

# Accurate parameter estimation of systematic odometry errors for two-wheel differential mobile robots

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**Abstract:** Odometry using incremental wheel encoder sensors provides the relative robot pose estimation. However, the odometry suffers from the accumulation of kinematic modeling errors of wheels as the robot's travel distance increases. Therefore, the systematic errors need to be calibrated. The University of Michigan Benchmark (UMBmark) method is a widely used calibration scheme of the systematic errors in two-wheel differential mobile robots. In this paper, the accurate parameter estimation of systematic errors is proposed by extending the conventional method. The contributions of this paper can be summarized as two issues. The first contribution is to present new calibration equations that reduce the systematic odometry errors. The new equations were derived to overcome the limitation of conventional schemes. The second contribution is to propose the design guideline of the test track for calibration experiments. The calibration performance can be improved by appropriate design of the test track. The simulations and experimental results show that the accurate parameter estimation can be implemented by the proposed method.

**Key words:** calibration; kinematic modeling errors; mobile robots; odometry; test tracks

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In autonomous navigation, the position of a mobile robot needs to be estimated accurately. Odometry information using the encoder is the most widely used navigation method for mobile robot positioning. However, the odometry has a well-known drawback: its errors accumulate over time as the robot moves. In order to reduce the odometry errors with the increase of travel distance, the kinematic modeling errors need to be corrected.

Odometry error sources are classified into systematic errors and non-systematic errors<sup>[1,2]</sup>. The systematic error sources include unequal wheel diameters, uncertainty of the effective wheelbase. These are vehicle specific and do not usually change during navigation. Therefore, it is essential to reduce these errors by calibrating kinematic error parameters.

The non-systematic error sources result from the environmental conditions, which are stochastic. Examples are uneven floors, wheel slippages, which are significant problems in the practical application of calibration experiments. These errors can be modeled by using the robot's absolute position from the external sensors<sup>[3,4]</sup>.

Calibration of the systematic odometry errors

have been discussed in many studies. Kelly<sup>[5]</sup> suggested a general solution for linearized systematic error propagation for an optional trajectory. Abbas<sup>[6]</sup> introduced a bi-directional circular path test, in which the robot is driven along a circular reference path. Bostani<sup>[7]</sup> suggested a simple method based on two experiments, in which the robot is programmed to move back and forth in a straight line to estimate the kinematic parameters.

The University of Michigan Benchmark (UMBmark) method<sup>[1]</sup> is the conventional calibration scheme of two-wheel differential mobile robots. The wheel radius error and wheelbase error can be calibrated by driving the robot along a bi-directional square path, and by using the final position errors. This paper proposes a new calibration strategy by extending the conventional UMBmark.

The first objective is to derive a new accurate calibration scheme by investigating the limitation of Ref. [1]. The calibration strategy in Ref. [1] was derived under the assumption that the wheel radius error and the wheelbase error are completely independent. The new calibration equations in this paper are derived from the coupled effect between the two errors.

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The second objective is to present the design guideline of the test track for calibration experiments. From the experiences, the authors recognized that the calibration performance can be significantly improved by appropriate design of the test track. The presented numerical simulations show that the appropriate selection of the track size is essential to improve the calibration accuracy.

This paper is organized as follows. Section 1 reviews the UMBmark in Ref. [1] and proposes new calibration equations. The requirements of the track design are explained. In section 2, we investigate the advantages of the proposed calibration scheme and importance of the track design through numerical simulations. Experimental verifications are shown in section 3. A conclusion is drawn in the end.

## 1 Accurate parameter estimation

### 1.1 UMBmark method in Ref. [1]

Fig. 1 shows the experimental motion that was proposed in UMBmark<sup>[1]</sup>. Two sources of systematic errors were assumed: wheel radius error and wheelbase error. In Ref. [1], it was assumed that two error sources independently affect the final positional errors of a robot. Two kinematic error parameters are calibrated from positional errors.

