Uncertainty analysis of ship model vertical center of gravity and transverse moment of inertia test

SHI Sheng-zhe, ZHENG Ya-xiong

(*High Speed Hydrodynamic Laboratory, Special Vehicle Research Institute of Aviation Industry Corporation of China, Jingmen 448035, China*)

Abstract: The usability of test results of ship model vertical center of gravity and transverse moment of inertia is generally depends on its uncertainty. Referring to the guidelines for uncertainty analysis in examination of liquid dynamic recommended by International Towing Tank Conference (ITTC), the results were analyzed, bias limits and precision limits were calculated and total uncertainty was estimated. The total uncertainty of six tests on ship model vertical center of gravity is is 0.16% of the mean value, and the total uncertainty of six tests on ship model transverse moment of inertia is 5.66% of the mean value. The test results show that the total uncertainty of both the multiple tests and the single test is from the precision limits of ship model vertical center of gravity and transverse moment of inertia tests. Thus, the improved measurement system stability can enormously decrease the total uncertainty of multiple tests and the single test.

Key words: ship model test; vertical center of gravity; transverse moment of inertia; uncertainty analysis

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Test uncertainty is quantitative analysis of the quanlity of test results. The usability of test results generally depends on uncertainty analysis[1]. In order to improve the test veracity of ship model towing tank, the international towing tank conference (ITTC) recommended tanks to offer test results and the usability of test results, too. China Ship Scientific Research Center (CSSRC) collected and translated the recommended procedures and guidelines of uncertainty analysis of resistance towing tank tests and computational fluid dynamics (CFD) of the ITTC[2]. ZHU et al.[3] calculated viscosity circumferential flow field of bare hull of submarine standard model SUBOFF and analyzed the uncertainty of CFD results using ITTC temporary procedure; ZHOU et al.[4-5] analyzed the uncertainty of form factor, Froude number and the breakwater resistance of a standard ship model; LIU et al.[6] mentioned a new method about uncertainty of wetted surface; SHI et al.[7] analyzed the uncertainty of form factor, wetted surface and Froude number, and presented the analysis results; SUN[8], YANG[9] and ZHOU et al.[10] made uncertainty analysis of CFD for ship and propeller using the procedures recommended by the ITTC.

Most uncertainty analysis of domestic ship model test was resistance test. No mention was made of ship model vertical center of gravity and transverse moment of inertia test. Referring to the guidelines for uncertainty analysis in examination of liquid dynamic of recommended by ITTC[11-14], this paper analyzes the uncertainty of ship model vertical center of gravity and transverse moment of inertia test and lays a foundation for the uncertainty analysis of rolling test in static water and transverse regular wave test at zero speed.

1 Test design

Preparative work of ship model test in static water and transverse regular wave at zero speed is the same as that in resistance test, except that the ballast of the ship model must to be moved to the required state

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Corresponding author: SHI Sheng-zhe (shishengzhe05011232@126.com)

(tonnage, fore draft and aft draft), and the vertical center of gravity and transverse moment of inertia of the ship model need to be adjusted. It is receivable to adjust the vertical center of gravity and transverse moment of inertia on simple edge shelves. The vertical center of gravity can be adjusted on the right-and-left edge shelf, and the transverse moment of inertia can be adjusted on the fore-and-aft edge shelf.

1.1 Principal dimensions of ship model

The resistance test of a 2.5 m fiberglass-reinforced plastics (FRP) ship model was conducted in a towing tank in China Special Vehicle Research Institute in 2012, and the ship model vertical center of gravity and transverse moment of inertia tests were conducted before the resistance test.

Table 1 Principal parameters of ship model

Definition	Value
Length	2.488 m
Breadth	1.032 m
Height	0.592 m
Breadth of waterline	0.768 m
Longitudinal center of gravity (station 0)	0.6592 m
Vertical center of gravity(base line)	0.284 m
Tonnage	113.92 kg

1.2 Measurement of vertical center of gravity

After the ballast of ship model moves to the required state (including test instrument in ship model) by right-and-left edge shelf, the support ship model moves weight for longitudinal balance of ship model horizontally, and then moves little weight along fore and aft to get vertical center of gravity by ship slope. Fig. 1 shows the measurement setup of vertical center of gravity.

