Suboptimal State Feedback H_2/H_∞ Controller Design with Spectrum Constraint for Discrete-time Stochastic Systems

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Abstract – **With the aid of the spectrum technique, a new concept named α-stabilizability ($0 \leqslant \alpha \leqslant 1$) is introduced and its sufficient and necessary conditions are also proposed. Especially, it is identical with the asymptotically mean square stabilizability when $\alpha = 1$. As an application, the suboptimal state feedback H_2/H_∞ controller that satisfies the additional spectrum constraint via solving a convex optimization problem is delt with.**

Key words – ***spectrum technique; asymptotically mean square stabilizability; α-stabilizability; H_2/H_∞ controller design***

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1 Introduction

For the deterministic systems, the problem of assigning poles has kept attention for a long time and a rich source of references is provided[1-4]. Direct linkage with the system dynamic behavior is one of the causes for great concern. Taking the continuous systems for instance, the further left the spectrum set is located to, the faster the system response rate is. It is well known that, the time-invariant system is asymptotically stable if and only if the all spectra belong to the open left-half complex plane. However, on the stability and the stabilization of the stochastic systems, some necessary and sufficient conditions are mainly obtained in terms of the stochastic algebraic Riccati equations and the linear matrix inequalities, which are derived from the application of the generalized Lyapunov equation-based approach[5-6]. References [7-10] have generalized some concepts and conclusions related to the spectrum of the deterministic systems to the stochastic case, such as the spectrum criterion for the asymptotically mean square stabilizability and the stochastic "PBH" criterion for the exact observability. The objective of this paper is to contribute to develop the corresponding results on the stability and stabilization for the discrete stochastic systems. Our primary goal is to define and investigate the α-stabilizability ($0 \leqslant \alpha \leqslant 1$), which can be considered as the extension of the asymptotically mean square stabilizability.

As one of the most fundamental and widely used tools in modern engineering, mixed H_2/H_∞ control theory has000000 been intensively studied for both the continuous and discrete stochastic dynamics, see Ref. [11-14] and references therein. It is concerned with the design of a controller that minimizes the H_2 performance of the system with respect to some input noises while guarantees certain worst-case performance with respect to other external disturbances. It has been show that the stochastic H_∞ synthesis can be formulated as a convex optimization problem involving the linear matrix inequalities. However, for stochastic systems, many authors deal with the sole H_2/H_∞ control, only a few papers consider the design of H_2/H_∞ controllers with additional specifications on the closed-loop poles. As another applications of the obtained α-stabilizability, we attempt to address the design of suboptimal state feedback H_2/H_∞ controller with the spectrum constraint, which is achieved via solving a convex optimization problem.

The content and scope of this paper are as follows: In section 2, we provide a spectrum criterion for the asymptotically mean square stabilizability of the linear time-invariant stochastic discrete systems. Besides, a notion named-stabilizability with $0 \leqslant \alpha \leqslant 1$ is defined and investigated, which may be regarded as the extension of the related results of the asymptotically mean square stabilizability. The following section addresses the mixed H_2/H_∞ synthesis with α-stability constraints. Section 4 ends this paper with some concluding remarks.

Notations. In this paper, we make use of the following notations: $R(C)$ is the set of all real (complex) numbers. $\boldsymbol{R}^n$ is n-dimensional real Euclidean space. $\boldsymbol{R}^{n\times m}$ is the space of all $n \times m$ matrices. $\boldsymbol{S}_n$ is the set of all $n \times n$ symmetric matrices, its components may be complex. $\boldsymbol{C}_n$ is the set of all nn complex matrices. $\boldsymbol{A}'$ is the transpose of a matrix $\boldsymbol{A}$. $\boldsymbol{A} \geqslant 0 (\boldsymbol{A} > 0)$ is $\boldsymbol{A}$ is a positive semidefinite (positive definite) matrix. $\boldsymbol{I}$ is the identity matrix. $\|\boldsymbol{A}\|$ is the norm of a matrix or vector $\boldsymbol{A}$. $|z|$ is the modular of

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a complex number z. $\sigma(L)$ is the spectrum set of an operator L, $D(q,\alpha):=\{z:z\in C, |z-q|<\alpha\}$. $N:=\{0,1,2,\cdots\}$.