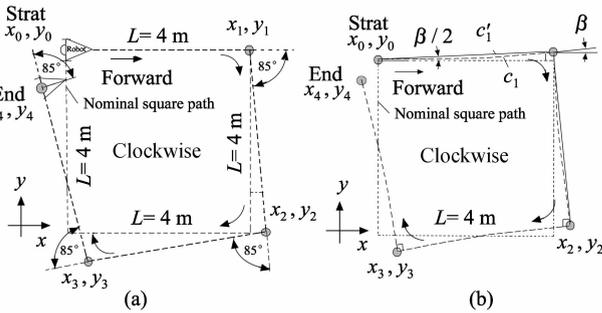


Fig. 1 Calibration tests in UMBmark<sup>[1]</sup>

In Ref. [1], the wheelbase error affects the orientation error during the rotating motion at corners and the wheel radius modeling errors cause the curved motion in straight path in test track as shown in Fig. 1. The final position of the robot is determined through highly nonlinear equations. Furthermore, there is a coupled effect between the radius error and the wheelbase error. In a strict sense, the principle of superposition in Ref. [1] is invalid. We propose a new calibration equation under the consideration of the simultaneous occurrence of the two errors. Also, the size of the test track was  $4\text{ m} \times 4\text{ m}$  without detailed explanation.

The kinematic model of a mobile robot is a two-wheel differential drive robot. The odometry pose

of a robot can be computed by following equations as in Ref. [8].

### 1.2 New calibration equations by considering the coupled effect of wheelbase error and unequal wheel diameters

To investigate the coupled effect between the wheel radius and wheelbase errors, we assume that two errors occur simultaneously. Fig. 2 shows that the orientation errors after a straight motion and a  $90^\circ$  turning motion in counter-clockwise (CCW) direction. The initial robot heading is  $0^\circ$ . The resultant robot orientation may contain some errors because of wheel radius error as well as wheelbase error. The simultaneous occurrence of two errors was not considered in Ref. [1].

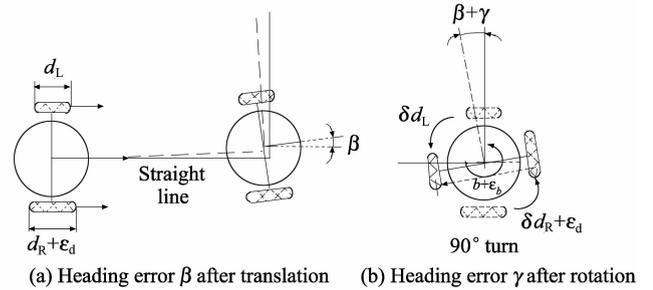


Fig. 2 The coupled effect in  $90^\circ$  rotation motion because of the wheelbase error and the wheel diameter errors

When the right wheel radius is larger than the radius of a left wheel as  $\epsilon_d$ , orientation error  $\beta$  takes place after a translational motion by 4 m as shown in Fig. 2(a). Then, the robot pose before the turn (gray) and the robot pose after the turn (black) are shown in Fig. 2(b). The orientation error  $\alpha$  for  $90^\circ$  rotational motion is newly defined as

$$\alpha = \frac{x_{cw} + x_{ccw}}{-4L} + \frac{\pi b(x_{cw} - x_{ccw})}{16L^2}, \quad (1)$$

$$\text{or} \quad \alpha = \frac{y_{cw} - y_{ccw}}{-4L} + \frac{\pi b(y_{cw} + y_{ccw})}{16L^2}. \quad (2)$$

The first terms in Eq. (1) and Eq. (2) are identical to the equations in Ref. [1]. The second terms are the additional errors that are newly added in this paper.

### 1.3 Design of the test track

One of the significant factors of odometry calibration is design of the test track. The shape of the test track is identical to the square path in Ref. [1]. However, there are no considerations on the size of the test track in Ref. [1]. The size of the test track should be carefully determined under the consideration of the wheel diameter, the kinematic modeling error and the calibration equations<sup>[9,10]</sup>.

