Novel LMS adaptive filtering algorithm with variable step size

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Abstract: By analyzing algorithms available for variable step size least mean square(LMS) adaptive filter, a new modified LMS adaptive filtering algorithm with variable step size is proposed, along with performance analysis based on different parameters. Compared with the existing algorithms through the simulation, the proposed algorithm has faster convergence speed and smaller steady state error.

Key words: adaptive filter; variable step size; least mean square(LMS)

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0 Introduction

Least mean square (LMS) algorithm was proposed by Bernard Widrow in 1965 as a class of estimated gradient adaptive filtering algorithm[1-3]. This algorithm was simple in structure and easy in implementation, and was widely used in the fields of system identification, signal processing and noise canceling etc. However, in LMS algorithm, convergence speed and steady-state error cannot be satisfied simultaneously. The performance of the algorithm was controlled by the step size. While using larger convergence step size, algorithm has a faster convergence rate, but has high steady state error. While using smaller convergence step size, steady state error would be smaller, but it would have slower convergence speed. In order to overcome this shortcoming, numbers of improved LMS algorithms were proposed. The variable step size LMS algorithm was proposed based on the following idea that a larger step can accelerate the convergence of the algorithm in the initial stage of algorithm. When the algorithm tends to be stable, the steady state error becomes smaller, and the smaller convergence step will make the steady state error smaller.

The basic adaptive filtering schematic diagram is shown in Fig. 1. The input signal was $\boldsymbol{x}(n)$ at the moment n, the error signal was $\boldsymbol{e}(\mathrm{n})$, the output was $\boldsymbol{y}(\mathrm{n})$, $\boldsymbol{d}(n)$ was the desired signal. The coefficient of the adaptive filter was based on error $\boldsymbol{e}(n)$ for modulation:

$$\boldsymbol{e}(n) = \boldsymbol{d}(n) - \boldsymbol{x}^{\mathrm{T}}(n)w(n), \tag{1}$$

$$w(n+1) = w(n) + 2\boldsymbol{u}(n)\boldsymbol{e}(\mathrm{n})\boldsymbol{x}(\mathrm{n}). \tag{2}$$

The adaptive filter order is M, then

$$\boldsymbol{x}(n) = \{x(n), x(n-1), x(n-2)\cdots, x(n-M-1)\}^{\mathrm{T}}, \tag{3}$$

$$w(n) = \{w(n), w(n-1), w(n-2)\cdots, w(n-M-1)\}^{\mathrm{T}}. \tag{4}$$

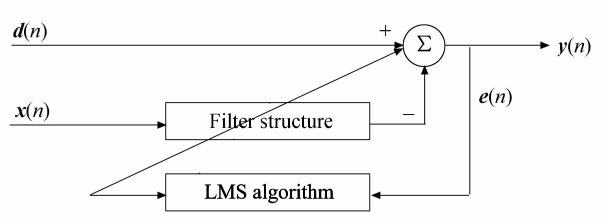

Fig. 1 Block diagram of adaptive filter

1 Analysis of different kinds of variable step size LMS algorithms

A class of adaptive filtering method was proposed in literature based on the traditional adaptive filter, the fixed step u was substituted by the variable step size, and the variable step size formula was[4]

$$u(n) = \beta(1/(1+\exp(-\alpha \mid \boldsymbol{e}(n) \mid^{3})) - 0.5). \tag{5}$$

This method in steady stage algorithm would still

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have a larger step change, causing larger algorithm steady-state error during the steady-state; the algorithm was improved aiming at the deficiency in the earlier research[6], the variable step size formula was

$$u(n) = \beta(1/(1 + \exp(-\alpha \mid \boldsymbol{e}(n)^2))). \quad (6)$$

A further improved method in Ref. [5] is presented compared to the one in Ref. [4]. To get

$$u(n) = \beta(1/(1 + \exp(-\alpha \mid e(n) \mid^3)) - 0.5),$$

simulation results show that the error of the method was much smaller than the former algorithm. At the same time, there were some other variable step size factor revision methods, as the step factor mentioned in Ref. [7] was written in the form of a piecewise function, the maximum and minimum values of step u was limited. The filter near the best weights generated a smaller mis-adjustment; The step factor proposed in Ref. [8] was rewritten as

$$u(n) = 1/a - 1/\sqrt{b \parallel \boldsymbol{e}(n)\boldsymbol{x}(n) \parallel + a^2}. \quad (7)$$

In Eq. (7), when $\boldsymbol{e}(n)$ closed to zero, $u(n)$ changed greatly, and the algorithm would produce greater steady-state error.

These above methods simply changed step length factor to achieve the purpose of improving algorithm. However, the new variable step LMS (NVLMS) algorithm mentioned in Ref. [7] realized the algorithm modified by changing the filter coefficient iterative formula way, but this algorithm had slow response in tracking of time-varying systems. Otherwise, a new convex combination of variable step-size LMS algorithm (VSCLMS) was proposed in Ref. [9]. The algorithm had a certain improvement in the convergence speed and the steady state error, but increased the computation.

