

# Design of Time-Domain Equalizer in Elastic Wave Earth Channel

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**Abstract** – In low-frequency elastic wave through-the-earth communication system, because of multipath transmission caused by characteristics of the layered earth, the time domain equalizer is different from other wireless communication systems. A modified LMS algorithm of variable step size is proposed based on improvement of traditional LMS. On the base of principle and simulation analysis, the improved Least Mean Square (LMS) algorithm is analyzed and the performances are compared between the improved LMS algorithm and traditional LMS algorithm. In the improved algorithm, the contradiction between convergence speed and the steady-state error is considered at the same time. Therefore, the improved algorithm has good convergence properties and channel-tracking performance.

**Key words** – *Inter-Symbol Interference (ISI); adaptive equalization; stratum channel; through-the-earth communication system*

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## 1 Introduction

In elastic wave through-the-earth communication system, the earth is communication medium. The reflection, refraction and scattering of signals caused by stratified structure of earth medium and inhomogeneity in layers can make severe multipath effect, and they are different from the multipath effect of elastic wave in some other channel. So more complex domain equalization should be used in this channel than the traditional methods.

In 1967, Austin proposed the decision feedback equalizer, and the study on time domain equalization has become the focus in the field of wireless communication from then on<sup>[2]</sup>. In 1969, Proakis and Miller put forward the adaptive equalization techniques based on Least Mean Square(LMS) algorithm, and it has become the fundamental theory in this field<sup>[3]</sup>. In past 10 years, fast convergence is important for all sorts of adaptive equalizers used in com-

munication system, so some new algorithms have been proposed<sup>[4-7]</sup>. Orthogonal LMS algorithm can achieve rapid convergence in the partial response system when the spectrum shape of received signal has been known<sup>[8]</sup>; and when the interference signals and input signals or the channel characteristics are unable to control, adaptive equalization based on Recursive Least Squares(RLS) which realizes fast tracking will be used<sup>[9]</sup>.

## 2 Multipath propagation model of the earth channel

The earth channel is a typical multipath fading channel. We can regard the received signal as superposition of multiple signal components from different paths with different delays and amplitudes.

Elastic wave signals sent by transmitting terminal diffuse to each direction in propagation process, therefore, signal propagations in each direction generate different conditions. We can regard the transmitting terminal as a spot radiation source. Propagation paths of elastic wave can be divided into three classes. The first kind is called direct wave, which arrives receiving point from the transmitting terminal directly. The second kind is called scattering wave. When arriving at the receiving point, it has refracted many times in medium because the medium is inhomogeneity. The last kind is called reflected wave. When arriving at receiving point, it has reflected many times in layered medium. Fig. 1 shows the multipath model of elastic waves propagating in the layered earth medium.

The superposition of multiple-path signals constitutes receipt signal. From Fig. 1, we know that signal of each path has different distance, so the times of signals arriving at receiver are different, and the signals' amplitudes change randomly in different paths. Both of these two reasons make signal

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broaden in time-domain, and amplitude emerges frequency selective decline, which is called multipath interference. Impulse response function of the earth related multipath channel can be set as

$$h(\tau) = A_0\delta(\tau - \tau_0) + \sum_{i=1}^{N-1} A_i\delta(\tau - \tau_i), \quad (1)$$

where  $A_i$  and  $\tau_i$  are propagation attenuation coefficient and relative delay in different paths respectively,  $N$  is the number of multipath components which can be distinguished. When the signals arrive at receiver, time delays and energy attenuation are different because the distances of different paths are dissimilar.

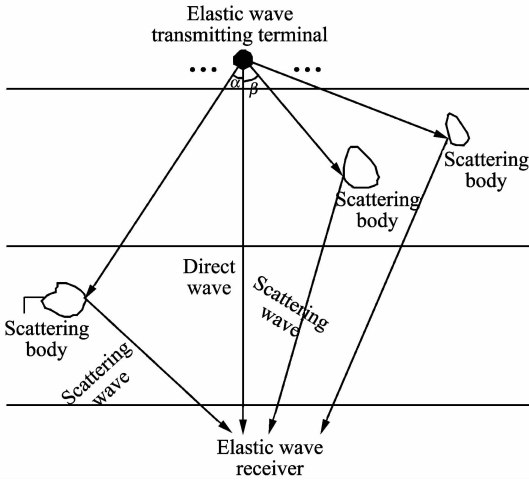


Fig. 1 Multipath model of elastic wave propagation in the earth

If elastic wave emission signal is set as  $Z(t)$ , the received waveform through the multipath channel is shown in Eq. (2)