In order to analyze the effect of the non-system-

atic errors on the calibration accuracy, we estimate the standard deviation of the non-systematic errors in the experimental test over square paths with the length of one side varying from 1 m to 4 m.

**Table 1 Non-systematic errors**

| Track size | Standard deviation/cm |
|------------|-----------------------|
| 1 m×1 m    | 5.7                   |
| 2 m×2 m    | 12.2                  |
| 3 m×3 m    | 22.9                  |
| 4 m×4 m    | 29.3                  |

Table 1 shows the estimated standard deviation, obtained through experiments, of the non-systematic errors under four different track sizes. The experimental robot in Ref. [11] is driven 30 times over square paths with the length of one side varying from 1 m to 4 m. The final position errors of the test were measured. The translational velocity of the robot is 0.2 m/s. The commercially available Stargazer system in Ref. [12] was used to measure the absolute position of the robot. The standard deviation of the final position errors in test tracks are applied to the numerical simulation.

## 2 Simulations

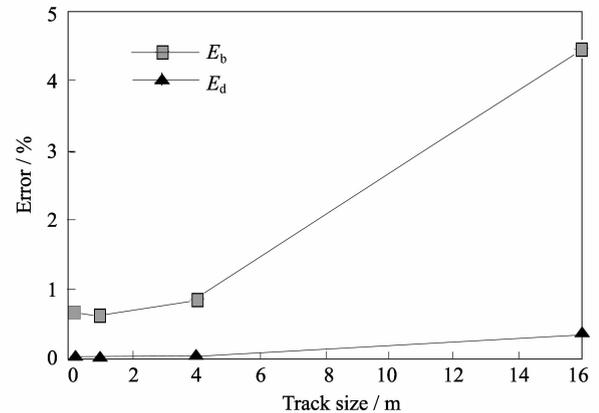
The aim of the numerical simulation is to clarify two contributions of the proposed calibration strategy. The first objective is to show the advantages of the presented calibration scheme in Eq. (1) and Eq. (2). The second objective is to establish the design guideline of a test track.

In simulations, the robot pose is numerically computed from the robot kinematics under kinematic modeling errors. The calibration performance is evaluated by the difference between the real and the estimated  $E_b$  and  $E_d$ . Fig. 3 shows the resultant error of  $E_b$  and  $E_d$  under four different track sizes. The track size is denoted by the length of a side of a square  $L$ . Four selected tracks were  $L = 0.2, 1, 4, 16$  m. Since y-axis represents the kinematic modeling error after calibration, smaller y value is preferable.

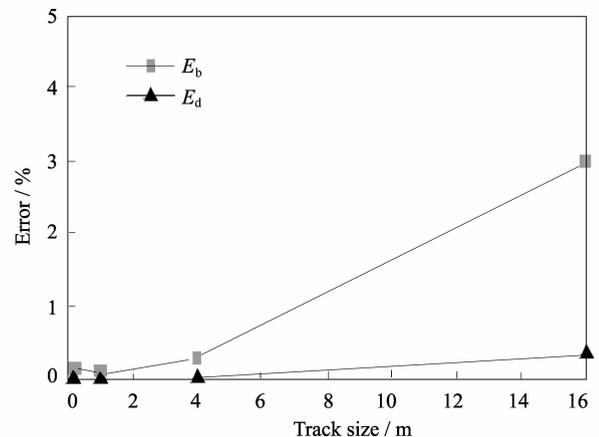
The initial condition was  $E_b = 0.99$ ,  $E_d = 0.99$ . Fig. 3(a) shows the calibration result by using the conventional UMBmark. It can be seen that the parametric errors increased when the track size is too large. From Fig. 3(a), it is clear that  $E_b$  still contains parametric error after calibration. Also, it is recommended to choose  $L \leq 4$  m.

Fig. 3(b) shows the calibration result on the basis of the proposed calibration scheme in Eq. (1) and Eq. (2). It is clear that parametric errors of  $E_b$  become remarkably smaller than in Fig. 3(a), which implies that the simultaneous occurrence of two errors should be modeled in the calibration equation.

