A novel mathematical model on Peer-to-Peer botnet

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Abstract: Peer-to-Peer (P2P) botnet has emerged as one of the most serious threats to Internet security. To effectively eliminate P2P botnet, a delayed SEIR model is proposed, which can portray the formation process of P2P botnet. Then, the local stability at equilibria is carefully analyzed by considering the eigenvalues' distributed ranges of characteristic equations. Both mathematical analysis and numerical simulations show that the dynamical features of the proposed model rely on the basic reproduction number and time delay τ. The results can help us to better understand the propagation behaviors of P2P botnet and design effective counter-botnet methods.

Key words: Peer-to-Peer (P2P) botnet; stability; SEIR model; time delay

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0 Introduction

Botnet is a network with thousands (or more) of compromised hosts running malicious software under the control of a bot-master, which usually recruits new vulnerable computers by running all kinds of malicious softwares, such as Trojan horses, worms and computer viruses, etc[1]. Botnets with a large number of zombie hosts are usually used for distributed denial-of-service attacks (DDoS), email spam and password cracking, etc[2]. Botnets have been turned out to be one of the most serious threats to Internet[3].

The first botnet is Internet relay chat (IRC) botnet where the attacker establishes an IRC server and opens a specific channel to transmit its commands and bots connect to the channel to accept commands[4]. This architecture is easy to be constructed and efficient for the communication between bots and their master. However, there is a severe default that shutting off servers of botnets will result in all bots losing contact with their bot-master. In addition, botnets are easily checked and cracked by defenders. In comparison, Peer-to-Peer (P2P) botnets employing a distributed command-and-control structure are more robust and difficult for security community to defend. Thus, P2P botnets, such as Trojan. Peacomm, Storm botnet[4], have emerged and become popular in recent years. Also, P2P botnets represent an escalation in the increasingly sophisticated technological antagonism between attackers and defenders. Further, the potential for more damage exits in the future.

Therefore, security researchers must develop new methods to mitigate the threat of P2P botnet. At present, there is a lot of research on P2P botnet[5-10]. Inspired by the Storm botnet, the authors[5] developed a stochastic model of P2P botnet formation, which provided insight into possible defense tactics. YAN et al.[6] mathematically analyzed the performance of a new type of P2P botnet - Anti-Bot from perspectives of reachability, resilience to pollution and scalability. Meanwhile, they developed a P2P botnet simulator to evaluate the effectiveness of theoretical analysis. Furthermore, the authors suggest some potential defense schemes for defenders to effectively destroy Anti-Bot operations. Kolesnichenko et al.[7] developed the mean-field model to analyze P2P botnet behaviors and compared it with simulations obtained from the Moebius tool. The results show that mean-field method is much faster than simulation for defending botnet. These existing researches provide wide insight for us to deeply understand propagation mechanism of P2P botnets. However, their work is only concentrated on a kind of specific botnet.

To describe the dynamics of P2P botnets in a more effective way, we employ the epidemic model of computer worms, which has been widely used by

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many researchers to study Internet malware propagation[11-20]. As many botnets are created by computer worms[21], it is reasonable to describe the prevalence of P2P botnets using the model of worm propagation. In this paper, the dynamics of *Leaching P2P botnet* are investigated. In a *Leaching P2P botnet*, bot-masters recruit new zombies on the whole Internet. For constructing this kind of P2P botnet, there are two steps: the first step is trying to infect new vulnerable hosts throughout the whole Internet, and the second step is joining newly compromised hosts into network and connecting them with other bots[22]. Directing at the formation of *Leaching P2P botnet*, a delayed SEIR model is proposed and studied in this paper.

