Kill probability modeling of multi-purpose guided missile against gunship and its application

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Abstract: Aimed at current deficiencies of multi-purpose guided missile kill probability model against gunship, the concept of the important coefficient of vulnerability blade unit is proposed in this paper. Laser fuze actuation model and warhead condition kill probability model of rotor blades are established by Monte Carlo method and kinetics theory with new ideas. Based on limited data, armor thickness of gunship is estimated, and a complete multi-purpose guided missile kill probability mathematical model is established, which provides necessary mathematical tool for the accurate and objective analysis of multi-purpose guided missile kill probability against gunship. Based on the establishment of the model, sensitivity analysis and optimal design of the main factors of multi-purpose guided missile kill probability are conducted, and the results show that the single multi-purpose guided missile lethality performance can be improved significantly by sensitivity analysis and optimization.

Key words: multi-purpose guided missile; gunship; rotor blade; the important coefficient of vulnerability blade unit; armor thickness; kill probability

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0 Introduction

With the continuous development of science and technology, multi-purpose guided missile becomes an important development direction of the next generation missile. There are two main methods to get kill performance of multi-purpose guided missile against gunship currently. One method focuses on computing kill probability of missile against the cabin of the gunship by using kinematics and ignoring rotors when the kill probability models of missile against gunship are established[1,2]; the other method focuses on the mechanical property changes and damage assessments of the cabin and the rotor blades by combining the finite element method with materials and dynamics to establish a collision model of gunship in the study of kinetic energy rod or other shape fragments collision with gunship[3-5].

This paper combines the advantages of the two methods, estimates and verifies armor thickness of gunship. We introduce the important coefficient of vulnerability blade unit and try out new ideas to establish laser fuze startup model of rotors of gunship and kill probability model of warhead against rotor blades. The kill probability model of multi-purpose guided missile against gunship is systematic and compact under the conditions to ensure a high analytical precision. Analysis of sensitivity and optimal designs of the main parameters that affect kill probability of multi-purpose guided missile against gunship are conducted with the model, which can provide basis for the design of multi-purpose guided missile.

1 Coordinate system

1.1 Cabin coordinates

We set AH-64D gunship as the example to establish the cabin coordinates of gunship $o_z x_z y_z z_z$, as shown in Fig. 1. Where o_z is the origin of coordinates, which locates in the gunship centroid; x_z axis points to the nose of gunship; y_z axis is perpendicular to the x_z axis and vertical upwards; z_z axis satisfies the right-hand rule.

According to the geometry distribution of AH-64D, the cabins of AH-64D are simplified as cuboid structure. Main parameters of cabins of AH-64D are shown in Table 1.

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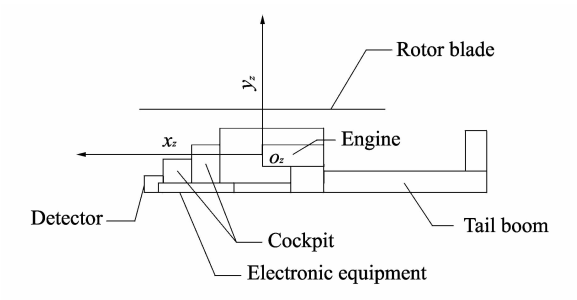

Fig.1 Diagram of cabin coordinates

Table 1 Equivalent data of key parts of AH-64D gunship

Key parts	Centroid coordinates(m)	$L\times W\times H$(m^3)
Detector	(4.3,0.5,0)	1.4×1.0×1.0
Electronic equipment(*L*)	(1.4,0.3,1.0)	3.7×0.7×0.9
Electronic equipment(*R*)	(1.4,0.3,−1.0)	3.7×0.7×0.9
Cockpit	(2.4,1.2.0)	2.4×1/2×1.4
Actuator and the fuel tank	(−0.7,1.7,0)	3.8×1.9×1.4
Engine(*L*)	(−1.3,1.4,1.2)	2.7×0.7×1.0
Engine(*R*)	(−1.3,1.4,−1.2)	2.7×0.7×1.0
Tail boom	(−5.0,0.7,0)	6.8×1.4×1.2

1.2 Rotor coordinates

It is assumed that four blades are in the same plane during the flight, and the missile velocity vector is consistent with the missile body axis. The rotor coordinates $o_r x_r y_r$ are defined in Fig.2.

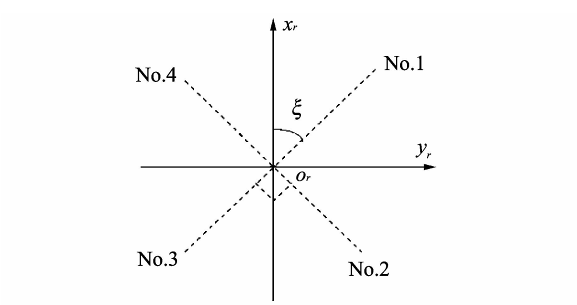

Fig.2 Rotor coordinates

It is assumed that the angle between No.1 rotor blade and x-axis of the rotor coordinates is ξ, when the furthest trigger point from the rotor shaft touches trigger side. Given that the randomness of the blade position and the consistency of the four blades properties, the angle ξ is subject to the uniform distribution [0°, 90°]. The parameters of AH-64D rotor geometry are shown in Table 2.