The vertical center of gravity is calculated by

$$z_g = H_{OK} - \frac{PLd_{OA}}{D_m \Delta_H}, \qquad (1)$$

where z_g is ertical center of gravity(m); P is the weight of the small moving object(kg); L is the distance of the small moving object(m); d_{OA} is the distance between right-and-left edge and point A(m), and its position in station 0 is the same height as right-and-left edge; D_m is the tonnage of ship model (kg); Δ_H is the vertical distance of point A when the small object moves before and after(m); H_{OK} is the height of right-and-left edge (m), and it is set at 0.33 m.

Fig. 1 Measurement setup of vertical center of gravity

1.3 Measurement of transverse moment of inertia

The support ship model moves a small object along fore and aft at the same height, and adjusts the ship model to transverse balance. There is an initial angle by holding ship model. After clocking ten periods by stopwatch, the ship model transverse moment of inertia is got.

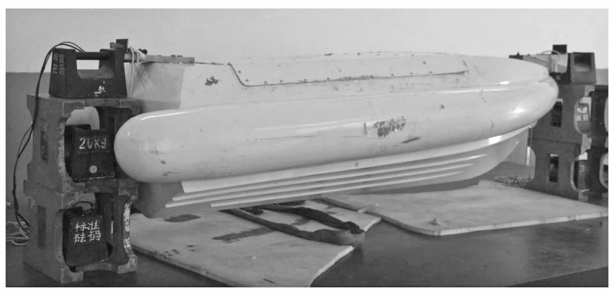

Fig. 2 Measurement setup of transverse moment of inertia

Based on pendulum principle, when ship model gets instantaneous transverse force, the transverse time which circles fore-and-aft edge is

$$T = 2\pi \sqrt{\frac{I + D_m (H_{OK'} - z_g)^2}{D_m g (H_{OK'} - z_g)}}, \qquad (2)$$

where $H_{OK'}$ is the height of fore-and-aft edge (m), and it is set at 0.587 m.

The transverse moment of inertia is

$$I = \left(\frac{T}{2\pi}\right)^2 D_m g (H_{OK'} - z_g) - D_m (H_{OK'} - z_g)^2. \qquad (3)$$

There are total bias limit and total precision limit described as

$$(U_{z_g})^2 = (B_{z_g})^2 + (P_{z_g})^2, \tag{4}$$

$$(U_I)^2 = (B_I)^2 + (P_I)^2. \tag{5}$$

where the total bias limits of vertical center of gravity and transverse moment of inertia of the ship model can be calculated by

$$(B_{z_g})^2 = \left(\frac{\partial z_g}{\partial H_{OK}}B_{OK}\right)^2 + \left(\frac{\partial z_g}{\partial D_m}B_{D_m}\right)^2 + \left(\frac{\partial z_g}{\partial P}B_P\right)^2 + \left(\frac{\partial z_g}{\partial L}B_L\right)^2 + \left(\frac{\partial z_g}{\partial d_{OA}}B_{d_{OA}}\right)^2 + \left(\frac{\partial z_g}{\partial \Delta_H}B_{\Delta_H}\right)^2, \tag{6}$$

$$(B_I)^2 = \left(\frac{\partial I}{\partial T}B_T\right)^2 + \left(\frac{\partial I}{\partial D_m}B_{D_m}\right)^2 + \left(\frac{\partial I}{\partial z_g}B_{z_g}\right)^2 + \left(\frac{\partial I}{\partial H_{OK'}}B_{H_{OK'}}\right)^2. \tag{7}$$

Whatever the precision limits of the single test or multiple tests are, the standard bias must come from multiple tests. If multiple tests can not be conducted, it is necessary to estimate a value for the precision limit by using reliable information. The precision limit of multiple tests is

$$P(M) = \frac{KS_{dev}}{\sqrt{M}}, \tag{8}$$

where M is test time which determines precision limit; S_{dev} is standard bias of multiple tests; K refers to the procedures by ITTC, $K=2$.

The precision limit of single test is

$$P(S) = KS_{dev}. \tag{9}$$

2 Uncertainty analysis

2.1 Bias limit

Bias limit is uncertainty part due to system effect, which can be got by the most allowable error of measurement equipment through combined standard uncertainty.