1 Delayed SEIR model

Assume that every node in the network is viewed as a host, the state of which can be S of the susceptible hosts, E of the infected but not infective hosts, I of the infective hosts, and R of the recovered hosts. As a result, we propose the following model that can be represented as a flow diagram (see Fig. 1) or as a set of coupled differential equations as follows:

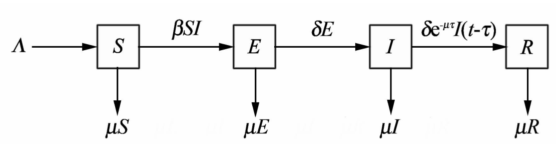

Fig. 1 Flow diagram for SEIR model with delay

$$\begin{cases}\dfrac{\mathrm{d}S}{\mathrm{d}t} = \Lambda - \beta S(t)I(t) - \mu S(t),\\ \dfrac{\mathrm{d}E}{\mathrm{d}t} = \beta S(t)I(t) - (\mu+\delta)E(t),\\ \dfrac{\mathrm{d}I}{\mathrm{d}t} = \delta E(t) - \mu I(t) - \delta \mathrm{e}^{-\mu\tau}I(t-\tau),\\ \dfrac{\mathrm{d}R}{\mathrm{d}t} = \delta \mathrm{e}^{-\mu\tau}I(t-\tau) - \mu R(t),\end{cases} \tag{1}$$

where Λ is the new number of the hosts; μ is the death rate of the hosts; δ is the transition rate from E to I, and β is the constant contact rate between I and S. The term $\delta \mathrm{e}^{-\mu\tau}I(t-\tau)$ represents that an individual has survived natural death in I before recovering.

System (1) needs to be analyzed with the initial condition

$$S(t)\geqslant 0,\ E(t)\geqslant 0,\ t\in[0,\infty] \text{ and}$$

$$I(t)\geqslant 0,\ R(t)\geqslant (0),\ t\in[-\tau,\infty).$$

Summing up the four equations of system (1) and denoting the number of the total hosts by $N(t)$, one can obtain the following equation

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = \Lambda - \mu N(t).$$

Hence, solutions of system (1) need to satisfy the following condition

$$S(t)+E(t)+I(t)+R(t)\leqslant \frac{\Lambda}{\mu}.$$

The feasible region of system (1) can be described as

$$D = \left\{(S,E,I,R)\,\middle|\,\begin{array}{l}0\leqslant S(t)\leqslant N(t),\ 0\leqslant E(t)\leqslant N(t),\ 0\leqslant I(t)\leqslant N(t),\ 0\leqslant R(t)\leqslant N(t),\\ S(t)+E(t)+I(t)+R(t)=N(t),\ S(t)+E(t)+I(t)+R(t)\leqslant \dfrac{\Lambda}{\mu}\end{array}\right\}.$$

2 Mathematical analysis of equilibria

In this section, we will analyze the dynamical behavior of system (1) by investigating the stability of its equilibria. Notably, the first three equations in system (1) do not depend on the fourth equation; therefore, without loss of generality, the fourth equation can be omitted and system (1) can be rewritten as

$$\begin{cases}\dfrac{\mathrm{d}S}{\mathrm{d}t} = \Lambda - \beta S(t)I(t) - \mu S(t),\\ \dfrac{\mathrm{d}E}{\mathrm{d}t} = \beta S(t)I(t) - (\mu+\delta)E(t),\\ \dfrac{\mathrm{d}I}{\mathrm{d}t} = \delta E(t) - \mu I(t) - \delta \mathrm{e}^{-\mu\tau}I(t-\tau).\end{cases} \tag{2}$$

Now, we analyze system (2) by finding the equilibria and their dynamical features. The equilibria of system (2) are given by solutions of

$$\begin{gathered}\Lambda - \beta S(t)I(t) - \mu S(t) = 0,\\ \beta S(t)I(t) - (\mu+\delta)E(t) = 0,\\ \delta E(t) - \mu I(t) - \delta \mathrm{e}^{-\mu\tau}I(t-\tau) = 0.\end{gathered} \tag{3}$$

System (2) always has a free-equilibrium

$$Q_0 = (S_0, E_0, I_0) = \left(\frac{\Lambda}{\mu}, 0, 0\right).$$

Further, denote

$$R_0 = \frac{\beta\Lambda\delta}{\mu(\mu+\delta)(\mu+\delta \mathrm{e}^{-\mu\tau})}. \tag{4}$$