$$s(t) = A_0Z(t - t_0) + \sum_{i=1}^{N-1} A_iZ(t - \tau_i) + n(t). \quad (2)$$

In Eq. (2),  $A_0Z(t - t_0)$  is direct wave,  $\sum_{i=1}^{N-1} A_iZ(t - \tau_i)$  is multipath expansion of reflected wave or transmission wave. If there is overlap between multipath expansion and direct wave in time, interferences will generate, then the waveform and amplitude of the composite signal will generate distortion. All of these reasons make the composite signal be different from the emission signal. Still in this equation,  $n(t)$  is interference noises, such as environmental noise and other interferences. If the delay between multipath expansion and direct wave is larger than code width, it will overlap with successive code waveform and generate interference, which is called multipath “intersymbol interference”. Otherwise, multipath “amplitude fading” will occur.

### 3 LMS algorithm of adaptive equalizer

The fundamental structure of adaptive equalizer is shown in Fig. 2. The subscript in the figure expresses discrete-time serial number.

In order to describe the adaptive equalized algorithm shown in Fig. 2, more convenient methods with vector and matrix are used. Input vector  $\mathbf{x}(n)$  of equalizer can be defined in Eq. (3).

$$\mathbf{x}_n = [x_n, x_{n-1}, \dots, x_{n-N+1}]^T. \quad (3)$$

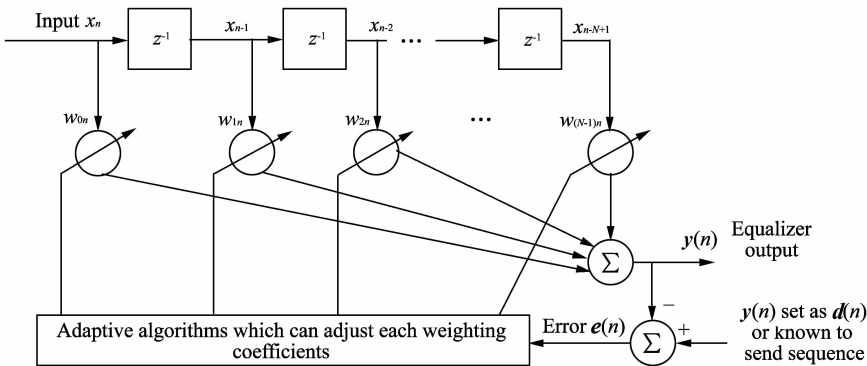


Fig. 2 The basic structure of adaptive equalizer

The weights vector  $\mathbf{w}(n)$  can be defined as

$$\mathbf{w}(n) = [w_{0n}, w_{1n}, \dots, w_{(N-1)n}]^T \quad (4)$$

Then, output of the equalizer is shown in Eq.

(5).

$$\mathbf{y}(n) = \mathbf{x}^T(n)\mathbf{w}(n) = \mathbf{w}^T(n)\mathbf{x}(n). \quad (5)$$

If the expected output  $d(n)$  of the equalizer is known, the error signal  $e(n)$  can be described as

$$\begin{aligned} e(n) &= d(n) - \mathbf{y}(n) = d(n) - \mathbf{x}^T(n)\mathbf{w}(n) \\ &= d(n) - \mathbf{w}^T(n)\mathbf{x}(n). \end{aligned} \quad (6)$$

Therefore, the mean square value of  $e(n)$  can be set as

$$E[e(n)] = [E d(n)] + \mathbf{w}^T(n)E[\mathbf{x}(n)\mathbf{x}^T(n)]\mathbf{w}(n) - 2E[d(n)\mathbf{x}^T(n)]\mathbf{w}(n). \quad (7)$$

And we assume that the autocorrelation matrix  $\mathbf{R}$  of the equalizer input sequence can be expressed as

$$\mathbf{R} = E[\mathbf{x}(n)\mathbf{x}^T(n)]. \quad (8)$$

$\mathbf{P}$  is the cross-correlation matrix between input

sequence and desired signal  $\mathbf{d}(n)$ , expressed by Eq. (9).

$$\mathbf{P} = E[\mathbf{d}(n)\mathbf{x}^T(n)]. \quad (9)$$

If  $\mathbf{d}(n)$  and  $\mathbf{y}(n)$  are stable, the elements of  $\mathbf{P}$  and  $\mathbf{R}$  are second-order statistics, and they do not change in the following time. So Eq. (7) can also be expressed as

$$E[e^2(n)] = E[\mathbf{d}^2(n)] + \mathbf{w}^T(n)\mathbf{R}\mathbf{w}(n) - 2\mathbf{P}^T\mathbf{w}(n). \quad (10)$$

The renewal equation of LMS algorithm can be expressed as

$$\mathbf{w}(n+1) = \mathbf{w}(n) + 2\mu e(n)\mathbf{x}(n). \quad (11)$$

In practical applications, the minimum value of Mean Square Deviation (MSE) is obtained by recursion according to LMS. The minimum MSE of LMS algorithm is made in iterative solution by statistical gradient algorithm, which is the simplest equalization one. Each iteration of LMS algorithm needs  $N+2$  times multiplication (used for renewal of equalizer) and  $N+1$  times multiplication (used for generating error signal).

