Experimental Verification of Doppler Effect with the Refraction Method

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Abstract – The traditional method of measuring Doppler Effect is either reflection or dispersion. This article clarifies that it can also verify the Doppler Effect with the refraction method. We have designed the experimental system with the method of optical heterodyne, using the refraction light beam from a prism, and made the experiment. The experimental results are in accordance with the theoretical calculation. It is very useful in some particular case, such as in Negative-Index Materials(NIM), to verify the Doppler Effect with this method.

***Key words** – refraction method*; *verification the Doppler Effect*; *NIM*

Manuscript Number: 1674-8042(2010)01-0058-03

dio: 10.3969/j.issn.1674-8042.2010.01.12

1 Introduction

The Doppler Effect, named after Austrian physicist Christian Johann Doppler, who proposed it in 1842, is the change of the frequency of a wave for an observer moving relative to the source of the wave[1]. It is commonly heard when a vehicle sounding a siren or horn approaches, passes, and recedes from an observer. The received frequency is higher (compared to the emitted frequency) during the approach, it is identical at the instant of passing by, and it is lower during the recession.

The hypothesis was tested for sound waves by Buys Ballot in 1845. He confirmed that the sound's pitch was higher than the emitted frequency when the sound source approached, and lower than the emitted frequency when the sound source receded. Hippolyte Fizeau discovered independently the same phenomenon on electromagnetic waves in 1848. In Britain, John Scott Russell made an experimental study of the Doppler effect (1848)[2].

Nowadays the Doppler Effect has been applied in many fields, such as astronomy, temperature measurement, radar, medical imaging, and flow measurement, and so on[3-4]. For example, the instrument such as Laser Doppler Velocimeter (LDV) has been developed to measure the velocities of the fluid flow. The LDV emits a light beam, and measures the Doppler shift in a wavelength of reflection from particles moving with the flow.

However, most applications are based on the reflected or scattered light beam to measure the velocity. With the development of science, more and more materials of some new characteristic have been appeared, such as the Negative-Index Materials (NIM). In 2003, N. Seddon and T. Bearpark proposed that the Doppler effect in the NIM could be abnormal theoretically, but nobody has testified it with experiments[5-6], because the traditional methods with reflected or scattered beam cannot be satisfied for this experiment verification. Therefore, an experimental system is designed to measure the Doppler effect with the refracted light beam. Several experiments show that this way can exactly measure the Doppler shift, and offer a credible reference for the further study for the NIM.

2 Principle of the Method

In order to verify the Doppler Effect with the method of refraction, the beam out of the prism is used to measure the Doppler frequency. Since the optical frequency is too high to be measured directly, an experimental system is designed by the means of optical heterodyne[7-8]. The diagram of the experimental system is shown in Fig.1.

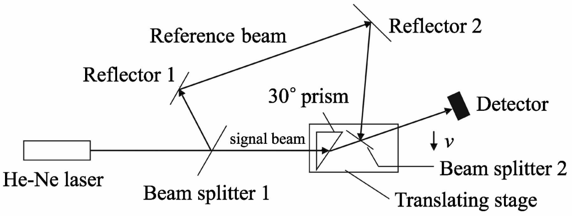

Fig.1 Diagram of the experimental system

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As shown in the Fig.1, the He-Ne laser is used as the light source. The laser is divided into two beams by the beam splitter 1. One is the signal beam and the other is reference beam. The signal beam is normally projected upon the side of the 30° prism. The prism is fixed on a translating stage, which can move at uniform velocity v along the direction as shown in Fig. 1. The reference beam goes though two reflectors to the beam splitter 2. Then two light beams will interfere with each other at the surface of the detector. During the whole experiment, the He-Ne laser is static and the surface of the detector is normal to the emergent light. When the translating stage moves, the Doppler Effect will occur. Because the splitter 2 is also fixed on the translating stage, the signal beam and the reference beam both produce the frequency shift, respectively.