Table 2 Rotor geometry parameters of AH-64D

Length of blade (m)	Chord length (m)	M (kg)	RPM (r·s^{-1})
6.1	0.53	272.4	4.82

2 Laser fuze triggering process

2.1 Calculation of cabin laser fuze triggering process

With applications of the "trigger line" method[6], each cabin of the gunship is meshed. If intersections of the grids are taken as trigger points and grids are treated as vulnerable units, size of the grid is 0.01 m×0.01 m in this paper.

The trajectory of trigger points in the body coordinates is

$$(x - x_{b_i})/\nu_1 = (y - y_{b_i})/\nu_2 = (z - z_{b_i})/\nu_3, \tag{1}$$

where ν_1, ν_2, ν_3 are the target relative speed components of the missile in the missile body coordinates; $x_{b_i}, y_{b_i}, z_{b_i}$ are the coordinates component of trigger points in the missile body coordinates which take the center of fuze as the origin point.

The equations of trigger side in the missile body coordinates which take the center of fuze as the origin point are

$$\begin{cases} |\arctan(z/y)| \leqslant \dfrac{\Omega_s}{2}, \\ y^2 + z^2 = (x\tan\varphi_s)^2, \quad x \in [0, R_s\cos\varphi_s]. \end{cases} \tag{2}$$

The corresponding time t_i that is intersection $(x_{c_i}, y_{c_i}, z_{c_i})$ of the trigger points trajectory with trigger side is calculated. The judgment of whether the point $(x_{c_i}, y_{c_i}, z_{c_i})$ is in the trigger side is that the intersections are not only within the fuze field of vision, but also within the fuze detection distance.

2.2 Calculation of rotor blade laser fuze triggering process[7]

The trajectory of rotor shaft endpoint in the missile body coordinates is

$$(x - x_a)/\nu_1 = (y - y_a)/\nu_2 = (z - z_a)/\nu_3 = T, \tag{3}$$

where x_a, y_a, z_a are coordinate components of rotor shaft endpoint in the missile body coordinates; ν_1, ν_2, ν_3 are the target relative speed components of missile in the missile body coordinates; T is the time when endpoint of rotor shaft touches the fuze trigger side.

Bending deformation during rotation of rotor blade is ignored. Time T can be obtained by calculating intersection (x_b, y_b, z_b) of the trajectory of rotor shaft endpoint when endpoint of rotor shaft touches the fuze trigger side, as shown in Fig. 3.

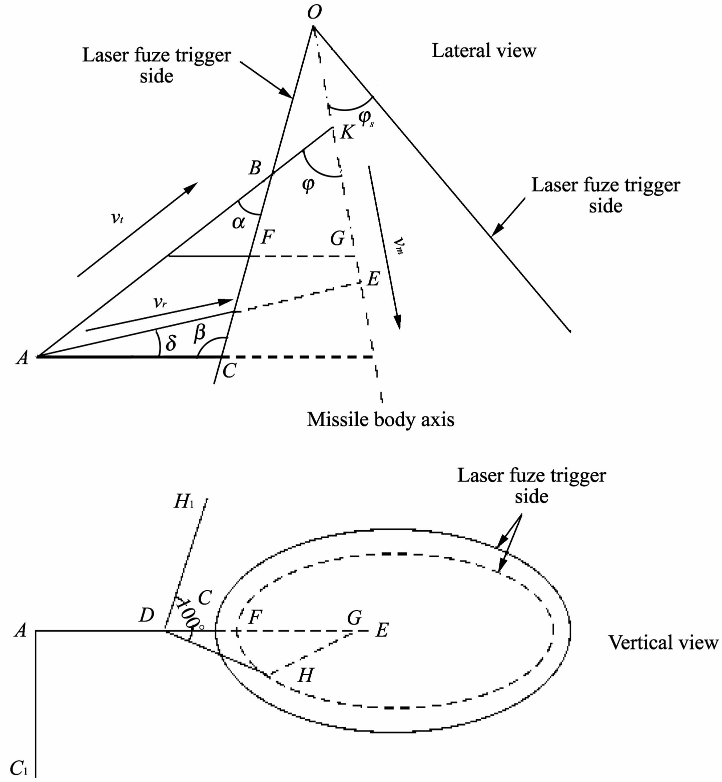

Fig. 3 Fuze start point of rotor blades geometric relationship

The relative motion of multi-purpose guided missile and the rotor blades is shown in Fig. 3. *OBC* is the fuze trigger side; δ is the angle between gunship velocity and the rotor plane; ν_r is the target relative velocity of the missile; ν_t is the gunship velocity; ν_m is the missile velocity respectively; γ is the intersection angle between missile and target; φ_s is the fuze inclination angle, $OC \leqslant R_s$; t is the time interval that the furthest trigger point of the adjacent rotor blade from the rotor blade shaft touches the trigger side; ω is rotor angular velocity; $AC = DH$ is the length of blade.