Distance is measured by altitudinal ruler and meter ruler. The most allowable error of altitudinal ruler, which takes for half width of distribution interval and has rectangular distribution, is 0.000 02 m. When the small object moves before and after, the bias limit of right-and-left edge, $B_{H_{OK}}$, and the bias limit of vertical distance of point A, B_{Δ_H}, are given by

$$B_{H_{OK}} = B_{\Delta_H} = 0.000\,02/\sqrt{3} = 0.000\,011\,6 \text{ m}.$$

The most allowable error of meter ruler is 0.001 m. The bias limit of distance of the small moving object, B_L, and bias limit of distance between right-and-left edge and point A, $B_{d_{OA}}$, are given by

$$B_L = B_{d_{OA}} = 0.001/\sqrt{3} = 0.000\,58\ m.$$

The resolution of digital dynamometer to measure the tonnage of ship model is 0.02 kg, with rectangular distribution. The bias limit of tonnage of the ship model is

$$B_{D_m} = 0.02/\sqrt{3} = 0.116 \text{ kg}$$

The resolution of digital dynamometer to measure the weight of the small moving object is 0.001 kg, with rectangular distribution. The bias limit of weight of the small moving object is

$$B_P = 0.001 \text{ kg}/\sqrt{3} = 0.000\,58 \text{ kg}$$

According to Eq. (1), the relevant parameters are set: $H_{OK}=0.330$ m, $D_m=113.92$ kg, $P=0.168$ kg, $L=2.30$ m, $d_{OA}=0.659$ m and $\Delta_H = 0.049\,3$ m. Thus, the sensitivity coefficients of the vertical center of gravity are given by

$$\frac{\partial z_g}{\partial H_{OK}} = 1,$$

$$\frac{\partial z_g}{\partial D_m} = -\frac{PLd_{OA}}{D_m^2\Delta_H} = -0.000\,4,$$

$$\frac{\partial z_g}{\partial P} = -\frac{Ld_{OA}}{D_m\Delta_H} = -0.271\,53,$$

$$\frac{\partial z_g}{\partial L} = -\frac{Pd_{OA}}{D_m\Delta_H} = -0.19\,83,$$

$$\frac{\partial z_g}{\partial d_{OA}} = -\frac{PL}{D_m\Delta_H} = -0.69\,22,$$

$$\frac{\partial z_g}{\partial \Delta_H} = -\frac{PLd_{OA}}{D_m\Delta_H^2} = -0.930\,96.$$

According to Eq. (6), the bias limit of vertical center of gravity, B_{z_g}, is $1.638\,1\times10^{-4}$ m. This value is 0.06% of z_g.

Table 2 shows the composition of total bias limit of ship model vertical center of gravity. It can be seen that the bias limits of the weight of the small moving object P and right-and-left edge H_{OK} account for 87.58% and 11.88% of total bias limit of vertical center of gravity, respectively. Improving measurement system precision can reduce total bias limit of the ship model vertical center of gravity B_{z_g}.

Table 2 Composition of total bias limit of ship model vertical center of gravity

Definitions	Value	Percent of $(B_{z_g})^2$
$(B_{z_g})^2$	2.832×10^{-8}	
$\left(\frac{\partial z_g}{\partial H_{OK}}B_{H_{OK}}\right)^2$	3.364×10^{-9}	11.88%
$\left(\frac{\partial z_g}{\partial D_m}B_{D_m}\right)^2$	2.153×10^{-11}	0.08%
$\left(\frac{\partial z_g}{\partial P}B_P\right)^2$	2.480×10^{-8}	87.58%
$\left(\frac{\partial z_g}{\partial L}B_L\right)^2$	5.291×10^{-14}	0.00%
$\left(\frac{\partial z_g}{\partial d_{OA}}B_{d_{OA}}\right)^2$	1.612×10^{-11}	0.06%
$\left(\frac{\partial z_g}{\partial \Delta_H}B_{\Delta_H}\right)^2$	1.166×10^{-10}	0.41%

The resolution of digital stopwatch to measures time is 0.1 s. The stopwatch measures the time of ten transverse periods, thus the most allowable error of transverse period is 0.01 s, with rectangular distribution. The bias limit of transverse period is

$$B_T = 0.01/\sqrt{3} = 0.005\ 8\ \text{s}.$$

The right-and-left edge is measured by altitudinal ruler, and the bias limit of right-and-left edge, $B_{H_{OK'}}$, is 0.000 011 6 m.

