on the sample's thermal diffusivity κ and the measuring time t.

So the minimum size of the sample must meet

$$W > 2D + 2a, \quad (17)$$

$$H > D, \quad (18)$$

where W is the minimum width of the sample and H is the minimum thickness of the sample.

In addition, in order to obtain a stable thermal conductivity and thermal diffusivity, according to the sensitivity coefficient theory, the measurement time should be between 1/3 of the characteristic time and the entire characteristic time[12],

$$0.3 < \frac{\kappa t}{a^2} < 1. \quad (19)$$

The size of the sample in this paper meets the above requirements. One probe with a radius of 6.4 mm was used to measure PMMA, wood, marble and stainless steel. The rest of materials were measured by another probe with a radius of 14.6 mm. The relations of Eqs. (16)−(19) are used together to select a suitable heating time and heating power[13].

4 Results and analysis

Firstly, the eight different materials were measured by the hot disk thermal constant analyzer. The temperature response data of the probe with different materials were obtained by the data acquisition system. Relying on the obtained temperature response data, the heat loss through the leads ΔP_l under different materials was calculated according to Eq. (13). The experimental results are shown in Table 1.

Table 1 Heat loss through leads under different materials

Sample material	Thermal conductivity ($W \cdot m^{-1} \cdot K^{-1}$)	Heating power (W)	Heat loss (W)	$\Delta P_l/P_0$(%)
Black rubber sheet	0.037	0.07	0.000 73	1.04
PMMA	0.2	0.1	0.000 16	0.16
Wood	0.445	0.1	0.000 12	0.12
Marble	1.822	0.1	0.000 072	0.072
Stainless steel	14.5	1	0.000 15	0.015
Lead	34.8	2	0.000 16	0.008
Industrial pure iron	74.4	3	0.000 17	0.005 7
Brass	119.1	4.5	0.000 23	0.005 1

The proportion of heat loss in heating power $\Delta P_l/P_0$ under different materials was calculated from Table 1. The results are shown in Fig. 4.

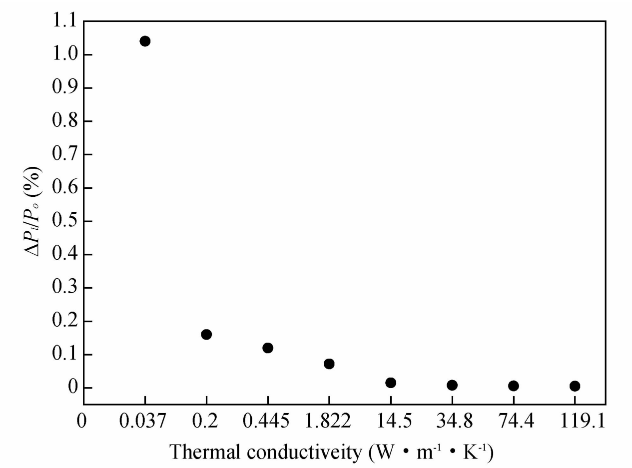

Fig. 4 Influence of heat loss through electrical leads on measurement under different materials

In line with Table 1 and Fig. 4, it can be found that the influence of the heat loss through the electrical leads on the thermal conductivity measurement (namely $\Delta P_l/P_0$) is different in the aforementioned eight materials. The influence of the heat loss increases when the thermal conductivity of the material decreases. The major reason of this finding is that the larger the thermal conductivity of the sample is, the faster the heat transfer rate is, the higher the heating power is required, and the corresponding temperature difference (the temperature difference between the hot disk and the leads) is relatively small. Eq. (9) shows that the heat loss is essentially proportional to the temperature difference between the hot disk and the leads, so $\Delta P_l/P_0$ is smaller. Therefore the following conclusions can be drawn: the influence of leads heat loss on the measurement increases with the decrease of the thermal conductivity of the material. When the thermal conductivity of the material is greater than 0.2 W/(m · K), the influence of leads heat loss on the measurement is less than 0.16%, which can be ignored. As for the materials with low thermal conductivity, the heat loss through the leads must be corrected.

Based on the above analysis, the thermal conductivities of black rubber sheet, XPS and polyurethane were measured respectively by the ideal probe temperature increase model and the corrected probe temperature increase model. The results of thermal conductivity values are listed in Table 2. Furthermore, the relative errors of thermal conductivity of black rubber sheet under different heating power from 0.07 W to 0.14 W with 0.01 W interval were studied with different models and different probes.

Table 2 Thermal conductivity measuring results with or without leads heat loss correction

Sample material	Probe radius (mm)	Ideal model		Corrected model	
		Thermal conductivity ($W \cdot m^{-1} \cdot K^{-1}$)	Relative error (%)	Thermal conductivity ($W \cdot m^{-1} \cdot K^{-1}$)	Relative error (%)
Black rubber sheet	14.6	0.038 29	3.50	0.037 91	2.46
XPS	14.6	0.033 27	3.98	0.032 89	2.78
Polyurethane	14.6	0.023 0	4.54	0.022 65	2.95

Columns 1, 3 and 5 of Table 2 indicate that the thermal conductivity values of the three kinds of insulation materials obtained by corrected model are less than that produced by ideal model. The column 6 shows that after the correction of the leads heat loss with Eq. (15), the relative errors are significantly reduced compared with the ideal model by 1.04%, 1.20%, 1.59% for the three materials respectively.