And there are

$$\begin{cases} \varphi = \arcsin[(\nu_t/\nu_r)\sin\gamma, \\ \theta = \pi - \gamma - \varphi + \delta, \\ \alpha = \varphi - \varphi_s, \\ \beta = \pi - \theta - \alpha, \\ AB = AC(\sin\beta/\sin\alpha), \\ AD = \nu_r t, \\ OB = \sqrt{x_b^2 + y_b^2 + z_b^2}, \\ DF = [(AB - AD)/AB] \cdot AC, \\ BC = \sqrt{AB^2 + AC^2 - 2AB \cdot AC\cos\theta}, \\ BF = (DF/AC) \cdot BC, \\ FG = [\sin\varphi_s/\sin(\beta - \varphi_s)] \cdot (BF + OB), \\ \sin(\omega t) = \dfrac{DH^2 + (DF + FG)^2 - GH^2}{2DH(DF + FG)}. \end{cases} \tag{4}$$

Since the time interval that rotor blade trigger point touches trigger side twice is short, so there is approximately relationship of $GH \approx FG$. To solve Eq. (4) can obtain the time interval t that the furthest trigger point of the adjacent rotor blade from the rotor blade shaft touches the trigger side. So the time t' that laser fuze begins to accumulate signal is

$$t' = T - AB/\nu_r + t \cdot \kappa, \tag{5}$$

where κ is a uniformly distributed random number between [0,1].

It is assumed that the laser fuze starts working when it is away from the target $L(L > 0)$ along the x-axis of relative velocity coordinates. The location of the start point of the laser fuze in the target relative coordinates is

$$\begin{cases} x = -L + \nu_r(t^n + \tau), \\ y = \rho\cos\eta_\theta, \\ z = \rho\sin\eta_\theta, \end{cases} \tag{6}$$

where ν_r is the relative velocity of missile and target; ρ, η_θ are miss distance and miss orientation respectively; τ is the average fuze delay time; $t^n = \min(t', t_i)$.

The angle between No. 1 rotor blade and x-axis of the rotor coordinates changes to $(\xi + \omega(t \cdot \kappa + \tau))$ when the laser fuze starts working.

3 Establishment of kill probability models

3.1 Modeling of kill probability of cabin

3.1.1 Equivalent armor thickness estimate of gunship

Armor thickness estimate requires thickness ratio of the composite armor, so the data acquisition is very difficult to obtained because of various reasons. Therefore, formulas and data are needed to be reverse derived and optimized to obtain armor thickness and thickness ratio which shoule be validated by experiments. According to U.S. military specifications for the bulletproof level requirements for the fifth class of lightweight ceramic/composites armor used for rotor aircraft: V_{50} is 1 600 ft/s, and incident angle is 0°[8].

Taking B_4C/Kevlar armor of AH-64D gunship as example, assuming that it is attacked by 12.7 mm flat tungsten alloy rod. The diameter of flat tungsten alloy rod is 12.7 mm; the length is 24 mm; penetration velocity is 487 m/s. The density of B_4C is 2 510 kg/m^3; the density of Kevlar is 1 650 kg/m^3; the ultimate tensile strength of Kevlar is 9.664×10^8 Pa; the maximum failure strain of Kevlar is 0.019.

Taking the areal density of B_4C/Kevlar as the objective function by the improved Ben-Dor formula[9], that is

$$\begin{aligned} &\text{find } h_1,\ h_2, \\ &\min\ \rho_1 h_1 + \rho_2 h_2, \\ &\text{s.t. } \frac{\alpha \varepsilon_2 \sigma_2 h_2 z[(\rho_1 h_1 + \rho_2 h_2)\pi(R + 2h_1)^2 + m]}{0.91\ \text{m}^2} = V_{50}, \end{aligned} \tag{7}$$

where V_{50} is the ballistic limit of B_4C/Kevlar; m is the quality of projectile; R is the radius of the tungsten alloy; h_1, h_2 are the thicknesses of panel and backplane; σ is ultimate tensile strength of Kevlar; ε is the maximum failure strain of Kevlar; ρ is the density; subscripts 1, 2 are panel and backplane; generally $a = 1$.

Using simulated annealing genetic algorithm (SAGA) to optimize the objective function, the optimization process is shown in Fig.4.

The optimum areal density is 63.95 kg/m^2; the optimum thicknesses of panel is 19.4 mm; the optimum thickness of backplane is 9.3 mm.

Equivalent thickness of duralumin h_d can be obtained after getting armor thickness ratio according to the Eq.(8)[10]

$$h_d = \eta_1 = \frac{\sigma_{b_1}}{\sigma_{bc}} h_1 + \eta_2 \frac{\sigma_{b_2}}{\sigma_{bc}} h_2, \tag{8}$$

where σ_{b_1}, σ_{b_2} are ultimate strength of panel and backplane; σ_{bc} is ultimate strength of standard duralumin; η_1, η_2 are equivalent correction factors of panel and backplane; h_1, h_2 are actual thicknesses of panel and backplane.