Let T=1.59 s and $H_{OK'}$=0.587 0. According to Eq. (3), the sensitivity coefficients of transverse moment of inertia are calculated by

$$\frac{\partial I}{\partial T} = 2\frac{T}{2\pi}D_m g(H_{OK'} - z_g)\frac{1}{2\pi} = 27.303\ 57,$$

$$\frac{\partial I}{\partial D_m} = \left(\frac{T}{2\pi}\right)^2 g(H_{OK'} - z_g) - (H_{OK'} - z_g)^2 = 0.098\ 73,$$

$$\frac{\partial I}{\partial z_g} = \left(\frac{T}{2\pi}\right)^2 D_m g + 2D_m(H_{OK'} - z_g) = -2.602\ 56,$$

$$\frac{\partial I}{\partial H_{OK'}} = \left(\frac{T}{2\pi}\right)^2 D_m g - 2D_m(H_{OK'} - z_g) = 2.602\ 56.$$

According to Eq. (7), the bias limit of transverse moment of inertia, B_I, is 0.158 36 kg·m^2, and this value is 1.41% of I.

Table 3 shows the composition of total bias limit of ship model transverse moment of inertia. It can be seen that the transverse period almost accounts for 100% of the total bias limit of ship model transverse moment of inertia. Improving time measurement precision is the most important, which can reduce the bias limit of transverse period T.

Table 3 Composition of total bias limit of ship model transverse moment of inertia

Definitions	Value	Percent of $(B_I)^2$
$(B_I)^2$	2.508×10^{-2}	
$\left(\frac{\partial I}{\partial T}B_T\right)^2$	2.508×10^{-2}	99.99%
$\left(\frac{\partial I}{\partial D_m}B_{D_m}\right)^2$	1.918×10^{-7}	0.01%
$\left(\frac{\partial I}{\partial z_g}B_{z_g}\right)^2$	1.918×10^{-7}	0.00%
$\left(\frac{\partial I}{\partial H_{OK'}}B_{H_{OK'}}\right)^2$	$9.114\text{E}\times10^{-10}$	0.00%

2.2 Precision limit

Precision limit is uncertainty part due to random effect, which results from lack of reiteration, such as random error, instability, impossible accurately re-installation test state, and so on.

For more exact precision limit, the ship model needs to be re-installed before every test, and then the standard bias of a series of tests are evaluated . There are six times in the test. This is the best method including random errors, such as installation error, longitudinal slope and transverse slope.

Table 4 Standard bias of ship model vertical center of gravity and transverse moment of inertia test

Test time	Δ_H (m)	z(m)	T(s)	I(kg·m^2)
1	0.048 3	0.284	1.59	11.251
2	0.048 5	0.284	1.60	11.527
3	0.050 3	0.285	1.60	11.523
4	0.050 1	0.285	1.63	12.358
5	0.049 1	0.284	1.57	10.704
6	0.049 2	0.284	1.55	10.165
Average	0.049 3	0.284	1.59	11.255
Standard bias		0.000 52		0.755 53

Table 4 shows the average and standard bias of ship model vertical center of gravity and transverse moment of inertia test.

Accodting to Eq. (8), the precision limit of multiple tests of ship model vertical center of gravity is

$$P_{\overline{z_g}} = \frac{KS_{\mathrm{dev}\overline{z_g}}}{\sqrt{M}} = \frac{2\times0.000\ 52}{\sqrt{6}} = 4.216\ 4\times10^{-4}\ \text{m}.$$

This value is 0.15% of z_g.

According to Eq. (9), the precision limit of single test of ship model vertical center of gravity is

$$P_{z_g} = K \times S_{\mathrm{dev}_{z_g}} = 2 \times 0.000\ 52 = 1.032\ 8 \times 10^{-3}\ \mathrm{m}.$$

This value is 0.36% of z_g.

According to Eq. (8), the precision limit of multiple tests of the transverse moment of inertia of the ship model is

$$P_I = \frac{KS_{\mathrm{dev}_I}}{\sqrt{M}} = \frac{2 \times 0.755\ 53}{\sqrt{6}} = 0.616\ 9\ \mathrm{kg \cdot m^2}.$$

This value is 5.48% of I.

According to Eq. (9), the precision limit of single test of ship model transverse moment of inertia is

$$P_I = K \times S_{\mathrm{dev}_I} = 2 \times 0.755\ 53 = 1.511\ 1\ \mathrm{kg \cdot m^2}.$$

This value is 13.43% of I.