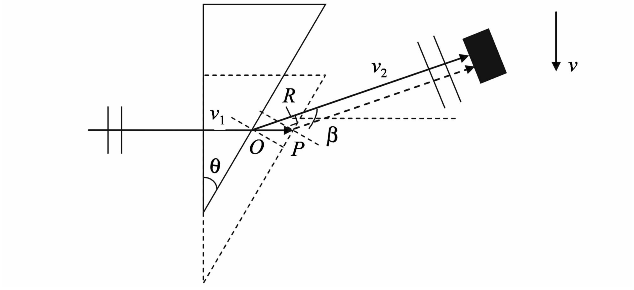

Fig.2 Diagram of the reference beam

The frequency shift of the signal beam can be calculated as follows. The schematic diagram of the light transmitting in the prism is shown in the Fig.2. The solid lines show the original position and the dashed lines show the position after moving. Firstly, as the prism moves towards the detector with the translating stage the optical path of the signal beam inside the prism will be shorter, ($\overline{OP}$ is the displacement of the prism), and then the first Doppler shift occurs. As v is the velocity of the translating stage, the velocity in the direction of light's transmission can be written as

$$v_1 = v\tan\theta.$$

If the frequency of He-Ne laser beam is f_0 , the shifting Doppler frequency at the interface of the prism where the light is refracted is

$$f_1 = f_0(1 - \frac{v_1}{c}n) = f_0(1 - \frac{v\tan\theta}{c}n),$$

where n is the index of the prism, $\theta = 30°$ is the angle of the prism, and c is the velocity of the light in vacuum.

Considering the relative movement between the prism and the detector, the output surface of the prism, which can be considered the light source, moves towards the detector. The velocity of the interface of the prism in the transmission direction of the outgoing light can be written as

$$v_2 = v_1\cos(\beta - \theta),$$

where β is the refraction angle of the outgoing light ($\beta = 49°$ since $n = 1.5163$).

Therefore, the observed frequency detected by the detector can be calculated as

$$f_2 = f_1(\frac{c}{c - v_2}) = f_1[\frac{c}{c - v\tan\theta\cos(\beta - \theta)}].$$

Because the splitter 2 moves together with the translating stage, the reference beam also experiences the frequency shift. The schematic diagram of the reference light system is shown in Fig.3. The beam splitter 2 is placed on the translating stage and its position has been adjusted in order to make the reflected light parallel to the refracted light passing through the prism. Similar to Fig.2, the solid lines show the original position of the beam splitter 2 and the dashed lines show the position after moving. $\overline{O'P'}$ is the displacement of the splitter 2. The velocity of the beam splitter 2 in the transmission direction of the ingoing light can be written as

$$v_3 = \frac{\overline{O'P'}}{\cos(\alpha/2)} = \frac{\cos(\pi/2 - \alpha/2 - \beta + \theta)}{\cos(\alpha/2)}v,$$

where α is the angle between the ingoing light and the reflected light. In the experiment, α is 28.5°. Therefore, the first shifting Doppler frequency at the beam splitter 2 surface is written as

$$f_1' = f_0(1 - \frac{v_3}{c}).$$

Then, the beam splitter 2 can be considered as an effective source of reflected light and its velocity in the transimission direction of the reflected light can be written as $v_4 = v_3\cos\alpha$. Therefore, the second shifting Doppler frequencies at the detector surface can be calculated as

$$f_2' = f_1'(\frac{c}{c + v_4}) = f_0(1 - \frac{v_3}{c})(\frac{c}{c + v_3\cos\alpha}).$$

The frequency differences between the signal beam and the reference beam can be written as

$$\Delta f = | f_2 - f_2' |.$$

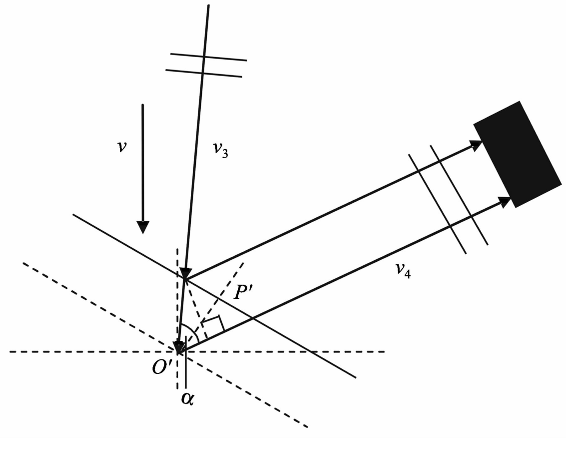

Fig.3 Diagram of the reference beam

3 Experimental results

Tab.1 shows the recorded signal from the detector at different velocity ratios.