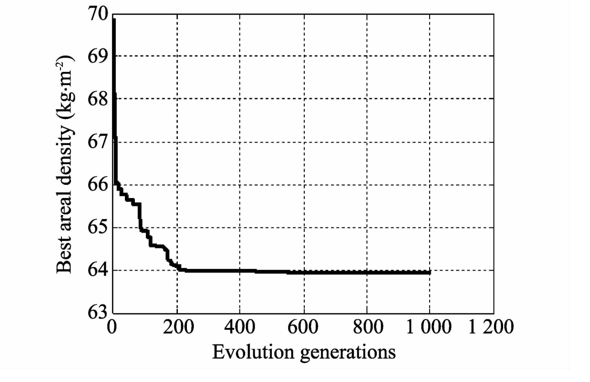

Fig. 4 Optimization process of surface density by SAGA

3.1.2 Kill probability of fragments

Residual velocity and residual mass of fragments estimation can be obtained by THOR equation based on the experimental data fitting[11]. THOR equation can apply to the steel fragments that velocity is less than 2 500 m/s, and the aspect ratio is less than 3 penetrate metallic and non-metallic target plate, it is

$$\begin{cases} v_r = v_s - 0.304\,8 \times 10^{c_{11}}(61\,023.75 h_d A_s)^{c_{12}} \times \\ \qquad (15\,432.1 m_s)^{c_{13}}(\sec\theta)^{c_{14}}(3.280\,84 \nu_s)^{c_{15}}, \\ m_r = m_s - 6.48 \times 10^{c_{21}-5}(61\,023.75 h_d A_s)^{c_{22}} \times \\ \qquad (15\,432.1 m_s)^{c_{23}}(\sec\theta)^{c_{24}}(3.280\,84 \nu_s)^{c_{25}}, \end{cases} \tag{9}$$

where ν_r is average residual velocity of the fragments after penetrating the target; m_r is average residual mass; ν_s is average dynamic striking velocity; h_d is equivalent thickness of duralumin; A_s is the contact area when penetrating the target; m_s is initial mass of the fragments; θ is the average bank angle when striking the target; $c_{11} - c_{25}$ are coefficients of THOR equation of different materials.

Kill Probability of fragments $P_{ac_1,i}$ can be calculated in accordance with the ratio dynamic energy formula in the case of lacking relevant experimental data

$$P_{ac_1,i} = F(E_i) \cdot K_i, \tag{10}$$

where $F(E_i)$ is the empirical formula; K_i is correction factor associated with vulnerability of the cabin.

Generally, cumulative kill probability is used to calculat kill probability of fragments $P_{ac_2,i}$ of vulnerable units. The vulnerable unit is not damaged when the number of hit fragments of i-th vulnerable unit K_i is less than n_i, but it is inevitable damaged when the number k_i is more than m_i.

$$P_{ac_2,i}(K_i) = \begin{cases} 0, & K_i \leqslant n_i, \\ (K_i - n_i)/(m_i - n_i), & n_i < K_i < m_i, \\ 1, & K_i \geqslant m_i, \end{cases} \tag{11}$$

where m_i and n_i are related with position of vulnerable unit on the rotor blade and the mass and velocity of the fragments.

Kill probability of fragments of the cabin P_{ac} is

$$P_{ac} = 1 - \prod_{i=1}^{N_{c_1}} (1 - P_{ac_1,i}) \prod_{i=1}^{N_{c_2}} (1 - P_{ac_2,i}), \tag{12}$$

where N_{c_1}, N_{c_2} are the numbers of vulnerable units mentioned above.

3.1.3 Kill probability of shockwave

Overpressure and specific impulse of shockwave can be obtained in Ref.[12]. Kill probability of shockwave of vulnerable units is

$$P_{bc,i} = \begin{cases} 1, & \Delta P_i \geqslant \Delta P^* \text{ and } I_i \geqslant I^* \text{ and } D_i \geqslant K, \\ 0, & \Delta P_i < \Delta P^* \text{ or } I_i < I^* \text{ or } D_i < K, \end{cases} \tag{13}$$

where ΔP_i, I_i are overpressure and specific impulse of shockwave on the i-th vulnerable unit; ΔP^*, I^* are critical overpressure and critical specific impulse of wavefront when the target is destroyed; $D_i = (\Delta P_i - \Delta P^*)(I_i - I^*)$, $K = \alpha(\Delta P^*/2 + I^*/2)^{\beta}$, α, β are related with materials of blade and the damage level, and they can be obtained experimentally.

Kill probability of shockwave P_{by} is

$$P_{by} = 1 - \prod_{i=1}^{N_c} (1 - P_{by,i}), \tag{14}$$

where $P_{by,i}$ is kill probability of shockwave of vulnerable units.

3.2 Modeling of kill probability of rotor blades

3.2.1 Important coefficient of vulnerability blade unit

This paper introduces the important coefficient of vulnerability blade unit λ. The coefficient is used to determine whether the blade is incapable when a vulnerable unit along the rotor blade is damaged. The coefficient is related with rotor structure, materials and spanwise location of the vulnerable units along the rotor blades, and can be obtained experimentally.

For the 60 vulnerable units along the blade spanwise, 50 rotor instability experiments after destruction are conducted respectively, and average results are shown in Table 3.