2 Stability, stabilizability and spectrum

In this section, with the help of the spectrum, we aim at studying the stabilizability of discrete stochastic systems governed by difference equations. First, a spectrum criterion is presented which is for testing asymptotically mean square stabilizability of systems mentioned above. A class of regional stabilizability called $D(0,\alpha)$-stabilizability is introduced and its necessary and sufficient condition is also investigated via a linear matrix inequality approach.

2.1 Stability and stabilizability

Consider that the system expressed by the stochastic difference equations

$$\begin{cases} x(t+1)=\boldsymbol{A}x(t)+\boldsymbol{B}u(t)+\\ \qquad \boldsymbol{C}x(t)w(t)+\boldsymbol{D}u(t)w(t), \\ x(0)=x_0\in R^n, t\in N, \end{cases} \tag{1}$$

where $x(t)\in R^n$ and $u(t)\in R^{n_u}$ stand for the system state and control input respectively; $x_0\in R^n$ is the initial condition which is deterministic. $\boldsymbol{A},\boldsymbol{B},\boldsymbol{C}$ and $\boldsymbol{D}$ are constant matrices of appropriate dimensions; $\{w(t)\in R, t\in N\}$ is a sequence of real random variables defined on the filtered probability space (Ω,F,P,F_t) with $F_t=\sigma\{w(s),s\in N_t\}$ sand is a wide sense stationary, second-order process with $E(w(t))=0$ and $E(w(t)w(s))=\delta_{st}$, where δ_{st} the Kronecker delta.

To deal with the stabilizability of system (1), we first give the following definitions.

Definition 2.1[5]: System (1) is called stabilizable in the mean square sense if there is a feedback control $u(t)=\boldsymbol{K}x(t)$ with $\boldsymbol{K}$ a constant matrix, such that for any $x_0\in R^n$, the closed-loop system

$$\begin{cases} x(t+1)=(\boldsymbol{A}+\boldsymbol{BK})x(t)+\\ \qquad (\boldsymbol{C}+\boldsymbol{DK})x(t)w(t), \\ x(0)=x_0, t\in N, \end{cases} \tag{2}$$

is asymptotically mean square stable, that is, the state corresponding to x_0 satisfies $\lim_{t\to\infty}E\|x(t)\|^2=0$.

When system (1) is stabilizable, we call $(\boldsymbol{A},\boldsymbol{B},\boldsymbol{C},\boldsymbol{D})$ stabilizable; When Eq. (2) is asymptotically mean square stable, we also say $(\boldsymbol{A}+\boldsymbol{BK},\boldsymbol{C}+\boldsymbol{DK})$ stable for short.

Definition 2.2: For any given feedback gain matrix $\boldsymbol{K}$, let L_K be a linear operator from S_n to S_n defined as

$$L_K: Z\in S_n \mapsto (\boldsymbol{A}+\boldsymbol{BK})Z(\boldsymbol{A}+\boldsymbol{BK})'+(\boldsymbol{C}+\boldsymbol{DK})Z(\boldsymbol{C}+\boldsymbol{DK})', \tag{3}$$

where the spectra of L_K is the set described by

$$\sigma(L_K)=\{\lambda_i\in C: L_K(Z)=\lambda_i Z_i, Z_i\in S_n, Z_i\neq 0, i=1,2,\cdots,\frac{n(n+1)}{2}. \tag{4}$$

We are now equipped to investigate the stabilizability of system (1) by the spectrum technique.

Theorem 2.1: System (1) is stabilizable if and only if there exists a $K\in R^{m\times n}$, such that the spectrum of L_K belongs to the circular region $D(0,1)$.

