## Relevance of zero lift drag coefficient and lift coefficient to Mach number for large aspect ratio winged rigid body

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**Abstract:** Synthetic analysis is conducted to the wind tunnel experiment results of zero lift drag coefficient and lift coefficient for large aspect ratio winged rigid body. By means of wind tunnel experiment data, the dynamics model of the zero lift drag coefficient and lift coefficient for the large aspect ratio winged rigid body is amended. The research indicates that the change trends of zero lift drag coefficient and lift coefficient to Mach number are similar. The calculation result and wind tunnel experiment data all verify the validity of the amended dynamics model by which to estimate the zero lift drag coefficient and lift coefficient for the large aspect ratio winged rigid body, and thus providing some technical reference to aerodynamics character analysis of the same types of winged rigid body.

Key words: winged rigid body; zero lift drag coefficient; lift coefficient; wind tunnel experiment; dynamic characteristics

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#### 0 Introduction

The research on relevance of zero lift drag coefficient and lift coefficient to Mach number for large aspect ratio winged rigid body is crucial to analysis of motion characteristics of large aspect ratio winged rigid body. On account of uncertainty factors effect such as systematic constraint condition on the motion of large aspect ratio winged rigid body, stochastic environment variation, diversity of configuration as well as motion velocity and attitude<sup>[1-3]</sup>. In addition, the high altitude aerodynamic characteristics of rigid motion along with the change of the height of sea dials have large differences<sup>[4]</sup>. The existing dynamics models which estimate the zero lift drag coefficient and lift coefficient for the large aspect ratio winged rigid body have some limitations and errors<sup>[5]</sup>.

Currently, the general practice in obtaining the accurate data of zero lift drag coefficient and lift coefficient is based on of wind tunnel experiment [6-9]. However, the expense of wind tunnel experiment for large aspect ratio winged rigid body often results in lack of the data to modify the dynamics model. This

paper combines theoretical analysis, numeric simulation with wind tunnel experiment to investigate the relationship between the lift coefficient and zero lift drag coefficient of large aspect ratio winged rigid body to Mach number. On the basis of wind tunnel experiment data of large aspect ratio winged rigid body, it amends its dynamics model which may provide technology reference to the aerodynamics feature study of the same product.

### 1 Aerodynamics feature analysis

#### 1. 1 Aerodynamics feature analysis

A large aspect ratio winged rigid body is generally composed of two components, as shown in Fig. 1. A flying winged rigid body receives action of air drag on the surface whose magnitude rests on configuration, motion velocity and attitude, and action of stochastic environment load<sup>[10-11]</sup>. The action of aerodynamic force on a winged rigid body will affect the motion distance and stability.

To investigate aerodynamics feature of a rigid

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body, lift coefficient and zero lift drag coefficient are studied firstly since they are the parameters characterizing the aerodynamics feature. In general, it is required to study separately aerodynamic parameters of every component firstly, and then to take into consideration the interference between components, lastly, to find out the aerodynamics feature parameters of assembly.

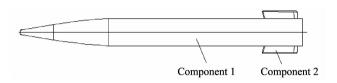


Fig. 1 Large aspect ratio winged rigid body

# 1. 2 Lift coefficient aerodynamics model of winged rigid body

Suppose a winged rigid body is composed of the body and the wing-tail-body.

1) The lift coefficient of the body,  $C_{YB}$ 

In subsonic range, i. e. M < 1, the lift coefficient of the body,  $C_{\rm YB}$ , is approximately a constant  $C_{\rm YB} = \delta$ , while in supersonic range, i. e. M > 1, the lift coefficient of the body is also approximately a constant  $C_{\rm YB} = 2.4\delta$ . Here, M is Mach number;  $\delta$  is attack angle.

2) The lift coefficient of wing-tail-body,  $C_{YW}$ 

$$C_{\text{YW}} = \frac{1.84\pi\lambda_{\text{W}}\delta K_{\text{YW}}}{2.4 + \lambda_{\text{W}}} \overline{S}_{\text{W}}, \text{ when } M \leqslant 0.5, \quad (1)$$

$$C_{\text{YW}} = \frac{1.84\pi\lambda_{\text{W}}\delta K_{\text{YW}}}{2.4 + \lambda_{\text{W}}\sqrt{1 - M^2}} \overline{S}_{\text{W}},$$

$$0.5 < M \leqslant 1,$$
 (2)

$$C_{\text{YW}} = 1.35 \delta K_{\text{YW}} \left( \lambda_{\text{W}} + \frac{1}{\sqrt{M^2 - 1}} \right) \overline{S}_{\text{W}},$$

$$M > 1$$
 and  $M < \sqrt{1 + \frac{1}{\lambda_{\mathrm{W}}^2}}$ , (3)

$$C_{\rm YW} = \frac{-4\delta K_{\rm YW}}{/M^2-1} \Big(1 - \frac{1}{2\lambda_{\rm W}~/M^2-1}\Big) \overline{S}_{\rm W} \,, \label{eq:CYW}$$

$$M > 1$$
 and  $M \geqslant \sqrt{1 + \frac{1}{\lambda_{\mathrm{W}}^2}}$ , (4)

where  $\lambda_W$  is aspect ratio,  $\lambda_W = L_W/b_{av}$ ;  $L_W$  is stretching length of cantilever section for the wing-tail-

body;  $b_{\rm av}$  is average chord length of wing;  $\overline{S}_{\rm W}$  is the relative area of a pair of wings,  $\overline{S}_{\rm W} = S_{\rm W}/S_{\rm M}$ ;  $S_{\rm W}$  is the area of cantilever section for the wing-tail-body;  $S_{\rm M}$  is the max area of the across section for the body;  $K_{\rm YW}$  is the modified coefficient of empennage.