Table 3 Part of instability experimental data of rotor vulnerable units after the destruction

x_i(m)	λ_{ij}	x_i(m)	λ_{ij}	x_i(m)	λ_{ij}
1.3	0.977	1.5	0.996	1.7	1.000
3.7	0.992	3.9	1.000	4.1	0.941
6.1	0.620	6.3	0.569	6.5	0.456

The least-squares curve is used to fit the experimental data, and the function prototype is

$$\lambda_{ij}(x_i) = 1 - 1/(1 + e^{-(ax_i+b)}), \tag{15}$$

where a, b are fitting constants; x_i is the distance of the i-th vulnerable unit of each rotor blade from the shaft; $i = 1,2,\cdots,N$ is the serial number of vulnerable units, N is the total number of vulnerable units of each rotor blade; $j = 1,2,3,4$ is the serial number of four rotor blades.

Fitting curve of the important coefficient λ_{ij} of vulnerability blade unit of AH-64D is shown in Fig.5.

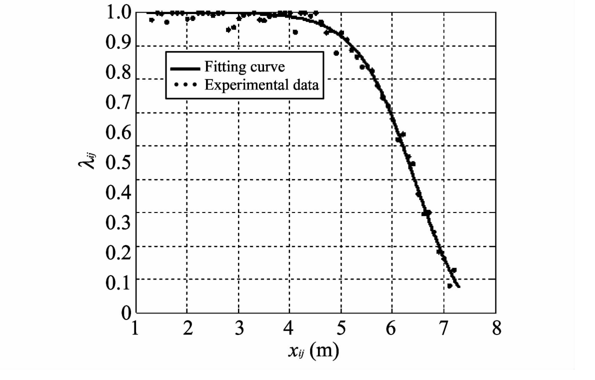

Fig. 5 Fitting curve of important coefficient λ_{ij} of vulnerability blade units

Fitting parameters are obtained as $a = 1.796, b = -11.55$, so

$$\lambda_{ij} = \lambda_{ij}(x_i) = 1 - 1/(1 + e^{-(1.796x_i - 11.55)}). \tag{16}$$

From Fig.5, the important coefficient of vulnerability blade unit λ decreases with an increasing distance along the blade spanwise, then it remains at about 1 m before 4.5 m and decreases obviously after 4.5 m, which means the closer the destroyed vulnerability unit from the shaft is, the greater the kill probability of the rotor blade is. And the best detonation point of warhead should be near the rotor shaft.

3.2.2 Kill probability of fragments

Kill probability of fragments P_{ar} can be obtained by the related theories after introducing the important coefficient of vulnerability blade unit λ. That is

$$P_{ar} = 1 - \prod_{i_1=1}^{N}(1-\lambda_{i_1}P_{a,i_1})\prod_{i_2=1}^{N}(1-\lambda_{i_2}P_{a,i_2})\prod_{i_3=1}^{N}(1-\lambda_{i_3}P_{a,i_3})\prod_{i_4=1}^{N}(1-\lambda_{i_4}P_{a,i_4}), \quad (17)$$

where P_{a,i_1}, P_{a,i_2}, P_{a,i_3}, P_{a,i_4} are kill probabilities of fragments of vulnerability units of the four blades; λ_{i_1}, λ_{i_2}, λ_{i_3}, λ_{i_4} are the important coefficients of vulnerability blade units.

3.2.3 Kill probability of shockwave

The destruction effect of rotor blade shockwave is divided into "overpressure-specific impulse" destruction and "bending-shearing" destruction. Kill probability of i-th blade vulnerability unit is

$$\begin{cases} P_{by,i} = \begin{cases} 1, & \Delta P_i \geqslant \Delta P^* \text{ and } I_i \geqslant I^* \text{ and } D_i \geqslant K, \\ 0, & \Delta P_i < \Delta P^* \text{ or } I_i < I^* \text{ or } D_i < K, \end{cases} \\ P_{bs,i} = \begin{cases} 1, & \sigma_{\max,i} > [\sigma] \text{ or } \tau_{\max,i} > [\sigma]/2 \text{ or } \sigma_{r_3,i} > [\sigma], \\ 0, & \sigma_{\max,i} \leqslant [\sigma] \text{ and } \tau_{\max,i} \leqslant [\sigma]/2 \text{ and } \sigma_{r_3,i} \leqslant [\sigma], \end{cases} \end{cases} \quad (18)$$

where ΔP_i, I_i, ΔP^*, I^*, K are the same with Eq.(13); $[\sigma]$ is allowable stress of the materials; $\sigma_{\max,i}$, $\tau_{\max,i}$ are the maximum normal stress and the maximum shear stress of the i-th vulnerability units; $\sigma_{r_3,i} = \sqrt{\sigma_{\max,i}^2 + r\tau_{\max,i}^2}$.

Kill probabilitiy of blade destruction is

$$\begin{cases} P_{by} = 1 - \prod_{i_1=1}^{N}(1-\lambda_{i_1}P_{by,i_1})\prod_{i_2=1}^{N}(1-\lambda_{i_2}P_{by,i_2})\prod_{i_3=1}^{N}(1-\lambda_{i_3}P_{by,i_3})\prod_{i_4=1}^{N}(1-\lambda_{i_4}P_{by,i_4}) \\ P_{bs} = 1 - \prod_{i_1=1}^{N}(1-\lambda_{i_1}P_{bs,i_1})\prod_{i_2=1}^{N}(1-\lambda_{i_2}P_{bs,i_2})\prod_{i_3=1}^{N}(1-\lambda_{i_3}P_{bs,i_3})\prod_{i_4=1}^{N}(1-\lambda_{i_4}P_{bs,i_4}) \end{cases} \quad (19)$$

where P_{by,i_1}, P_{by,i_2}, P_{by,i_3}, P_{by,i_4} are the kill probabilities of "overpressure-specific impulse" destruction of four blades; P_{bs,i_1}, P_{bs,i_2}, P_{bs,i_3}, P_{bs,i_4} are the kill probabilities of "bending-shearing" destruction of four blades.