This implies that, on the one hand, the leads heat loss does have a great impact on the measurement of low thermal conductivity materials, and on the other hand, the proposed corrected model can effectively improve the measurement accuracy. The foregoing results are consistent with the previous conclusion that the influence of leads heat loss on the measurement increases with the decrease of thermal conductivity.

In the light of Fig. 5, regarding the same probe, the relative errors of the thermal conductivity of the black rubber sheet after correction are reduced. With regard to different probes, the relative errors of the thermal conductivity calculated by the ideal model with small size probe are greater than that with large size probe. The relative errors of the thermal conductivity calculated by the corrected model with different size probes are basically the same. This means that the proposed corrected model is suitable for probes with different sizes. For both probes, either before or after the correction for heat loss, the relative errors of thermal conductivity of black rubber sheet measured at different heating power vary little. Hence, the influence of the heat loss on the measurement of thermal conductivity is basically unchanged under different heating power.

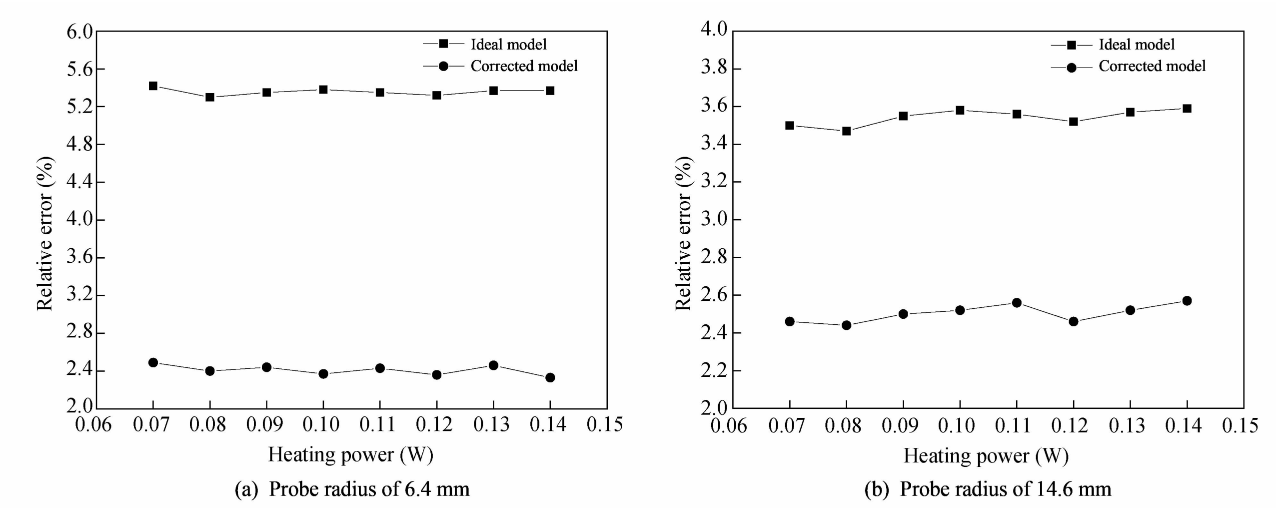

Fig. 5 Relative error of black rubber sheet thermal conductivity with or without leads heat loss correction

5 Conclusion

In this work, the heat loss through the electrical leads, which affects the measurement accuracy of thermal conductivity, has been studied. A model for correcting the heat loss is proposed. Based on a series of experiments, the following conclusions are drawn:

1) The influence of the heat loss on the measurement accuracy increases when the thermal conductivity of the material decreases, and the influence on materials with thermal conductivity greater than 0.2 W/(m·K) can be ignored.

2) When measuring the low thermal conductivity materials, it is necessary to correct the heat loss through the electrical leads, which can effectively improve the measurement accuracy. In addition, it is best to use a larger size probe so as to reduce the proportion of heat loss in the measurement, and thereby reducing the influence of heat loss through the probe leads on the measurement.

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探头引线热损失对 TPS 法测量导热系数的影响

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摘　要： 瞬态平面热源(Transient plane source，TPS)法是一种近年来发展起来的用于测量材料导热系数的方法。在测量过程中，加热功率受到探头引线传热的影响，进而会影响导热系数的测量准确度。针对这个问题，本文研究了测量过程中探头引线热损失对加热功率的影响，推导了热损失的数学计算公式，并提出了相应的修正模型。利用 hot disk 热常数分析仪对不同材料进行了一系列测量实验。实验结果表明引线热损失对测量的影响随着测量材料以及测试探头尺寸的不同而发生变化。当材料的导热系数大于 0.2 W/(m·K)时，探头引线热损失的影响小于 0.16%，可以忽略不计；但对于低导热系数材料的测量，对引线热损失进行补偿可以有效地提高导热系数的测量准确度。

关键词： 瞬态平面热源法；导热系数；引线热损失；加热功率

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