Proof: Let $X(t)=Ex(t)x'(t)$. Notice that the vectors $(\boldsymbol{A}+\boldsymbol{BK})x(t)$ and $(\boldsymbol{C}+\boldsymbol{DK})x(t)$ are uncorrelated with respect to $w(t)$, which are F_{t-1} measurable. In view of $Ew((t))=0$, we have

$$\begin{cases} X(t+1)=(\boldsymbol{A}+\boldsymbol{BK})X(t)(\boldsymbol{A}+\boldsymbol{BK})'+\\ \qquad (\boldsymbol{C}+\boldsymbol{DK})X(t)(\boldsymbol{C}+\boldsymbol{DK})', \\ X(0)=X_0=x_0x_0', t\in N. \end{cases} \tag{5}$$

By the same discussion as Theorem 4 in Ref. [10], there is a unique matrix $\boldsymbol{L}(K)\in R^{n(n+1)/2\times n(n+1)/2}$ such that Eq. (5) is equivalent to

$$\tilde{X}(t+1)=\boldsymbol{L}(K)\tilde{X}(t), \tilde{X}(0)=\tilde{X}_0, \tag{6}$$

where $\tilde{X}\in R^{n(n+1)/2}$. Moreover, $\sigma(L_K)=\sigma(\boldsymbol{L}(K))$. According to stability theory on discrete-time control system, system (6) is stable if $\sigma(L_K))\in D(0,1)$. This concludes the proof of Theorem 2.1.

Theorem 2.1: can be named a spectrum criterion for testing the stability of real stochastic discrete-time systems. Whereas, it should be pointed out that the criterion does not hold for complex stochastic systems.

Corollary 2.1: (A,C) is stable (i.e, $K=0$ in Eq. (1)) if and only if the spectra of system (1) belong to $D(0,1)$.

Example 2.1: Let the coefficient matrices of system (1) be

$$\boldsymbol{A}=\begin{bmatrix}2&1\\0&1\end{bmatrix}, \boldsymbol{B}=\begin{bmatrix}6&7/2\\1&-3\end{bmatrix},$$

$$\boldsymbol{C}=\begin{bmatrix}1&1\\0&1\end{bmatrix}, \boldsymbol{D}=\begin{bmatrix}-5&4\\0&-5/2\end{bmatrix}.$$

We take

$$\boldsymbol{K}=\begin{bmatrix}1/4&1/2\\0&1/2\end{bmatrix}, \boldsymbol{X}=\begin{bmatrix}x_{11}&x_{12}\\x_{12}&x_{22}\end{bmatrix},$$

then Eq. (5) becomes

$$\begin{bmatrix}\frac{5}{16}x_{11}-\frac{1}{2}x_{12}+\frac{5}{16}x_{22} & \frac{1}{8}x_{11}-\frac{1}{8}x_{22}\\ \frac{1}{8}x_{11}-\frac{1}{8}x_{22} & \frac{1}{16}x_{11}+\frac{1}{16}x_{22}\end{bmatrix}=\lambda\begin{bmatrix}x_{11}&x_{12}\\x_{12}&x_{22}\end{bmatrix},$$

which is equivalent to the following matrix characteristic equation

$$\boldsymbol{L}(K)\begin{bmatrix}x_{11}\\x_{12}\\x_{22}\end{bmatrix}=\lambda\begin{bmatrix}x_{11}\\x_{12}\\x_{22}\end{bmatrix}, \tag{7}$$

where

$$\boldsymbol{L}(K)=\begin{bmatrix}5/16&-1/2&5/16\\1/8&0&-1/8\\1/16&0&1/16\end{bmatrix}.$$

By solving Eq. (7), the spectra of system (1) are $\frac{1}{4}, \frac{1}{16}+\mathrm{j}\frac{\sqrt{7}}{16}, \frac{1}{16}-\mathrm{j}\frac{\sqrt{7}}{16}$, where $\mathrm{j}^2=-1$. By means of Theorem 2.

1, we claim that the system denoted by Example 1 is stabilizable.

Remark 2.1: Matrix $\boldsymbol{L}(K)$ emerged in the proof of Theorem 2.1 and Example 2.1 may be called an induced matrix by L_K .

Definition 2.3 The operator

$$L_K^* : Z \in S_n \mapsto$$

$$(\boldsymbol{A}+\boldsymbol{BK})'Z(\boldsymbol{A}+\boldsymbol{BK})+(\boldsymbol{C}+\boldsymbol{DK})'Z(\boldsymbol{C}+\boldsymbol{DK}) \quad (8)$$

is called the adjoint operator of L_K with the inner product $\langle Z,Y\rangle = \mathrm{trace}(\boldsymbol{Z}^* Y)$ for any $Z,Y \in S_n$, where trace $(Z^* Y)$ stands for the trace of the matrix $\boldsymbol{Z}^* Y$.