Kill probability of shockwave P_{br} is

$$P_{br} = 1 - (1-P_{by})(1-P_{bs}). \quad (20)$$

3.2.4 Collision analysis of rotor blades against missile

Collision injury of missile against the rotor blades is mainly caused by relative speed. Fracture of the blades caused by the collision is considered only. Impact damage of material is related with fracture toughness K_{IC}, cross section of the blades and impact value a_K. In the district of V-gap platform[13]

$$\begin{cases} \left(\dfrac{K_{IC}}{q_s}\right)^2 = 0.52\left(\dfrac{CVN}{q_s} - 0.02\right) \\ a_K = \dfrac{CVN}{S} \\ E_\nu = a_K S' \end{cases} \quad (21)$$

where q_s is yield strength of the material; CVN is charpy all-sample impacting energy of the material; S is cross section of the charpy sample; S' is cross section of the rotor blades; E_v is energy consumption when the rotor blades are broken.

So kill probabilities of the rotor blades collided with missile P_{dr} is

$$P_{dr} = \begin{cases} 1, & E > E_v, \\ E/E_\nu, & E \leqslant E_v, \end{cases} \quad (22)$$

where E is kinetic energy of rotor blades collided with the missile.

It is supposed that the warhead detonates when the missile collides with the rotor blades.

4 Calculation process of kill probability model

Based on the analysis mentioned before, condition kill probability $P_{K,i}$ of the i-th Monte Carlo sampling is

$$P_{K,i} = 1 - (1-P_{ar,i})(1-P_{br,i})(1-P_{dr,i})(1-P_{ac,i})(1-P_{bc,i}), \quad (23)$$

where the calculation of $P_{ar,i}$, $P_{br,i}$, $P_{dr,i}$, $P_{ac,i}$, $P_{bc,i}$ can be obtained by Eqs.(11)-(22); If the missile hits the target directly, so $P_{K,i} = 1$.

Kill probability of the multi-purpose guided missile against gunship can be obtained by the models deduced before, that is

$$P_K = \sum_{i=1}^{N}\frac{P_{K,i}}{N}, \quad (24)$$

where N is the total number of Monte Carlo sampling.

Kill probability calculation process of multi-purpose guided missile against gunship is shown in

Fig. 6 with specific steps as follows:

1) Inputting main parameters of fuze and warhead of multi-purpose guided missile: fuze inclination angle φ_s, average fuze delay time τ, filling coefficient of warhead K_a, number of fragments N, mass of single fragment m, static scattering angle of fragments φ_α and static scattering direction angle of fragments φ_β.

2) Inputting guidance precision of multi-purpose guided missile, and conducting guidance precision sampling and position of rotor blades sampling.

3) Calculating laser fuze start time t''.

4) Calculating detonation point of warhead (x, y, z).

5) Estimating armor thicknesses of gunship h_1, h_2; Estimating armor equivalent thickness of duralumin h_d; Calculating the important coefficients of vulnerability blade units $\lambda_1, \lambda_2, \lambda_3, \lambda_4$.

6) Calculating kill probabilities of fragments of the cabin and rotor blades $P_{ac,i}$, $P_{ar,i}$.

7) Calculating the parameters of shockwave of warhead ΔP, I, ΔP^*, I^*; Calculating shockwave kill probabilities of the cabin and rotor blades $P_{bc,i}$, $P_{br,i}$.

8) Calculating the kinetic energy of rotor blades collided with missile E; calculating the energy consumption E_v when the rotor blades are broken; calculating the kill probability of rotor blades collided with missile $P_{dr,i}$.

9) Calculating the condition kill probability of the i-th Monte Carlo sampling $P_{K,i}$.

10) Judging whether the sampling frequency reached the maximum limit, if yes it is transferred to 11), otherwise transferred to 2).

11) Clculating kill probability of the multi-purpose guided missile against gunship P_K.