If $R_0>1$, system (2) has a endemic-equilibrium

$$Q^* = (S^*, E^*, I^*)$$

where

$$S^* = \frac{\Lambda\mu - \Lambda\beta\delta - \mu(\mu + \delta e^{-\mu\tau})(\mu + \delta)}{\beta\delta\mu},$$

$$E^* = \frac{\Lambda\beta\delta - \mu(\mu + \delta e^{-\mu\tau}(\mu + \delta)}{\beta\delta(\mu + \delta)},$$

$$I^* = \frac{\Lambda\beta\delta - \mu(\mu + \delta e^{-\mu\tau}(\mu + \delta)}{\beta(\mu + \delta)(\mu + \delta e^{-\mu\tau})}.$$

2.1 Stability of free-equilibrium Q_0

The characteristic equation of system (1) at Q_0 is

$$\det\begin{pmatrix} -\mu-\lambda & 0 & -\beta\dfrac{\Lambda}{\mu} \\ 0 & -(\mu+\delta)-\lambda & \beta\dfrac{\Lambda}{\mu} \\ 0 & \delta & -(\delta e^{-\mu\tau}+\mu)-\lambda \end{pmatrix} = 0, \tag{5}$$

which is equivalent to

$$(\mu + \lambda)(\lambda^2 + a\lambda + b) = 0, \tag{6}$$

where $a = 2\mu + \delta + \delta e^{-\mu\tau}$,

$$b = (\mu + \delta)(\mu + \delta e^{-\mu\tau}) - \delta\beta\frac{\Lambda}{\mu}.$$

Eq. (6) always has a negative characteristic root: $\lambda = -\mu$. Other roots of Eq. (6) are determined by the following equation

$$\lambda^2 + a\lambda + b = 0. \tag{7}$$

Obviously, when $R_0 < 1$, in accordance with the relationship between roots and coefficients of quadratic equation, there is no positive real part characteristic root of Eq. (7). Hence, the following theorem holds.

Theorem 1 If $R_0 < 1$, the free-equilibrium point Q_0 is locally asymptotically stable; if $R_0 > 1$, Q_0 is unstable.

2.2 Dynamical properties of endemic-equilibrium Q^*

In this subsection, using time delay as the bifurcation parameter, we investigate the Hopf bifurcation for system (2). The Jacobian matrix of system (2) at Q^* is

$$\boldsymbol{J}(Q^*) = \begin{pmatrix} -\mu - \beta I^* - \lambda & 0 & -\beta S^* \\ \beta I^* & -(\mu + \delta) - \lambda & \beta S^* \\ 0 & \delta & -\delta e^{-\mu\tau} e^{-\lambda\tau} - \mu - \lambda \end{pmatrix}. \tag{8}$$

The corresponding characteristic equation of $J(Q^*)$ has the following form

$$\lambda^3 + p_0(\tau)\lambda^2 + p_1(\tau)\lambda + p_2 + (p_3(\tau)\lambda^2 + p_4(\tau)\lambda + p_5(\tau))e^{-\lambda\tau} = 0, \tag{9}$$

where $p_0(\tau) = 3\mu + \beta I^* + \delta$, $p_1(\tau) = 3\mu^2 + 2\delta\mu + \beta I^* + \beta\mu I^* + \delta\beta I^*$,

$$p_2(\tau) = \mu(\mu + \delta)(\mu + \beta I^*) + \delta\beta^2 e^{-\mu\tau} S^* I^*, \quad p_3(\tau) = \delta e^{-\mu\tau},$$

$$p_4(\tau) = (\delta + 2\mu + \beta I^*)\delta e^{-\mu\tau}, \quad p_5(\tau) = (\mu + \delta)(\mu + \beta I^*)\delta e^{-\mu\tau},$$

When $\tau = 0$, Eq. (9) can be simplified to

$$\lambda^3 + q_0(0)\lambda^2 + q_1(0)\lambda + q_2(0) = 0, \tag{10}$$

where $q_0(0) = p_0(0) + p_3(0)$,

$$q_1(0) = p_1(0) + p_4(0), !$$

$$q_2(0) = p_2(0) + p_5(0).$$

According to the Routh-Hurwitz criterion, the real parts of all roots of Eq. (10) are negative if and only if $q_i(0) > 0$, $(i = 0, 1, 2)$ and $q_0(0)q_1(0) - q_2(0) > 0$ hold.

Lemma 1 If $R_0 > 1$, the endemic-equilibrium Q^* of system (2) is locally asymptotically stable when $\tau = 0$ and $q_0 q_1 - q_2 > 0$.