Tab.1 The values of beat frequencies with different velocities

Velocity ($\mathrm{mm \cdot s^{-1}}$)	Beat frequency (Hz)		Error(%)
	theoretical	experimental	
0.01	11.588 3	11.485 9	0.8
0.04	46.353 0	45.940 1	0.89
0.11	15.882 5	113.499 1	2.05

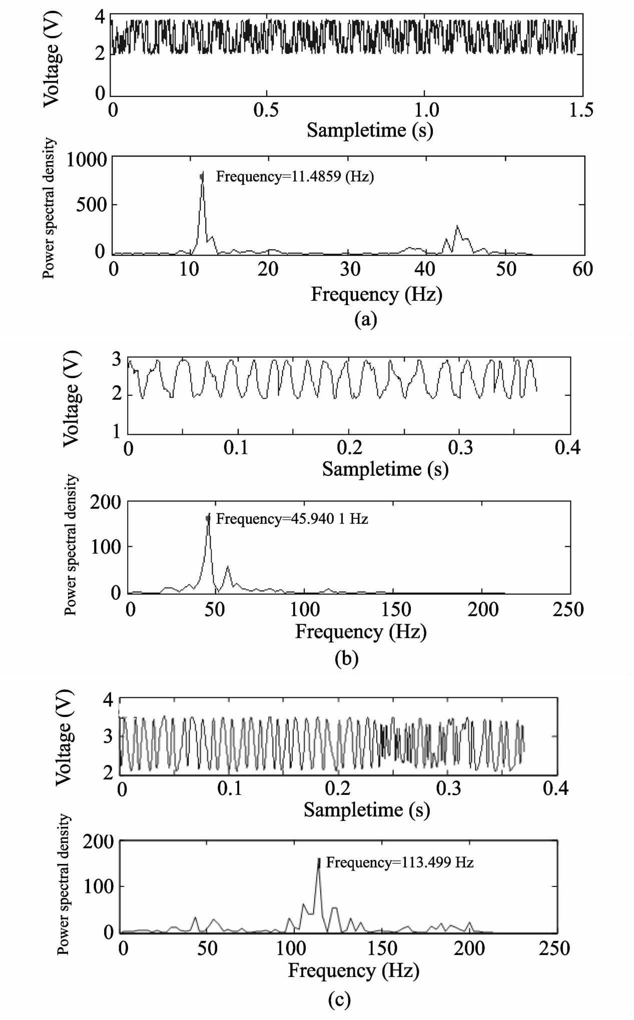

Fig.4 Power density of FFT at different velocities: (a) $\nu=0.01$ mm/s, (b) $\nu=0.04$ mm/s, (c) $\nu=0.1$ mm/s

The experimental beat frequencies can be calculated by using the Fast Fourier Transform (FFT) from the experiment data. The experimental and theoretical results are quite similar, and the errors are less than 3%. This result clearly indicates that the method of refraction to measure the Doppler Effect is effective. In order to explain the experimental results of power vividly, the figures density by FFT are shown in Fig.4.

4 Conclusion

The refraction method for verifying the Doppler Effect is described in the paper. Firstly, design of the experimental system for the method is explained clearly, and then the principle of the method and the calculations of the theoretical values are given. In the experiments, it has been found that the recorded beat frequencies that the detector receives are much close to the results of theoretical calculations, whatever the velocity of the translating stage. This method is very effective to verify the Doppler Effect, especially for NIM.

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