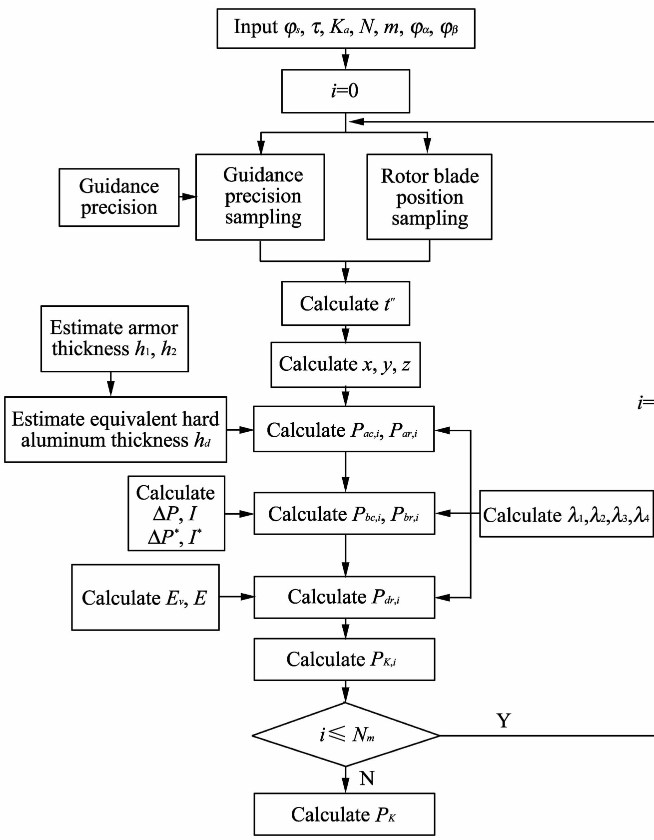

Fig. 6 Kill probability calculation process of multi-purpose guided missile against gunship

5 Numerical examples

It is supposed that directions of the missile velocity and missile body axis are the same, miss distance of missile $\rho \sim N(8.0\ \mathrm{m},\ 1.0\ \mathrm{m})$, miss orientation of missile $\eta_\theta \sim N(74.5°,\ 33°)$; missile velocity ν_m is 450 m/s, and gunship velocity v_t is 103 m/s within the horizontal plane. The selected sensitivity analysis and optimization design variables are: fuze inclination angle φ_s, average fuze delay time τ, filling coefficient of warhead K_a, number of fragments N, mass of single fragment m, static scattering angle of fragments φ_α and static scattering direction angle of fragments φ_β.

5.1 Analysis of sensitivity

This paper selects range analysis method of orthogonal experiment to determine sensitivity of each factor S_i with the kill probability model of multi-purpose guided missile against gunship. The factors should be normalized and dimensionless, so

$$r' = \frac{r-a}{b-a}, \quad (25)$$

where a, b are the range of influence factors, and they are shown in Table 4; r, r' are factors pre- and post- normalized and dimensionless.

From Table 4, descending order of factors affecting kill probability is: $K_a > m > \tau > \varphi_s > \varphi_\alpha > \varphi_\beta > N$. Coefficients associated with the warhead are predominant, so filling coefficient of warhead K_a mainly influences destruction degree caused by fragments and shockwave of warhead, while mass of single fragment mainly influences penetration effect of fragments; the second important aspect is fuze inclination angle φ_s, average fuze delay time τ, static scattering angle of fragments φ_α and static scattering direction angle of fragments φ_β, which mainly influence warhead detonation time to make warhead detonated nearby the best position.

Table 4 Range of influence factors a, b

Factor	φ_s(°)	τ(s)	K_a	N	m(g)	φ_α(°)	φ_β(°)
a	45	0	0.3	400	2.0	15	50
b	90	0.020	0.8	2 500	10.0	50	100

Table 5 Results of orthogonal experiment

	τ(s)	φ_β(°)	φ_α(°)	K_a	m(g)	N	φ_s(°)	P_K
1	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.752 6
2	0.3	0.3	0.3	0.7	0.7	0.7	0.7	0.711 2
3	0.3	0.7	0.7	0.3	0.3	0.7	0.7	0.803 2
4	0.3	0.7	0.7	0.7	0.7	0.3	0.3	0.688 3
5	0.7	0.3	0.7	0.3	0.7	0.3	0.7	0.965 9
6	0.7	0.3	0.7	0.7	0.3	0.7	0.3	0.619 2
7	0.7	0.7	0.3	0.3	0.7	0.7	0.3	0.919 6
8	0.7	0.7	0.3	0.7	0.3	0.3	0.7	0.655 9
K_{1_i}	2.955 3	3.048 9	3.039 3	3.441 3	2.830 9	3.062 7	2.979 7	–
K_{2_i}	3.160 6	3.067 0	3.076 6	2.674 6	3.285 0	3.053 1	3.136 2	–
R_i	0.205 3	0.018 1	0.037 3	0.766 7	0.449 6	0.009 6	0.156 5	–
S_i	0.513 3	0.045 3	0.093 3	1.916 8	1.124 0	0.024 0	0.391 3	–

5.2 Parameters optimization of fuze and warhead

Taking kill probability of multi-purpose guided missile P_K and mass of warhead M as the optimization objectives with intelligent single particle optimizer(ISPO)[14], the objective function F_{fit} is

$$\min(F_{\text{fit}}) = A \times (1 - P_K) + B \times M, \quad (26)$$

where A and B are weight coefficients of P_K and M, here $A = 10, B = 0.1$. Range of design variables are shown in Table 4.

Parameters comparison of fuze and warhead between U.S. CM-501G multi-purpose guided missile and values of corresponding design variables are shown in Table 6, of which kill probability is 0.862 1 and mass of warhead is 12.50 kg. Optimizing CM-501G multi-purpose guided missile with ISPO to redesign the fuze and warhead, the optimization process of objective function F_{fit} is shown in Fig.7. Optimization process of P_K and M are shown in Fig.8 and Fig.9. P_K after optimization is 0.975 7, and M is 11.95 kg.