The lift efficiency depends on configuration parameters and motion feature<sup>[12-14]</sup>. In addition, due to the aerodynamic force interference between two components of winged rigid body, the aerodynamic feature of the wing-tail-body in adjacent area of the body will be affected by the aerodynamic feature in the body area, and vice versa. Consequently, the computation of the lift coefficient of assembly  $C_{YBW}$  should take into consideration the lift coefficient in the bonding district of two components. We may just as well use an interference modified coefficient  $C_{Ky}$  to represent the lift coefficient in the bonding district.

By combining the theoretical analysis with experiment modification, the interference modified coefficient formula of  $C_{\rm Ky}$  could be obtained by

$$C_{\text{Ky}} = -0.006 \ 3(1 - D_{\text{b}}^2/D^2),$$
 (5)

where D is the diameter of cross section of the body;  $D_b$  is the bottom diameter of the body, thus, the lift coefficient of assembly is

$$C_{\text{YBW}} = C_{\text{YB}} + C_{\text{YW}} + C_{\text{Ky}}.$$
 (6)

# 1. 3 Zero lift drag coefficient aerodynamics model of winged rigid body

Zero lift drag coefficient of winged rigid body is the drag coefficient when the attack angle is zero.

1) Zero lift drag coefficient of the body  $C_{XOB}$ 

When the assembly flies at supersonic speed, the zero lift drag coefficient of the body  $C_{\rm XOB}$  is composed of its head section wave drag coefficient  $C_{\rm Xln}$ , tail section wave drag coefficient  $C_{\rm Xlb}$ , friction drag coefficient  $C_{\rm XlB}$  and increased drag coefficient  $\Delta C_{\rm XF}$  from the head section variation of the body, where the configuration in the tail section is the principal factors affecting the tail section drag [15-17]. When the assembly flies at subsonic speed, zero lift drag coefficient of the body,  $C_{\rm XOB}$ , has no head section wave drag and tail section wave drag,

$$C_{\text{XOB}} = C_{\text{XIn}} + C_{\text{XIt}} + C_{\text{XIb}} + C_{\text{XfB}} + \Delta C_{\text{XF}},$$
 (7) where

$$C_{\text{Xln}} = \left[ \left( 0.001 \ 6 + \frac{0.002}{M^2} \right) (\beta_0^{1.7}) \right] \left[ 1 - \frac{196\lambda_n^2 - 16}{14(M+18)\lambda_n^2} \right], \tag{8}$$

$$C_{\text{Xlt}} = \left[ \left( 0.001 \ 6 + \frac{0.02}{M^2} \right) (\beta_{\text{t}}^{\text{l.7}}) \right] (1 - \overline{S}_{\text{b}})^{0.5}, \tag{9}$$

$$C_{\text{Xlb}} = \begin{cases} 0.029 (D_{\text{b}}/D_{\text{M}})^2 / \ / \overline{C_{\text{XlB}}} \ 0.029 (D_{\text{b}}/D_{\text{M}})^2 / \ / \overline{C_{\text{XlB}}} \ , & M < 1 , \\ 0.85 k_1 (2 - k_1) \overline{S}_{\text{b}} / M^2 \ , & k_1 < 1 , \\ 0.85 \overline{S}_{\text{b}} / M^2 \ , & k_1 \geqslant 1 , \end{cases}$$

(15)

$$k_1 = M / \sqrt{\lambda_{\rm B}/S_{\rm b}}, \qquad (10)$$

$$C_{\text{XfB}} = 0.031 \, 5 \text{Re}^{-0.145} \eta_{\lambda} (1 + 0.12 M^2)^{-0.5} \, \frac{S_{\text{f}}}{S_{\text{M}}},$$
 (11)

$$\Delta C_{XF} = C_{XF} S_F / S_M, \qquad (12)$$

where  $\beta_0$  is circular cone half-angle of the body in the head section, and  $\beta_t$  is circular cone half-angle in the tail section;  $\lambda_{\rm n}$  is aspect ratio of body,  $\overline{S}_{\rm b} = (D_{\rm b}/$  $D_c)^2$ ;  $D_c$  is column diameter of body, and  $D_M$  is max diameter;  $\lambda_B$  is aspect ratio of assembly;  $\eta_{\lambda}$  is shape modified coefficient of body; Re is Reynolds number;  $S_{\rm f}$  is lateral surface area of body, and  $S_{\rm F}$  is cross section area in the head section;  $C_{XF}$  is drag coefficient in head section.