2 Proposed variable step-size LMS algorithm

A larger step could accelerate the convergence of the algorithm in the initial stage of algorithm, when the algorithm tending to be stable, the steady state error became smaller, and the smaller convergence step would make the steady state error smaller. Based on the step-size adjustment principle of the algorithm new LMS adaptive filtering algorithm with variable step size was proposed.

$$\boldsymbol{e}(n) = \boldsymbol{d}(n) - \boldsymbol{x}^{\mathrm{T}}(n)w(n), \quad (8)$$

$$u(n) = \begin{cases} 2b, & a\pi \mid \dfrac{\boldsymbol{e}(n)}{\boldsymbol{e}(0)} \mid > \pi, \\ b(1 - \cos(a\pi \mid \dfrac{\boldsymbol{e}(n)}{\boldsymbol{e}(0)} \mid)), & a\pi \mid \dfrac{\boldsymbol{e}(n)}{\boldsymbol{e}(0)} \mid < \pi, \end{cases} \quad (9)$$

$$w(n+1) = w(n) + 2u\boldsymbol{e}(n)\boldsymbol{x}(n). \quad (10)$$

According to the LMS algorithm convergence condition $0 < u(n) < 1/\lambda_{\max}$, $u(n)$ should meet the following relations $0 < u(n) < 1/\lambda_{\max}$, and b should meet the condition $0 < b < 2\lambda_{\max}$, where $\lambda_{\max}$ was the largest eigen value of the input signal autocorrelation matrix.

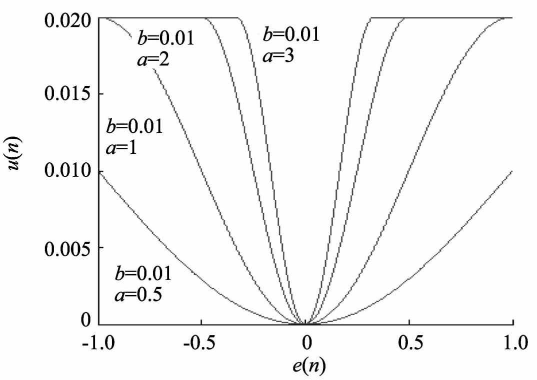

Fig. 2 Relationship curve of $e(n)$ and $u(n)$ when a is different

The relationship between $u(n)$ and $\boldsymbol{e}(n)$ was shown as in Fig. 2, the value of b was taken as 0.01, the value of a was taken as 0.5, 1, 2, 3. $u(n)$ should maintain a larger step size value when the error signal $\boldsymbol{e}(n)$ was larger as seen from the figure. $u(n)$ should maintain a smaller step size value when algorithm tends to be stable and steady state error $\boldsymbol{e}(n)$ maintains smaller value. As the value decreases, the curve would become more and more smooth so that the requirements of small steady state error algorithm could be met by adjusting a value. If b is constant, the convergence speed of the algorithm could be accelerated through increasing a value.

The relationship between $u(n)$ and $\boldsymbol{e}(n)$ was shown in Fig. 3, the value of a was taken as 0.01, the value of b was taken as 0.5, 1, 2, 3. When a was a constant, the maximum value of step u decreases as b decreases as shown in Fig. 3. When $\boldsymbol{e}(n)$ was smaller, $u(n)$ became smoother. Therefore, a suitable convergence step could be got through selecting the appropriate a and b.

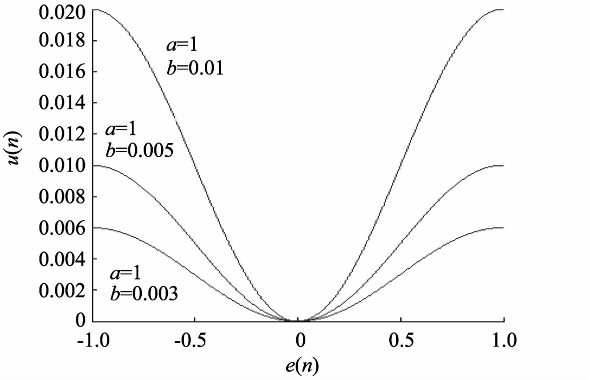

Fig. 3 Relationship curve of $e(n)$ and $u(n)$ when b is different

3 Simulation of the algorithm

In order to verify the proposed algorithm, it was compared with the algorithm in Ref. [5]. In this paper, an adaptive filter was designed as shown in Fig. 4. Using the proposed algorithm, a 32-order filter is designed, a is 10 and b is 0.005; A 32-order filter was designed using the algorithm in Ref. [5], the step factor was given by Eq. (5), where β was 0.01, α was 10. The input of the filter was signal $\boldsymbol{x}(n)$ with noise, the reference input was $\boldsymbol{d}(n)$. As shown in Fig. 5(a), filter removed noise signal through the error signal $\boldsymbol{e}(n)$ to adjust the coefficient of the adaptive filter. Signal in Fig. 5(b) was obtained by adaptive filter. Filter error curve is shown in Fig. 6.