In virtue of the fact that $\sigma(L_K)=\sigma(L_K^*)$, we have

Corollary 2.2: System (2) is asymptotically mean square stable if and only if its dual system

$$\begin{cases} x(t+1)=(\boldsymbol{A}+\boldsymbol{BK})'x(t)+ \\ \qquad (\boldsymbol{C}+\boldsymbol{DK})'x(t)w(t), \\ x(0)=x_0, t\in N, \end{cases} \quad (9)$$

is asymptotically mean square stable.

2.2 $D(0,\alpha)$ -stabilizability

Below, by means of the spectrum technique, we define a class of regional stabilizability named $D(0,\alpha)$ -stabilizability.

Definition 2.4: System modeled by Eq.(1) is said to be $D(0,\alpha)$ -stabilizable with $0<\alpha\leqslant 1$, if there is a constant matrix $\boldsymbol{K}\in\boldsymbol{R}^{m\times n}$, such tiat $\sigma(L_K)\subset D(0,\alpha):=\{\lambda: |\lambda|<\alpha\}$.

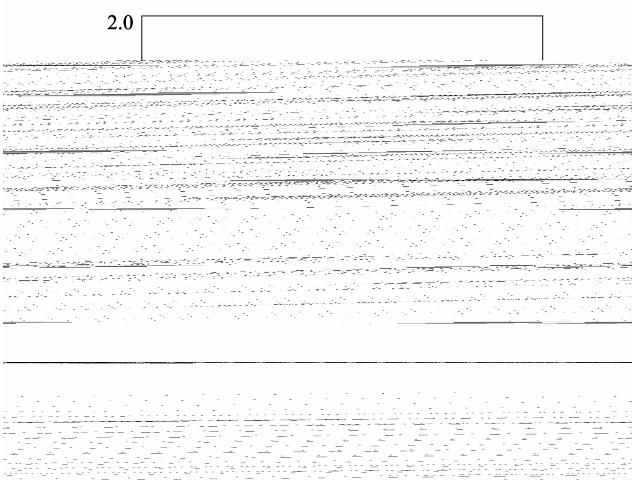

Fig.1 Circular region $D(0,\alpha)$

The following theorem presents a necessary and sufficient condition for Eq.(1) to be $D(0,\alpha)$ -stabilizable via a linear matrix inequality approach.

Theorem 2.2: System (1) is $D(0,\alpha)$ -stabilizable if and only if the following linear matrix inequality

$$\begin{bmatrix} \bigstar & \frac{1}{\sqrt{\alpha}}BY & \frac{1}{\sqrt{\alpha}}DY \\ \frac{1}{\sqrt{\alpha}}(BY)' & -P & 0 \\ \frac{1}{\sqrt{\alpha}}(DY)' & 0 & -P \end{bmatrix} < 0, \quad (10)$$

admits solutions $P>0$ and Y , where $\bigstar=-P+\frac{1}{\alpha}APA'+\frac{1}{\alpha}CPC'+\frac{1}{\alpha}AY'B'+\frac{1}{\alpha}BYA'+\frac{1}{\alpha}CY'D'+\frac{1}{\alpha}DYC'$. In this case, the corresponeing feedback gain $K=YP^{-1}$.

Proof By Theosem 2.1 and Corollary 2.2, system (1) is $D(0,\alpha)$ -stabilizable if and only if $\sigma(L_K^\alpha)\subset D(0,1)$,where $L_K^\alpha:=\frac{1}{\alpha}L_K$, where $L_K^\alpha:=\frac{1}{\alpha}L_K$, which is equivalent to

$$\begin{cases} x(t+1)=\frac{1}{\sqrt{\alpha}}(A+BK)x(t)+ \\ \qquad \frac{1}{\sqrt{\alpha}}(C+DK)x(t)w(t), \\ x(0)=x_0, t\in N, \end{cases} \quad (11)$$