Next, we will discuss the properties of Q^* when $\tau > 0$. If $\lambda = i\omega (\omega > 0)$ is a solution of Eq. (9), separating real and imaginary parts, we derive that

$$-\omega^3 + p_1(\tau)\omega + p_4(\tau)\omega\cos^{\omega\tau} + (p_3(\tau)\omega^2 - p_5(\tau))\sin^{\omega\tau} = 0,$$

$$-p_0(\tau)\omega^2 + p_2(\tau) + p_4(\tau)\omega\sin^{\omega\tau} + (p_5(\tau) - p_3(\tau))\cos^{\omega\tau} = 0. \tag{11}$$

Squaring and adding the two equations of Eq. (11), it follows that

$$\omega^6 + g_1\omega^4 + g_2\omega^2 + g_3 = 0, \tag{12}$$

where $g_1 = p_0^2 - 2p_1 - p_3^2$,

$$g_2 = p_1^2 - 2p_0p_2 - p_4^2 + 2p_3p_5,$$

$$g_3 = p_2^2 - p_5^2.$$

Let $z = \omega^2$ and rewrite Eq. (12) as

$$z^3 + g_1 z^2 + g_2 z + g_3 = 0. \tag{13}$$

Denote

$$h(z) = z^3 + g_1 z^2 + g_2 z + g_3.$$

Since $\lim_{z\to\infty} h(z) = +\infty$, Eq. (34) at least has a positive real root when $g_3<0$. Suppose

H(1): $$g_3 < 0.$$

In the following, we will discuss the distributions of the positive roots of Eq. (13).

Lemma 2 Define

$$\Delta = \frac{4}{27}g_2^3 - \frac{1}{27}g_1^2 g_2^2 + \frac{4}{27}g_1^3 g_3 - \frac{2}{3}g_1 g_2 g_3 + g_3^2.$$

Then, the necessary and sufficient conditions for Eq. (13) to have one simple positive real root for z are

(i) either $g_1>0$, $g_2\geqslant 0$ and $g_1^2>3g_2$, or $g_2<0$;

(ii) $\Delta < 0$.

(Detailed proof can be seen in Ref. [23]).

Assuming that z_0 is a positive root of Eq. (34), then Eq. (12) has a positive root $\omega_0 = \sqrt{z_0}$. It follows from Eq. (11) that

$$\cos^{\omega_0 \tau_0} = \frac{h_1 + h_2 + h_3 + h_4}{h_5^2 + h_6^2}.$$

where

$$h_1 = p_4 \omega^4,$$

$$h_2 = - p_1 p_4 \omega^2,$$

$$h_3 = p_0(p_3 - p_5)\omega^2,$$

$$h_4 = p_2(p_3 - p_5),$$

$$h_5 = p_4 \omega,$$

$$h_6 = p_3 - p_5.$$

Therefore

$$\tau_0^j = \frac{1}{\omega_0}\left[\arccos\left(\frac{h_1 + h_2 + h_3 + h_4}{h_5^2 + h_6^2}\right)\right],$$

$$j = 0,1,2,\cdots.$$

Then $\pm\, \mathrm{i}\omega_0$ is a pair of purely imaginary roots of Eq. (9) with τ_0^j Define

$$\tau_0 = \min\{\tau_0^j\}. \tag{14}$$

Further, we need verify the transversality condition

$$\left.\frac{\mathrm{dRe}(\lambda(\tau))}{\mathrm{d}\tau}\right|_{\tau=\tau_0} \neq 0. \tag{15}$$

Taking the derivative of λ with respect to τ in Eq. (9), it is easy to obtain

$$\frac{\mathrm{d}\lambda}{\mathrm{d}\tau} = \frac{\lambda \mathrm{e}^{-\lambda\tau}(p_3\lambda^2 + p_4\lambda + p_5)}{3\lambda^2 + 2p_0\lambda + p_1 + [2p_3\lambda + p_4 - \tau(p_3\lambda^2 + p_4\lambda + p_5)]\mathrm{e}^{-\lambda\tau}}. \tag{16}$$