Iterative steps of LMS algorithm can be summarized in equations below ( $n$  is serial number of iterative process)

$$\mathbf{y}(n) = \mathbf{x}^T(n)\mathbf{w}(n), \quad (12)$$

$$\mathbf{e}(n) = \mathbf{d}(n) - \mathbf{y}(n), \quad (13)$$

$$\mathbf{w}(n+1) = \mathbf{w}(n) + 2\mu e(n)\mathbf{x}(n). \quad (14)$$

In Eq. (14),  $\mu$  is step and controls convergence rate and stability of the algorithm. In a real system, in order to make the equalizer convergence, the primary condition is that propagation equalizer delay  $(N-1)T$  should be larger than the largest relative delay of the channel. The value of  $\mu$  should meet conditions as shown in Eq. (15) in order to prevent equalizer instability.

$$0 < \mu \leq \frac{1}{\sum_{k=1}^N E[\mathbf{x}^2(n)]}, \quad (15)$$

where  $\sum_{k=1}^N E[\mathbf{x}^2(n)]$  is total power of the input signal, which is usually known.

## 4 Modified LMS algorithm based on the elastic wave earth channel

Advantages of LMS algorithm are smallest MSE and comparatively simple algorithm. But the rate of convergence is comparative slow because only the information of present code is considered, especially in the condition of relatively great eigenvalue dispersion. When the channels change quickly, RLS algorithm is used for tracking the changes rapidly. For the earth channel, changes are relatively slow and requirement for real-time is not too high, but strong noise causing error accumulation is more serious. So requirement for tracking performance of al-

gorithm is not too high, but the stable algorithm which has small accumulation is required. Therefore, a fast and easy convergence to realize algorithm is used for equalizer in the earth channel. In this paper, one kind of modified LMS algorithm is used for the earth channel, and the purpose is faster convergence.

One variable step size LMS algorithm is got by step changing. In order to obtain a reasonable compromise between convergence rate and steady-state error, equations are established by setting up constraint relation between error and step. And it is shown as

$$\mu(n) = \frac{1}{a} - \frac{1}{a + b\sqrt{\frac{e^2(n) + e^2(n-1)}{2}}}. \quad (16)$$

From Eq. (16), we can know that when  $e(n)$  and  $e(n-1)$  are big enough, the posterior part of this equation could be ignored; then  $\mu = 1/a$ , condition of convergence is  $\lambda_{\max} < a$ ,  $\lambda_{\max}$  is the maximum eigenvalue of input signal  $\mathbf{x}(n)$  autocorrelation matrix.

The basic relations of modified LMS algorithm are shown as follows

1) Filtering output

$$\mathbf{y}(n) = \mathbf{x}(n)^T\mathbf{w}(n); \quad (17)$$

2) Estimation error or error signal is

$$\mathbf{e}(n) = \mathbf{d}(n) - \mathbf{y}(n); \quad (18)$$

3) Variable step size factor is

$$\mu(n) = \frac{1}{a} - \frac{1}{a + b\sqrt{\frac{e^2(n) + e^2(n-1)}{2}}}; \quad (19)$$

4) Tap weight vectors is

$$\mathbf{w}(n+1) = \mathbf{w}(n) + 2\mu e(n)\mathbf{x}(n). \quad (20)$$

## 5 Simulation and analysis

Whether convergence rate is improved or not could be validated by simulation. Therefore, in a certain SNR condition, the performance is compared by the error curve of equalizer's output symbols.

The channel model of this paper is that of fading obeying Rayleigh distribution. Rayleighchan function, which is self-brought by Matlab, was used for simulation model of channel. The parameters of multipath Rayleigh fading channel using the measured parameters of earth model when Doppler shift is set at 0. And it is assumed that the signal passes through three paths of Rayleigh fading channel. Besides, the noise of this system is additive white Gaussian noise. Based on the above assumptions, adaptive channel equalizer is studied, and the parameters of simulation are: SNR is 25 dB; the length of original information sequence, 100; iteration number,

500; the number of DFE forward filter coefficient, 31; the number of feedback filter coefficient, 5; the frequency of data transmission, 1 000 Hz; data transmission rate, 500 bit/s. Bit stream, which is generated from random signal source, becomes QPSK symbols by QPSK modulation mapping firstly, and this mapping relation is shown in Tab. 1.