or its dual system

$$\begin{cases} x(t+1)=\frac{1}{\sqrt{\alpha}}(A+BK)'x(t)+ \\ \frac{1}{\sqrt{\alpha}}(C+DK)'x(t)w(t), \\ x(0)=x_0, t\in N, \end{cases} \quad (12)$$

being asymptotically mean square stable. In addition, as a direct application of Proposition 2.2[16], Eq.(12) is asymptotically mean square stable if and only if the following inequality

$$-P+\frac{A+BK}{\sqrt{\alpha}}P\frac{(A+BK)'}{\sqrt{\alpha}}+\frac{C+DK}{\sqrt{\alpha}}P\frac{(C+DK)'}{\sqrt{\alpha}}<0, \quad (13)$$

exists a solution $P>0$. By Schur's complement [17], Eq.(13) is further equivalent to that there are matrices $P>0$ and Y such that Eq.(10) is feasible. The proof of Theorem 2.2 is concluded.

When $\alpha=1$, i.e. $\sigma(L_K)\subset D(0,1)$, system (1) is asymptotically mean square stabilizable, and Definition 2.4 and Theorem 2.2 coincide with Definition 2.1 and Theorem 2.1, respectively.

When system (1) is $D(0,\alpha)$ -stabilizable, we call $(\frac{A}{\sqrt{\alpha}},\frac{B}{\sqrt{\alpha}},\frac{C}{\sqrt{\alpha}},\frac{D}{\sqrt{\alpha}})$ stabilizable; When Eq.(2) is $D(0,\alpha)$ -stable, we also say $(\frac{A+BK}{\sqrt{\alpha}},\frac{C+DK}{\sqrt{\alpha}})$ stable for short.

Corollary 2.3 (A,C) is $D(0,\alpha)$ -stable if and only if for some $Q>0$, the following equation

$$-P+\frac{A}{\sqrt{\alpha}}P\frac{A'}{\sqrt{\alpha}}+\frac{C}{\sqrt{\alpha}}P\frac{C'}{\sqrt{\alpha}}=-Q$$

admits a solution $P>0$.

3 H_2/H_∞ controller design with spectrum constraint

This subsection addresses the state-feedback synthesis with a combination of H_2/H_∞ performance and the spectrum assignment specifications.

Consider the following discrete-time stochastic perturbed system

$$\begin{cases} x(t+1) = Ax(t) + Bu(t) + Cx(t)w(t) + \\ \qquad B_0 v(t)w(t), \\ z(t) = \begin{bmatrix} C_1 x(t) \\ D_1 u(t) \end{bmatrix}, \\ D_1' D_1 = I, x(t) = x_0 \in R^n, t \in N, \end{cases} \tag{14}$$

where $u(t) \in l_w^2(N, R^{n_u})$ is the control input and $v(t)$ is an external disturbance. Given a disturbance attenuation level $\gamma > 0$, define two associated performances

$$J_1^\infty(u,v) = \sum_{t=0}^{\infty} E[\gamma^2 \| v(t) \|^2 - \| z(t) \|^2 |,$$

and

$$J_2^\infty(u,v) = \sum_{t=0}^{\infty} E \| z(t) \|^2.$$

The infinite horizon stochastic H_2/H_∞ control of system (14) is stated as follows:

Given a scalar $\gamma > 0$, find, if possible, a control law $u^*(t) \in l_w^2(N, R^{n_u})$ such that

1) $u^*(t)$ stabilizes the system (14) internally, i.e. when $v(t) = 0, u(t) = u^*(t)$, the trajectory of (14) with any initial value $x(0) = x_0$ satisfies $\lim_{t\to\infty} E \| x(t) \|^2 = 0$.

2)

$$\| L_{u^*} \| = \sup_{\substack{v \in l_w^2(N, R^{n_v}) \\ v \neq 0, x_0 = 0}} \frac{(\sum_{t=0}^{\infty} E[\| Cx(t) \|^2 + \| u^*(t) \|^2])^{1/2}}{(\sum_{t=0}^{\infty} E[\| v(t) \|^2])^{1/2}} < \gamma,$$

3) When the worst case disturbance $v^*(t) \in l_w^2(N, R^{n_v})$, if exist, is implemented in Eq. (14), $u^*(t)$ minimizes the output energy

$$J_2^\infty(u, v^*) = \sum_{t=0}^{\infty} E \| z(t) \|^2.$$

If the above (u^*, v^*) exist, we say that the infinite horizon stochastic control problem is solvable.

