Modeling and simulation of capacitance weighing system

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Abstract: In some cases necessary to weigh vehicles, an electromechanical weighing system has been developed, which consists of automobile leaf springs, capacitance sensors together with measuring circuits and other major components. To increase the modeling accuracy, it is necessary to consider not only mechanical freedom, but also the electrical freedom. In this paper, the dynamic model based on the Lagrange-Maxwell equation is presented. Simulation results and test data show that the modeling is feasible and effective. One feature of the system is that it can provide static and dynamic measurements.

Key words: modeling; capacitance weighting system; electromechanical; Lagrange-Maxwell; simulation

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Over-loading is a problem in some developing countries. This causes many problems, e.g. dangerous and more accidents, and damaging road surface. To solve the over-loading problems in some developing countries, mechanical balance platforms and axle-load detectors are commonly used as load measurement systems on highway. The disadvantages of those systems are expensive to purchase, difficult to install and used in fixed locations[1,2]. In this research a capacitance weighing system has been developed. A feature of this system is that it can provide both static and dynamic load measurements.

1 Capacitance weighing system

The capacitance weighing system consists of two on-vehicle devices and an external monitoring device. The on-vehicle devices send signals to the external monitoring device, which performs data analysis and processing. An on-vehicle device consists of a capacitance sensor, a power supply, a measuring circuitry, an automotive leaf spring system and a shielding system. A capacitance sensor consists of an upper plate and a lower plate. It is mounted upon each axle of a vehicle. The lower unit is mounted on the axle, and the upper unit under the vehicle frame (see Fig.1). As different vehicle-loads cause different pressures on the leaf springs, the distance between the upper and lower units would change and hence the capacitance would change with the vehicle-load. The capacitance measuring circuit with a four-phase charge transfer circuit[3] converts the capacitance into a voltage as a measured signal.

To reduce the effect of slanting on measurement, two capacitance sensors are mounted in the centre of the front and back axles. When the upper unit slants, the distance between the upper unit and the lower unit in the centre does not change. More details of the system can be read on our previous publications[4].

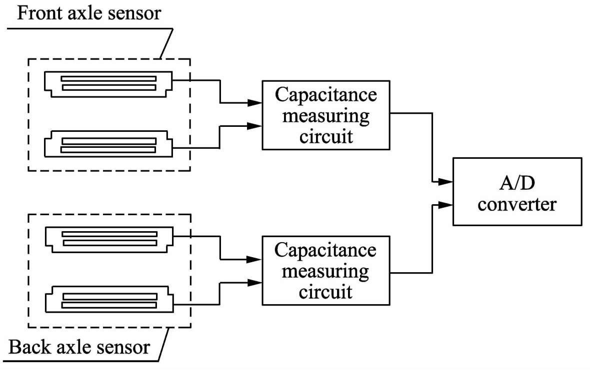

Fig.1 Capacitance weighing system

2 Dynamic model of capacitance weighing system

To measure load accurately, a model for the on-vehicle devices and the capacitance weighing system is needed, because the good model best represents reality. An automobile leaf spring consists of many

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leaf springs, which would be deformed under a load. Assuming that x is the deformation, the potential energy of flexibility of an automobile leaf spring is

$$V = \frac{1}{2}k(x)x^2, \tag{1}$$

where $k(x)$ is the stiffness of the leaf spring.

With different loads, the distance between the capacitance plates would change, and hence the output voltage of the capacitance sensor of the capacitance weighing system would change. The kinetic energy of the mechanical parts of the capacitance plates is

$$T = \frac{1}{2}m\dot{x}^2. \tag{2}$$

Let $E_0\sin\omega t$, r and $\beta(x)$ be the power, the resistance and the coefficients of self-inductance respectively. Let x and e be the displacement of capacitance plate and the charge on the capacitor. The electromagnetic energy and the electric energy are

$$W_{\mathrm{m}} = \frac{1}{2}\beta(x)\dot{e}^2, \tag{3}$$

$$W_{\mathrm{e}} = \frac{1}{2C(x)}e^2. \tag{4}$$

The capacitance weighing system is actually an electromechanical dynamical system with two degrees of freedom. According to the method of Refs.[5] and [6], by introducing the generalized coordinates x and e, a Lagrange function can be written as

$$L = T + W_m - W_e - V =$$
$$\frac{1}{2}m\dot{x}^2 + \frac{1}{2}\beta(x)\dot{e}^2 - \frac{1}{2}k(x)x^2 - \frac{1}{2C(x)}e^2. \tag{5}$$

The dissipative function is

$$F = \frac{1}{2}r\dot{e}^2 + \frac{1}{2}h\dot{x}^2, \tag{6}$$

where h is the coefficient of viscous friction associated with the displacement of the moving plate.

The virtual work of the external and dissipative forces is

$$\delta A = E_0\delta e\sin\omega t + Q\delta x, \tag{7}$$

where Q is the external force.