Substituting $\lambda = \mathrm{i}\omega_0$ into Eq. (16), one has

$$\left.\frac{\mathrm{d}\lambda}{\mathrm{d}\tau}\right|_{\tau=\tau_0} = \frac{k_1 + \mathrm{i}k_2}{k_3 + \mathrm{i}k_4},$$

where $$k_1 = p_5\omega_0\sin^{\omega\tau} - p_3\omega_0^3\sin^{\omega\tau} - p_4\omega_0^2\cos^{\omega\tau},\ k_2 = p_5\omega_0\cos^{\omega\tau} + p_4\omega_0^2\sin^{\omega\tau} - p_3\omega_0^3\cos^{\omega\tau},$$

$$k_3 = p_1 - 3\omega_0^2 + p_3\omega_0\sin^{\omega\tau} + p_4\cos^{\omega\tau} + p_3\tau\omega_0^2\cos^{\omega\tau} - p_4\tau\omega_0\sin^{\omega\tau} - p_5\tau\cos^{\omega\tau},$$

$$k_4 = 2p_0\omega + 2p_3\omega_0\cos^{\omega\tau} - p_4\sin^{\omega\tau} - p_3\tau\omega_0^2\sin^{\omega\tau} - p_4\tau\omega_0\cos^{\omega\tau} + p_5\tau\sin^{\omega\tau}.$$

Therefore

$$\left.\frac{\mathrm{dRe}(\lambda(\tau))}{\mathrm{d}\tau}\right|_{\tau=\tau_0} = \frac{k_1k_3 - k_2k_4}{k_3^2 + k_4^2}.$$

Suppose

H(2): $$\left.\frac{\mathrm{dRe}(\lambda(\tau))}{\mathrm{d}\tau}\right|_{\tau=\tau_0} \neq 0.$$

Summarizing the above analysis, it is easy to obtain the following theorem.

Theorem 2 If Lemmas 1 and 2, H(1) and H(2) satisfied, by combining Ref. [24], the following results hold: there exists τ_c such that the endemic-equilibrium Q^* of system (1) is asymptotically stable for $0\leqslant \tau < \tau_c$, and is unstable for $\tau \geqslant \tau_c$, with a Hopf bifurcation occurring when $\tau = \tau_c$.

3 Numerical simulations

In this section, we perform some numerical simulations to verify our mathematical analysis. In the first part, to check Theorem 1, we choose a set of parameters as follows: $\Lambda = 20$, $\mu = 0.3$, $\beta = 0.02$ and $\delta = 0.7$. By calculations, we can get $R_0 = 0.953\,1$. Fig. 2 demonstrates that simulation results are consistent with the mathematical analysis. From Fig. 2, we can see that the P2P botnet will be completely controlled when $R_0 < 1$. The conclusion agrees with Theorem 1.

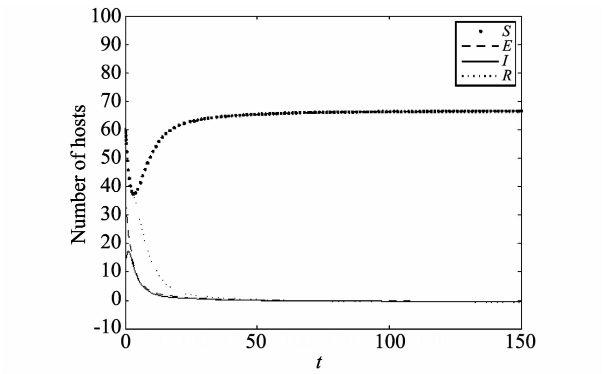

Fig.2 P2P botnet propagation results with $R_0 = 0.953\ 1$

Moreover, we choose $\Lambda = 10$, $\mu = 0.12$, $\beta = 0.02$ and $\delta = 0.6$ to verify Theorem 2. By calculations, we have $\tau_c = 2.532\ 3$ and $R_0 = 2.467\ 9$. We depict the dynamic properties of the endemic-equilibrium Q^* of system (1) at $\tau = 2$ and $\tau_c = 2.532\ 3$ in Figs.3 and 4, respectively.