It has been shown that the existence of the H_2/H_∞ controller is equivalent to the solvability of four coupled matrix-valued equations. We list the theorem and the reader may consult[11] for a detailed discussion.

Theorem 3.1: For system (14), suppose the following four coupled matrix-valued equations have solutions $(P_1, P_2; K_1, K_2)$ with $P_1 < 0$ and $P_2 > 0$.

$$\begin{cases} -P_1 + (A+BK_2)'P_1(A+BK_2) + C'P_1C \\ - C_1'C_1 - K_2'K_2 - K_3 H_1(P_1)^{-1}K_3' = 0, \\ H_1(P_1) > 0. \end{cases}$$

$$K_1 = -H_1(P_1)^{-1}K_3',$$

$$-P_2 + A'P_2A + (C+B_0K_1)'P_2(C+B_0K_1) + C_1'C_1 - K_4H_2(P_2)^{-1}K_4' = 0,$$

$$K_2 = -H_2(P_2)^{-1}K_4',$$

where

$$K_3 = C'P_1B_0,$$
$$H_1(P_1) = \gamma^2 I + B_0'P_1B_0,$$
$$K_4 = A'P_2B,$$
$$H_2(P_2) = I + B'P_2B.$$

If $[A, C \mid C_1]$ and $[A, C + B_0K_1 \mid C_1]$ are exactly observable, then the H_2/H_∞ control problem has a pair of solutions

$$u^*(t) = K_2x(t),\ v^*(t) = K_1x(t).$$

It is generally difficult to solve the above four coupled matrix-valued equations, analytically. The following theorem presents a numerical algorithm to design a suboptimal H_2/H_∞ controller, which satisfies the requirements 1) and 2) of the H_2/H_∞ control but with a suboptimal H_2 performance, namely, $\min_{u \in l_w^2(N, R^{n_u})} J_2^\infty(u, v^*) \leqslant x_0'P_2x_0$. Denote

$$\begin{bmatrix} U & C'P_2B_0 \\ B_0'P_2C & \gamma^2 I - B_0'P_2B_0 \end{bmatrix} > 0, \tag{15}$$

and

$$\begin{bmatrix} -U & C'P_2B_0 & C'P_2B_0 \\ B_0'P_2C & B_0'P_2B_0 - \gamma^2 I & 0 \\ B_0'P_2C & 0 & 2B_0'P_2B_0 - \gamma^2 I \end{bmatrix} < 0, \tag{16}$$

Where

$$U = P_2 - A'P_2A - C'P_2C + A'P_2\bar{B} + \bar{B}'P_2A - \prod(P_2) - C_1'C_1,$$
$$\bar{B} = [B \quad 0],$$
$$\prod(P_2) = \begin{bmatrix} I + B'P_2B & 0 \\ 0 & 0 \end{bmatrix}.$$

Theorem 3.2: For Eq. (14), a suboptimal stochastic H_2/H_∞ controller can be obtained by solving the following convexoptimization problem

$$\min_{\substack{\text{subject to (15),(16)} \\ \text{and } P_2 > 0}} Tr(P_2),$$

with

$$u^*(t) = -(I + B'P_2B)^{-1}B'P_2Ax(t),$$
$$v^*(t) = (\gamma^2 I - B_0'P_2B_0)B^{-1}B_0'P_2Cx(t),$$

Combining Theorem 2.2 and Theorem 3.2, we below propose the main result.