The Lagrange-Maxwell equations are

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{\partial L}{\partial \dot{e}}\right) - \dfrac{\partial L}{\partial e} + \dfrac{\partial F}{\partial \dot{e}} = E_0\sin\omega t, \\ \dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{\partial L}{\partial \dot{x}}\right) - \dfrac{\partial L}{\partial x} + \dfrac{\partial F}{\partial \dot{x}} = Q. \end{cases} \tag{8}$$

Now the equations of motion of the system can be written as

$$\begin{cases} \beta(x)\ddot{e} + r\dot{e} + \dfrac{e}{C(x)} = E_0\sin\omega t, \\ m\ddot{x} + h\dot{x} - \dfrac{1}{2}\dfrac{\partial\beta(x)}{\partial x}\dot{e}^2 + \dfrac{1}{2}\dfrac{\partial k(x)}{\partial x}x^2 + \\ \qquad \dfrac{1}{2}e^2\dfrac{\partial}{\partial x}\left(\dfrac{1}{C(x)}\right) = Q. \end{cases} \tag{9}$$

Eq. (9) can be used for static load measurement. However, it needs to be simplified to be a model. Assuming $k(x) = k$, $\beta(x) = \beta$, $Q = 0$, $h = 0$ and $C(x) = \dfrac{A}{s - x}$, the following equations of motion for the capacitance weighing system can be obtained

$$\begin{cases} \beta\ddot{e} + r\dot{e} + \dfrac{1}{A}(s - x)e = E_0\sin\omega t, \\ m\ddot{x} + k\dot{x} - \dfrac{1}{2A}e^2 = 0. \end{cases} \tag{10}$$

where s is the maximum value of the distance between the upper and lower plate, and A is a constant.

3 Simulation of capacitor weighing system and verification by test

Based on the model given by Eq. (10), let's consider the practice conditions of the on-vehicle devices with $\beta = 0.18$, $r = 0.2$, $s = 2$, $A = 2.5$, $\omega = 100\pi$, $m = 1$, $k = -6.29$ and $E_0 = 4$.

To make sure that the upper and lower units do not collide with each other, the distance between the inner surfaces of the upper and lower units is chosen to be 10 – 20 mm larger than the worst cast of distortion of the leaf springs.

Fig. 2 shows the simulation result using Eq. (10), indicating the effect of the change in distance on the measured voltage, in which the solid line denotes the coefficient of viscous friction $h = 0$ and the external force $Q = 0$. Numerical calculations show that the output value of capacitance sensor increases with the decrease of the distance between the upper and lower plate.

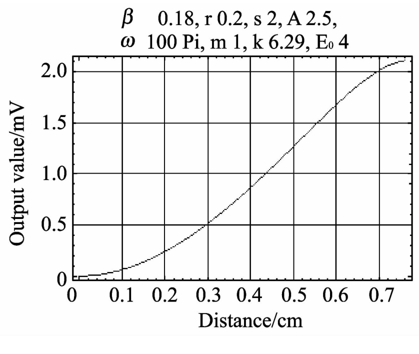
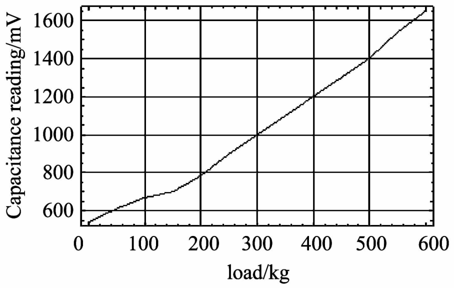


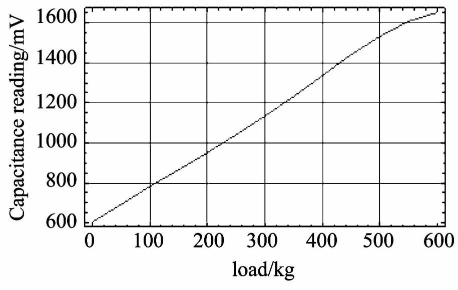
Fig. 2 Simulation result

To verify the above simulation, static tests were carried out on a pick-up truck with a rated load of 500 kg. The truck has 2 axles, 4 wheels and leaf springs. During the static tests, the truck was kept level, with two wheels of each axle on a platform scale vertically. The axle weight was measured by the capacitance weighing system. From Fig. 3(a), it can be seen that if the load increases gradually from 0 up to 600 kg, with an increment of 50 kg, the output of the capacitance sensor increases from the lowest point to the highest point, along the upward curve. If the load decreases gradually, 50 kg reduction each time, the output decreases from the highest point to the lowest point, along the downward curve as shown in Fig. 3(b). The upward cure and the downward curve do not coincide with each other. The average curve is shown in Fig. 3(c), which is used as the static relationship between the transducer output and the load.

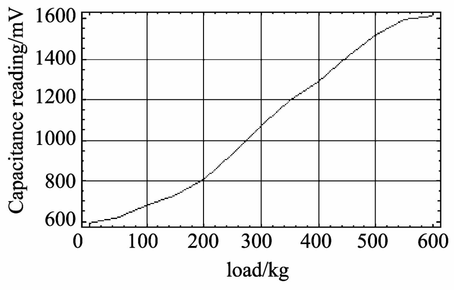
Fig. 3(d) shows the difference between the simulation result as shown in Fig. 2 and the measured average result as shown in Fig. 3(c), with a maximum error of 6.2%, indicating that the dynamic model of capacitance weighing system is effective.



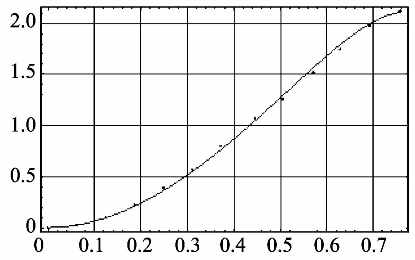
(a) Upward curve



(b) Downward curve



(c) Average curve



(d) Comparison between simulation result and the measured result

Fig. 3 Capacitance reading against load

4 Conclusion

In this work, modeling and simulation of a capacitance weighing system are studied, based on the Lagrange-Maxwell equation. It can be seen that the numeric simulation result is very similar to the testing average result with a maximum error of 6.2%. Static tests show that the hysteresis behavior is quite serious, due to the automotive leaf spring, the measuring circuit, and other reasons. Further study on the hysteresis of the capacitance weighing system and software compensation are needed[7]. To improve the accuracy of the model, several factors such as acceleration, humidity and braking need to be considered. Further development of the model for dynamic load measurement using the capacitance weighing system remains another future work.

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