2) Zero lift drag coefficient of the wing-tail-body,  $C_{\text{XOW}}$ 

When  $M \ge 1$ , zero lift drag coefficient of wing-tailbody  $C_{\text{XOW}}$  is composed of profile drag coefficient  $C_{\text{XfW}}$ and thickness wave drag coefficient  $C_{XPW}$ ; when M <1, the thickness wave drag coefficient will not exist.

$$C_{\text{XOW}} = C_{\text{XfW}} + C_{\text{XPW}},$$
 (13)  
 $C_{\text{XfW}} = 2 \times 0.031 \, 5 \text{Re}^{-0.145} (1 + 0.12 M^2)^{-0.5} \, \eta_c,$  (14)  
 $C_{\text{XPW}} = \frac{4\bar{c}^2 K}{\sqrt{M^2 - 1}} \overline{S}_{\text{W}},$  (15)

where  $\eta_c$  is modified coefficient when considering the thickness effect of wing-tail-body, the relative thickness  $c = c/b_r$ , c is the thickness of the wing-tail-body, and  $b_r$  is the root chord length of the wing-tail-body; K is the section plane shape modified coefficient in

The zero lift drag coefficient of assembly  $C_{X0}$  is the sum of zero lift drag coefficients of the body and the

the airflow direction of the wing-tail-body.

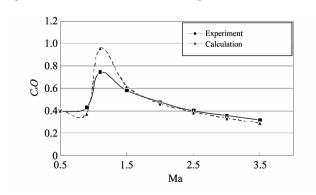
wing-tail-body. The drag which the wing-tail-body provides to the assembly is proportional to N, which is the pair number of the wing-tail-body.

The zero lift drag coefficient of the assembly formula is

$$C_{X0} = C_{X0B} + NC_{X0W} \frac{S_W}{S_M}.$$
 (16)

#### Calculation examples, result analysis 2 and experiment verification

This paper takes a kind of large aspect ratio winged rigid body as an example to calculate its lift coefficient and zero lift drag coefficient. The comparison results of the calculation result and the wind tunnel experiment data are shown in Figs. 2 and 3.



Relationship between zero lift drag coefficient of assembly and Mach number

From the figures, the calculation results and the wind tunnel experiment data indicate that the change trends of zero lift drag coefficient and lift coefficient of assembly to Mach number are similar. In transonic region, the zero lift drag coefficient and lift coefficient all present sharp mutations. While in subsonic and supersonic regions, the changes are relatively stable, especially in the region of 1.5Ma to 3.5Ma, the received load of assembly descends in relatively stable mode.

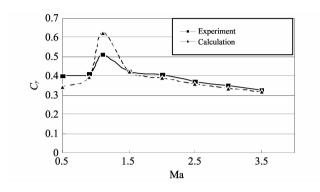


Fig. 3 Relationship between lift coefficient of assembly and Mach number

In supersonic region of 1.5Ma to 3.5Ma, the calculation results reflecting the relationship of zero lift drag coefficient and lift coefficient of assembly to Mach number bear fairly good coincidence to the wind tunnel experiment result; While in subsonic and transonic regions, the calculation results have big error compared with the wind tunnel experiment result, because the received load change of assembly and the effect of airflow disturbance change are fairly complex while the existing dynamics model of zero lift drag coefficient and lift coefficient do not take into account reasonably of the received load change of assembly and the effect of airflow disturbance change.

#### 3 Conclusion

The research has indicated that in the region of 0.5Ma to 3.5Ma the change trends of zero lift drag coefficient and lift coefficient of assembly to Mach number are similar, and the theoretical analysis and the wind tunnel experiment results verify the conclusion.

By means of wind tunnel experiment data, the dynamics model of zero lift drag coefficient and lift coefficient for the large aspect ratio winged rigid body is amended. The calculation results and the wind tunnel

experiment results all verify the validity of the amended dynamics model. Consequently, under the condition of lack of aerodynamics experiment data, it is possible to utilize the configuration parameters to amend the theoretical model for estimaion of the aerodynamics character, which can provide some technical reference to aerodynamics design for the same types of the large aspect ratio winged rigid body<sup>[18-19]</sup>.

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## 大长径比带翼刚体的零升阻力系数和 升力系数与马赫数的相关性

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摘 要: 对大长径比带翼刚体的零升阻力系数和升力系数的风洞试验数据进行了综合分析,利用风洞试验数据修正了带翼刚体升力系数和零升阻力系数的动力学模型。研究结果表明,大长径比带翼刚体的零升阻力系数和升力系数随马赫数的变化趋势基本相似,数值计算数据与风洞试验数据都验证了修正后的动力学模型对于估算大长径比带翼刚体的升力系数和零升阻力系数是可行的,为同类型带翼刚体的空气动力特性分析提供了技术参考。

关键词: 带翼刚体; 零升阻力系数; 升力系数; 风洞试验; 动力学特性

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