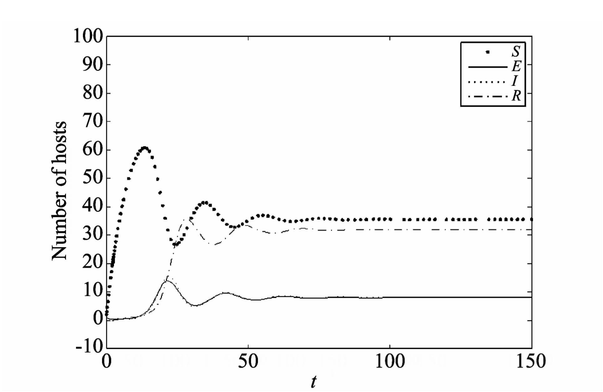

Fig.3 P2P botnet propagation results with $R_0 = 2.467\ 9$ and $\tau = 2$

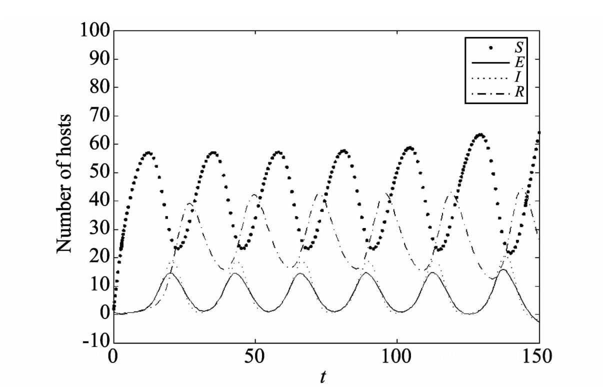

Fig.4 P2P botnet propagation results with $R_0 = 2.467\ 9$ and $\tau_c = 2.532\ 3$

Fig.3 shows that the trajectory converges to the endemic-equilibrium when $\tau = 2$. System (1) is periodic solutions when $\tau_c = 2.532\ 3$ in Fig.4. The conclusion agrees with Theorem 2. To well demonstrate Hopf bifurcation, we depict the phase space at $\tau_c = 2.532\ 3$ in Fig.5.

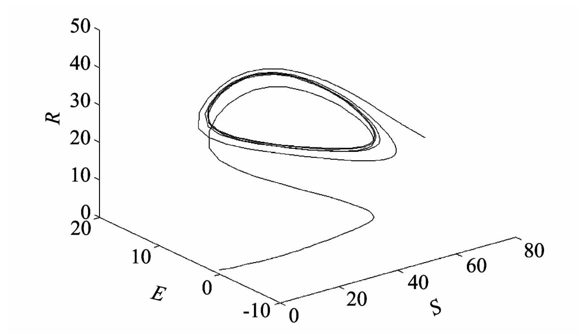

Fig.5 Phase space with $\tau_c = 2.532\ 3$

From the conclusion of Theorems 1 and 2, we learn that it is necessary for eliminating P2P botnet prevalence in networks to make by corresponding countermeasures.

4 Conclusion

According to the formation of *Leaching P2P botnets*, a novel P2P botnet propagation model is proposed in this paper. It has been demonstrated that Hopf bifurcation occurs when the time delay passes through a critical value. In addition, stability has been investigated by considering different distributed ranges of R_0. Furthermore, numerical simulations have been carried out to verify the correctness of mathematical analysis. The results have also shown that the dynamical features of system (1) rely on the basic reproduction number R_0 and time delay τ. The value of τ will impact on the stability and Hopf bifurcation of equilibria, and $R_0 < 1$ can guarantee the extinction of P2P botnet. Decreasing the value of Λ or δ can lead to $R_0 < 1$, which gives us inspiration for predicting and preventing P2P botnet prevalence on the Internet.

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P2P 僵尸网络的新型数学模型

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摘　要： P2P 僵尸网络已成为互联网安全领域最严重的威胁之一。为了有效地遏制 P2P 僵尸网络，本文提出刻画 P2P 僵尸网络形成过程的一种新模型，该模型是带时滞的 SEIR 模型。基于特征方程特征值的分布范围，分析了模型在平衡点的局部稳定性。理论分析和数值模拟结果都表明，该时滞模型的动力学特征依赖于基本再生数 R_0 和时间延迟 τ。本文的结果有助于更好地了解 P2P 僵尸网络的传播行为，并据此设计有效的反制措施。

关键词： P2P 僵尸网络；稳定性；SEIR 模型；时滞

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