To analyze the calibration accuracy from the test track size, we compare the final position errors under the systematic error condition,  $E_b = 0.97$  and  $E_d = 0.98$ .



(a)  $E_b$  and  $E_d$  (UMBmark method)

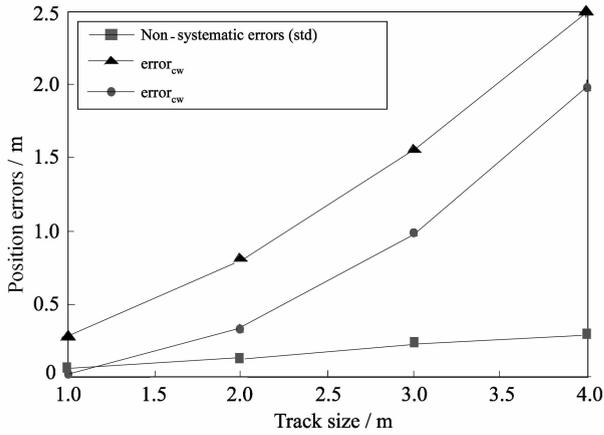


(b)  $E_b$  and  $E_d$  (Proposed method)

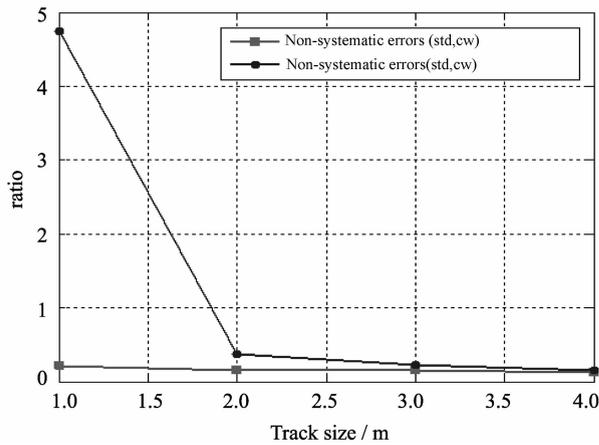
**Fig. 3 Kinematic parameter errors after calibration along four different tracks  $L = 0.2, 1, 4, 16$  m**

Fig. 4 shows the comparison of the final position errors and the standard deviation of the non-systematic errors after CW, CCW runs over square paths with the length of one side varying from 1 m to 4 m. The final position errors increase as the track size increases. However, from Fig. 4(b), it can be seen that the relative size of the non-systematic errors in CW and CCW increase as the track size decreases.

Fig. 5 shows the relative size of the final position errors when the robot was driven along the same  $4 \text{ m} \times 4 \text{ m}$  square path after calibration using different track sizes. The systematic error condition is  $E_b = 0.96$  and  $E_d = 0.98$ . It shows that the relative size of final position errors obtained through calibration by  $2 \text{ m} \times 2 \text{ m}$  track size is smaller than others. Therefore, we get the conclusion that the calibration accuracy is improved when  $2 \text{ m} \times 2 \text{ m}$  track size is applied to odometry calibration.



(a) Comparison of two errors



(b) Relative size of non-systematic errors

Fig. 4 Comparison of the final position errors and the standard deviation of the non-systematic errors

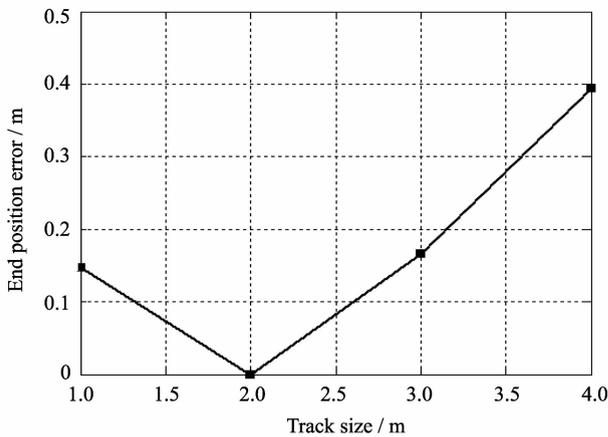


Fig. 5 Comparison of the relative final position errors for a 4 m x 4 m test path of 2 m x 2 m track with different track sizes

## 3 Experimental results

### 3.1 Experimental setup

In this section, we present experimental results

that validate the proposed design of the track size for improving the calibration accuracy.