Theorem 3.3: The state-feedback H_2/H_∞ design with $D(0,\alpha)$-stable constraint for system (14) can be computed as the global minimum of the following linear matrix inequalities optimization problem

$$\min_{\substack{\text{subject to (10),(15),(16)} \\ \text{and } P_2 > 0}} Tr(P_2),$$

with

$$u^*(t) = -(I + B'P_2B)^{-1}B'P_2Ax(t),$$
$$v^*(t) = (\gamma^2 I - B_0'P_2B_0)^{-1}B_0'P_2Cx(t).$$

Example 3.2: Consider the following two-dimensional discrete time stochastic system

$$\begin{cases} x(t+1) = Ax(t) = \\ \qquad Bu(t) + Cx(t)w(t) + B_0v(t)w(t), \\ \boldsymbol{z}(t) = \begin{bmatrix} C_1x(t) \\ u(t) \end{bmatrix}, x(0) = x_0 \in \boldsymbol{R}^n, t \in \boldsymbol{N}, \end{cases} \tag{17}$$

where

$$\boldsymbol{A} = \begin{bmatrix} -0.35 & 0.1 \\ 0 & -0.4 \end{bmatrix},\ \boldsymbol{B} = \begin{bmatrix} 0.25 \\ -0.3 \end{bmatrix},$$

$$\boldsymbol{C} = \begin{bmatrix} 0.3 & 0 \\ 0 & -0.35 \end{bmatrix}, \boldsymbol{B}_0 = \begin{bmatrix} -0.52 \\ -0.45 \end{bmatrix},$$
$$\boldsymbol{C}_1 = [-0.65 \quad -0.74]$$

Set $\gamma = 2.3, \alpha = 0.25$. By applying the LMI Control Toolbox, the solution to the convex optimization problem is given by

$$\boldsymbol{P}_2 := \begin{bmatrix} 2.790\,2 & 0.211\,4 \\ 0.211\,4 & 0.798\,9 \end{bmatrix},$$
$$\boldsymbol{u}^*(t) := [0.182\,7 \quad -0.113\,7]x(t),$$
$$\boldsymbol{v}^*(t) := [-0.108\,5 \quad 0.038\,4]x(t),$$

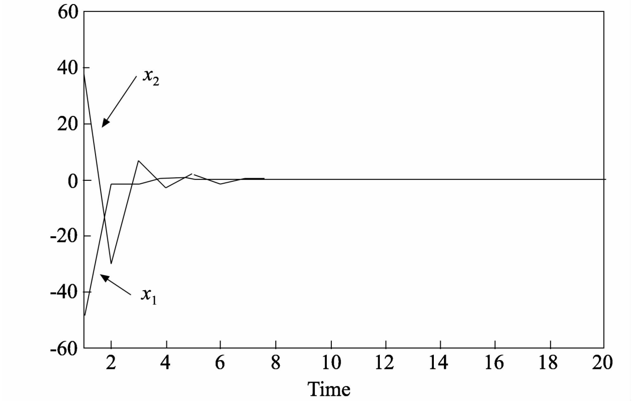

Fig. 2 Trajectories of x_1 and x_2

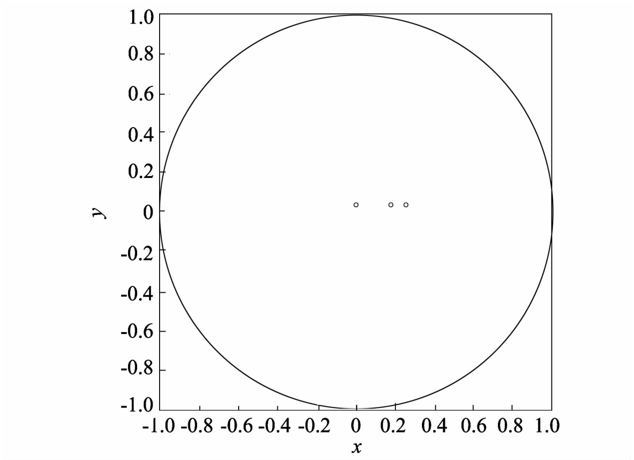

Fig. 3 Spectra of system (17)

4 Conclusion

Different from the continuous case, we confine our discussion on regional stabilization of discrete systems in the unit circle. Asymptotically mean square stabilization can be considered as a special case of $D(0,\alpha)$-stabilization with $0 < \alpha \leqslant 1$, which is defined and investigated still by means of the spectrum technique. Just because the validity of $D(0,\alpha)$-stabilization is equivalent to the feasibility of a linear matrix inequality, we combine a suboptimal H_2/H_∞ design with a spectrum placement constraint. Software like MATLAB's LMI Control Toolbox is available to solve such LMIs in a fast and user-friendly manner[18].

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