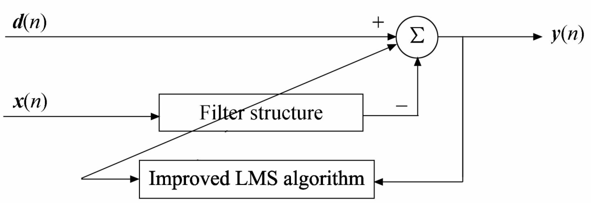

Fig. 4 Adaptive filter block diagram

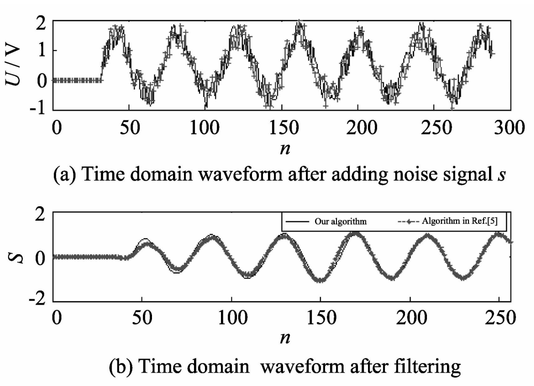

Fig. 5 Adaptive filter results

The simulation is carried out to verify influence on convergence speed and steady-state error when a or b takes different values. A 32-order filter was designed still using the filter structure as shown in Fig. 4. The simulation results in Fig. 7 takes the same value of a, and different values of b. The convergence speed gradually accelerated with the b increasing as seen from the simulation results, but the steady-state error also increased. The simulation results shown in Fig. 8 was taking the same value of b, the different values of a. The convergence speed gradually accelerated with the a increasing seen from the simulation results. Algorithm convergence speed and steady-state error could be guaranteed by selecting the appropriate a and b.

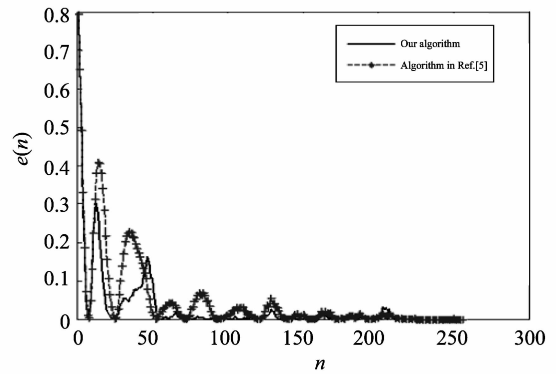

Fig. 6 Error curve of the adaptive filter

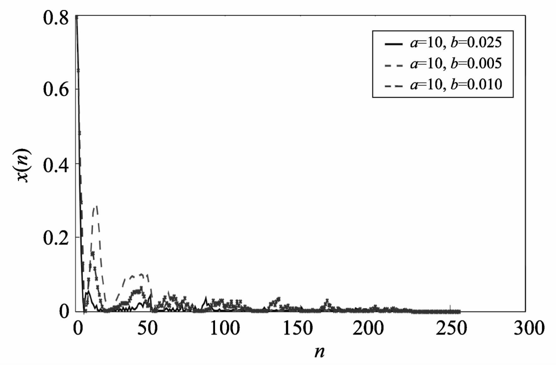

Fig. 7 Error curve of adaptive filter when b is different

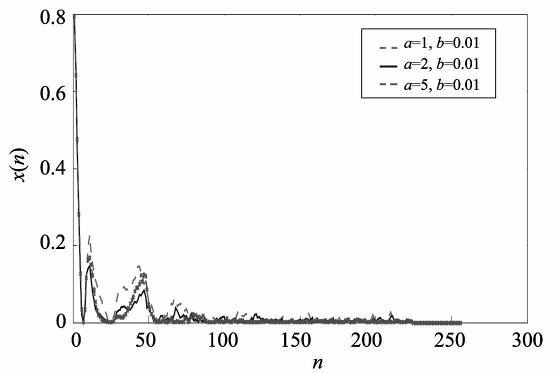

Fig. 8 Error curve of adaptive filter when a is different

4 Conclusion

The proposed algorithm could effectively eliminate the noise in the signal through the simulation. In the algorithm, convergence speed and steady-state error requirements could be satisfied by adjusting a and b values. However, the shortcoming was that a and b values could not be determined by theoretical calculations. Only a large number of tests could determine the appropriate a and b values to meet the requirements of the convergence speed and steady-state error of the system.

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