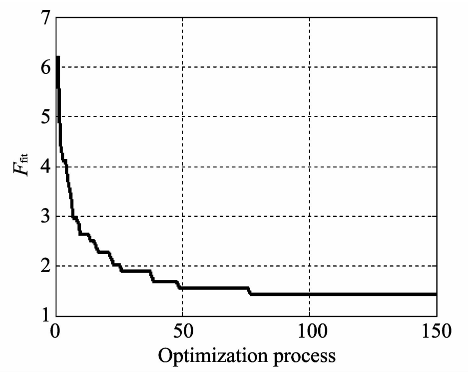

Fig.7 Optimization process of objective function F_{fit}

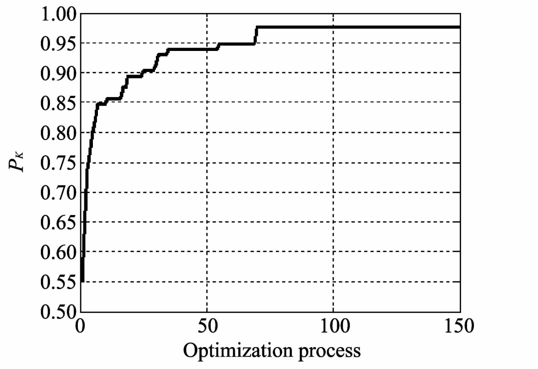

Fig. 8 Optimization results of kill probability P_K

From Table 6 we can know that the mass of warhead decreases 4.4% after optimization while kill probability increases 13.17%, and the optimization effect is obvious; the filling coefficient K_a is changed from 0.52 to 0.68, which means the rotor blades are more sensitive than blast warhead; the total number of fragments N decreases from 2 000 to 450; the mass of single fragment m increases from 3.0 g to 8.5 g; the static scattering angle of fragments φ_α decreases from 40° to 22.5°. To some extent, the decrease of static scattering angle of fragments φ_α offsets the density reduction of fragments which is caused by decrease of the total number of fragments N, but the kinetic energy of single fragment increases significantly, which improves the penetration ability of single fragment directly.

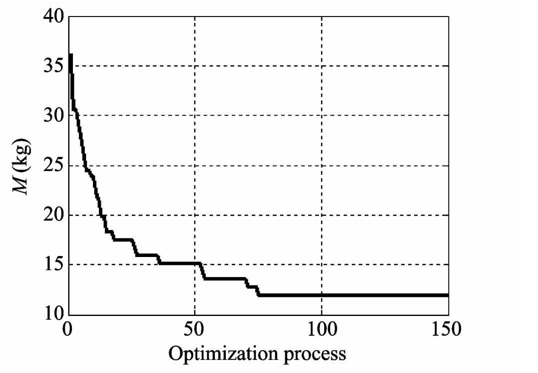

Fig. 9 Optimization results of warhead mass M

Table 6 Preliminary design parameters of CM-501G and optimal design parameters

Name	φ_s(°)	τ(s)	K_a	N	m(g)	φ_α(°)	φ_β(°)	M(kg)	P_K
Parameters	80	0.008	0.52	2 000	3.0	40	85	12.50	0.862 1
Optimal values	73	0.004 5	0.68	450	8.5	22.5	74	11.95	0.975 7

6 Conclusion

A complete mathematical model of kill probability of multi-purpose guided missile against gunship is established; laser fuze actuation model and warhead condition kill probability model of rotor blades are structured with new ideas and the important coefficient λ of vulnerability blade unit is introduced well to calculate kill probability of warhead against rotor blades, which makes the results more consistent with the theoretical analysis.

Sensitivity analysis is conducted for the main factors influenced on kill probability of multi-purpose guided missile against gunship to find out the most influential factors. After the U.S. CM-501G multi-purpose guided missile is optimized, the results show that the killing performance of the missile increases 13.17% and mass of warhead decreases 4.4%. When the filling coefficient, the total number of fragments and the mass of single fragment change significantly are optimized, we can know that the killing effect of a blast warhead with greater mass and smaller scattering angle fragments to attack gunship is better than fragmentation warhead with similar mass.

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多用途导弹对武装直升机杀伤概率建模与应用

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摘　要： 针对多用途导弹对武装直升机单发杀伤概率建模中存在的问题，本文提出了直升机旋翼桨叶易损单元重要性因子的概念，并综合运用蒙特卡罗方法和运动学原理采用新思路建立旋翼激光引信启动模型和战斗部对旋翼桨叶条件杀伤概率模型。根据有限的数据对武装直升机装甲厚度进行优化估算并通过实验进行验证，最终建立了多用途导弹对武装直升机完整的单发杀伤概率数学模型，为准确客观地分析多用途导弹对武装直升机目标的杀伤概率提供了必要的数学工具。基于所建立的单发杀伤概率模型，对影响多用途导弹杀伤概率的主要因素进行灵敏度分析及优化设计，分析及优化结果明显改善了单枚多用途导弹对武装直升机的杀伤性能。

关键词： 多用途导弹；武装直升机；旋翼桨叶；桨叶重要性因子；装甲厚度；杀伤概率

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