2.3 Total uncertainty

According to Eqs. (4) and (5), combining bias limits and precision limits of single test and multiple tests, total uncertainty are evaluated.

The total uncertainty of the multiple tests of ship model vertical center of gravity is

$$U_{\bar{z}_g} = ((B_{z_g})^2 + (P_{\bar{z}_g})^2)^2 = 4.544\ 0 \times 10^{-4}.$$

This value is 0.16% of z_g.

The total uncertainty of the single test of ship model vertical center of gravity is

$$U_{z_g} = ((B_{z_g})^2 + (P_{z_g})^{2\frac{1}{2}} = 1.046\ 4 \times 10^{-3}.$$

This value is 0.37% of z_g.

The total uncertainty of the multiple tests of ship model transverse moment of inertia is

$$U_I = ((B_I)^2 + (P_I)^2)^{\frac{1}{2}} = 0.686\ 9.$$

This value is 5.66% of I.

The total uncertainty of the single test of ship model transverse moment of inertia is

$$U_I = ((B_I)^2 + (P_I)^2)^{\frac{1}{2}} = 1.519\ 3.$$

This value is 13.50% of I.

Table 5 shows the composition of total uncertainty. It is helpful to systematical analysis of the total uncertainty of ship model vertical center of gravity and transverse moment of inertia test.

Table 5 Composition of total uncertainty

Definitions	Value	Percent (%)	Annotation
$(U_{\bar{z}_g})^2$	2.061×10^{-7}		
$(B_{z_g})^2$	2.832×10^{-8}	13.74	Percent of $(U_{\bar{z}_g})^2$
$(P_{\bar{z}_g})^2$	1.778×10^{-7}	86.26	Percent of $(U_{\bar{z}_g})^2$
$(U_{z_g})^2$	1.095×10^{-6}		
$(B_{z_g})^2$	2.832×10^{-8}	2.59	Percent of $(U_{z_g})^2$
$(P_{z_g})^2$	1.067×10^{-6}	97.41	Percent of $(U_{z_g})^2 x$
$(U_I)^2$	4.056×10^{-1}		
$(B_I)^2$	2.508×10^{-2}	6.18	Percent of $(U_I)^2$
$(P_I)^2$	3.806×10^{-1}	93.82	Percent of $(U_I)^2$
$(U_I)^2$	2.308		
$(B_I)^2$	2.508×10^{-2}	1.09	Percent of $(U_I)^2$
$(P_I)^2$	2.283	98.91	Percent of $(U_I)^2$

3 Conclusion

1) Improving measurement system precision for the weight of small moving object and right-and-left edge can reduce the total bias limit of ship model vertical center of gravity. Improving time measurement precision is the most important, which can reduce the bias limit of transverse period, and furthermore, can reduce the total bias limit of ship model transverse moment of inertia.

2) Whatever total uncertainty of multiple tests or single test is, primary part of ship model vertical center of gravity and transverse moment of inertia comes from precision limit. Impoving the stability of measurement system can reduce precision limit, and furthermore, can reduce total uncertainty of multiple tests and single test.

3) This test is conducted six times repeatedly. The total uncertainty of multiple tests is half of single test. Therefore, it can be proven that the method of

multiple tests is more effective to reduce total uncertainty than single test.

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船模重心高度和横向转动惯量测量试验的不确定度分析

史圣哲，郑亚雄

（中国特种飞行器研究所 高速水动力实验室，湖北 荆门 448035）

摘　要： 船模重心高度和横向转动惯量测量试验测量结果的可用性很大程度上取决于其不确定度的大小。参照 ITTC 推荐规程中试验流体动力学不确定度分析规范，对船模重心高度和横向转动惯量测量试验进行了不确定度分析，给出了船模重心高度和横向转动惯量的偏差极限、精密度极限和总不确定度。重心高度的六次试验平均值的总不确定度占平均值的 0.16%，横向转动惯量的六次试验平均值的总不确定度占平均值的 5.66%。船模重心高度和横向转动惯量测量试验多次试验平均值的总不确定度和单次试验的总不确定度主要来自精密度极限，提高测量系统的稳定性可以极大的降低多次试验平均值的总不确定度和单次试验的总不确定度。

关键词： 船模试验；重心高度；横向转动惯量；不确定度分析

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