Tab. 1 The symbols mapping relation

Bit	QPSK symbols
00	1
01	$i$
10	-1
11	$-i$

Finally, we observed the effect of schemes given above. Scheme 1 contained modified LMS algorithm and DFE equalizer without channel estimation, and scheme 2 contained channel estimation, which was the difference between scheme 1 and 2. The output symbol error curve of DFE equalizer of two schemes is illustrated in Fig. 3.

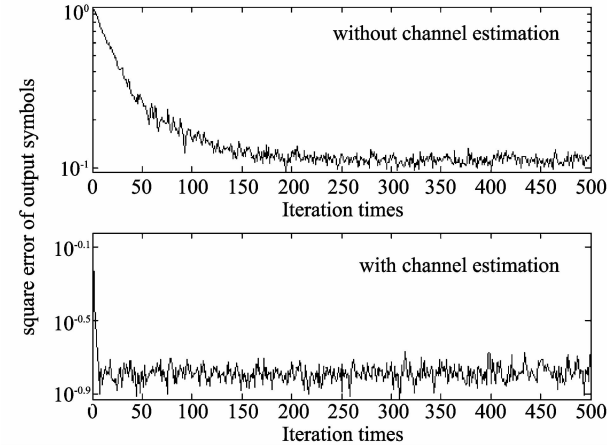


Fig. 3 The output symbols error curve of DFE equalizer under two schemes

From simulation results we know that the scheme of equalizer designed in this paper greatly accelerates the convergence speed in a certain SNR. The improvement scheme with channel estimation can be stable within 50 times iteration, while the scheme without channel estimation needs about 150 times iteration. So the convergence rate of improved scheme can save more than half of time against scheme 1. And as seen from the comparison of vertical axis, square error of output symbol has decreased a lot. Errors of output have decreased a lot for initial value of channel estimation at the beginning of initialization. Consequently, under the condition of better starting initial value, the channel achieves stability faster, and convergence speed is improved.

In this paper, the initial value is obtained by the way of training sequence to research related channel estimation and initialize the tap coefficient

of equalizer, and some information of channel is contained in the estimation value of channel, which is easier to track channel. Therefore, speed of the system to reach steady working state is accelerated greatly. Changes of channel are tracked better in the process of adaptation and modified LMS algorithm is optimized.

On the other hand, the equalization performance is judged by constellations graphs which are before and after equalization. Multipath Rayleigh fading channel is simulated in the modulation mode of QPSK as shown in Fig. 4 and Fig. 5. From these simulation results, we can see that the constellation after equalization is more concentrated than before. In other words, constellation points after equalization using the adaptive equalizer designed in this paper are more concentrated. Data recovery is more accurate and equalization result is better too.

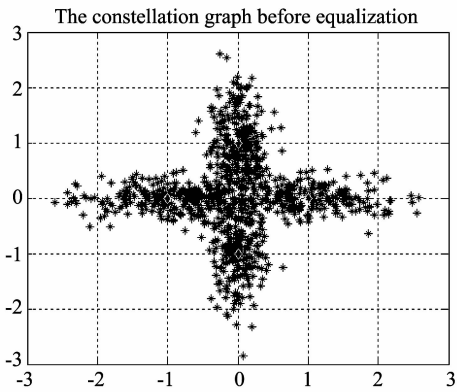


Fig. 4 The constellation graph before equalization

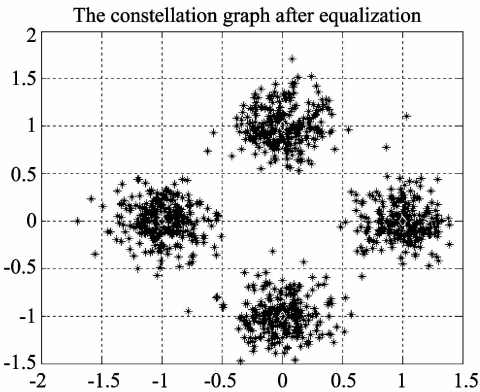


Fig. 5 The constellation graph after equalization

## 6 Conclusions

We can draw a conclusion that the equalizer with channel estimation designed in this paper reduces intersymbol interference and remove multipath effect obviously, which greatly improves performance of the previous equalizer. For the above reasons, the scheme proposed in this paper is suit-

able for elastic wave through-the-earth communication system. The equalized scheme of channel estimation combined with adaptive decision feedback accelerates the equalizer convergence speed, and the error rate value meets requirement of SNR in performance. Overall, adaptive equalizer in this paper has better performance in convergence speed, equalization effect and bit error rate than ordinary equalizer. In addition, the system achieves certain equalization effect, and the receiver works normally.

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