### 3.2 Calibration experiments and performance comparison

Fig. 6 shows the two-wheel differential driving robot from Ref. [11] used for the experiments in this study. The configurations of the robot are: wheel diameter is 150 mm, wheelbase is 385 mm, encoder resolution is 10 000 pulses/rev; and sampling time of encoder signal is 0.1 s.

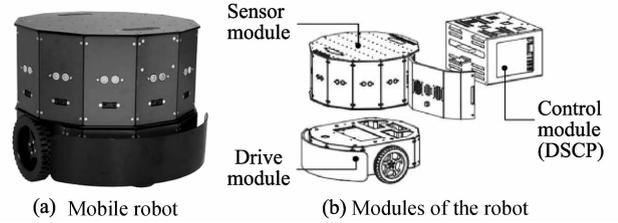


Fig. 6 Experimental setup

The robot is driven by open loop control along the square path. The moving directions of a robot should include both CW and CCW. If the calibration is successful, the final pose after calibration converges to the origin. The pose distribution around the center represents the stochastic non-systematic errors. The final positions during the calibration experiments are plotted in Fig. 7.

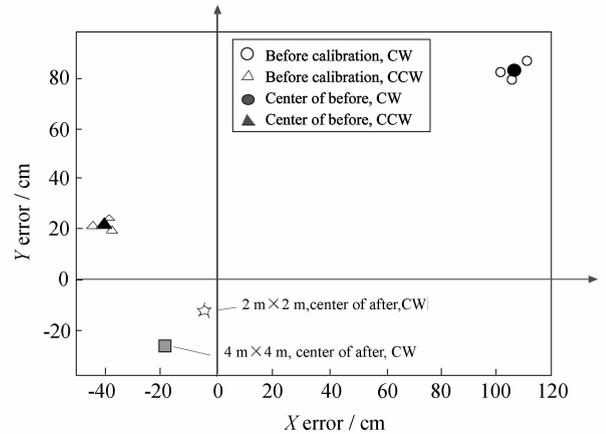


Fig. 7 Comparison between proposed method (2 m x 2 m) and UMBmark method (4 m x 4 m)

Fig. 7 and Table 2 show the experimental results in CW direction. The final positional error before calibration was 134.5 cm. After application of the conventional UMBmark in Ref. [1], the error was reduced to 32.4 cm. Therefore, the odometry accuracy was increased by four times by the UMBmark.

The final pose error after the application of the proposed scheme was 12.8 cm. The odometry accuracy of the proposed scheme is 2.5 times higher than that of the conventional UMBmark approach. This

result clearly shows the advantages of the proposed calibration scheme over the prior approach.

**Table 2 Results of calibration experiments**

|                 | Track size | Before Calibration | After Calibration |
|-----------------|------------|--------------------|-------------------|
| UMBmark method  | 4 m×4 m    | 134.5 cm           | 32.4 cm           |
| Proposed method | 2 m×2 m    | 134.5 cm           | 12.8 cm           |

## 4 Conclusion

This paper proposes the accurate parameter estimation scheme for calibration of the systematic odometry errors in two-wheel differential mobile robots. The first contribution is to derive new calibration equations by considering the coupled effect of wheel diameter errors and wheelbase errors. The presented simulations and experiments clearly show that the proposed scheme provides more accurate calibration results than the conventional scheme. The second contribution is the suggestion of the appropriate size of the test track for calibration. The proposed scheme is experimentally verified through quantitative comparison.

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