Error modeling and analysis of inclinometer based on digital accelerometer

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Abstract: Measuring accuracy of inclinometer based on accelerometer is mainly influenced by the adopted accelerometer sensor. To improve the measuring accuracy of the inclinometer, the structure of the measuring system is given and measuring principle is analyzed, and the error model is established in this paper. Furthermore, the model is verified by simulation and experiment, which not only gives the smallest errors of the measured pitch and roll, but also lays foundation for sensor selection, error analysis and error compensation. The results show that the error model is of practical value.

Key words: inclinometer; accelerometer; error modeling; error analysis

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An inclinometer is an instrument for measuring the angles of scope or depression of an object with respect to gravity, including simple pendulum inclinometer, gyroscope inclinometer, inclinometer based on accelerometer, etc. The simple pendulum inclinometer has complex construction, hard manufacturing technology, small measure range, bad transient response and slow stable speed[1-4]. High accuracy gyroscope inclinometer has high measuring accuracy, but the error will accumulate with time and its price is high. Compared with the above two kinds of inclinometers, the inclinometer based on accelerometer has such advantages as all-weather, small interference and high accuracy, so it is widely used to measure the slope angles in practical applications.

The inclinometer based on accelerometer has high measuring accuracy, but the error of acceleration sensor will influence measuring accuracy of the inclinometer. To study the effect of acceleration sensor's measuring error on measuring accuracy of the inclinometer, error analysis of inclinometer is needed. On the basis of error sources analysis, the error model is established, and then it is simulated by Matlab and verified through experiment finally.

1 Structure and measuring principle of inclinometer

The inclinometer discussed in this paper consists of triaxial accelerometer, A/D converter, microcontroller unit (MCU) and output interface, as shown in Fig.1.

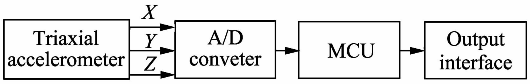

Fig.1 Composition of inclinometer based on accelerometer

The gravitational field around the earth has characteristics such as all-weather, constant orientation and small variation, which makes it possible to measure the inclination of the object in it. Thus, the navigation coordinate system o-ned (n coordinate system) and body coordinate system o_b-$x_by_bz_b$ (b coordinate system) are defined, as shown in Fig.2.

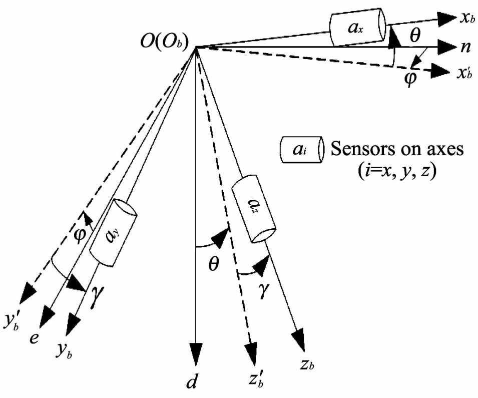

Fig.2 Sensors installation and definition of navigation coordinate system

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According to the definition of the coordinat systems, the transformation from o-ned to o_b-$x_b y_b z_b$ is complished as follows: the inclinometer rotates for φ degree to o-$x_b' y_b' d$ around d axis firstly, then it rotates for θ degree to o-$x_b y_b' z_b'$ coordinate system around y_b' axis, finally, it rotates for γ degree to o-$x_b y_b z_b$ coordinate system around x_b axis. Transformation matrices are shown in Eq. (1). In these matrices, φ is yaw, θ is pitch and γ is roll.

$$\boldsymbol{C}_n^b = \boldsymbol{C}_1 \boldsymbol{C}_2 \boldsymbol{C}_3, \tag{1}$$

where

$$\boldsymbol{C}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & \sin\gamma \\ 0 & -\sin\gamma & \cos\gamma \end{bmatrix},$$

$$\boldsymbol{C}_2 = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix},$$

$$\boldsymbol{C}_3 = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The relationship between measuring values a_x, a_y, a_z obtained by sensors and gravitional acceleration g in n coordinate system is given as

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \boldsymbol{C}_n^b \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}. \tag{2}$$

The pitch θ and roll φ can be solved from Eq. (2), their expressions are

$$\theta = \arcsin\frac{-a_x}{g}, \tag{3}$$

$$\gamma = \arctan\frac{a_y}{a_z}. \tag{4}$$

The above two equations show the basic measuring principle of inclination based on accelerometers. The inclination can be got when the accelerometers' measuring values are taken into the equations. It can also be seen that accuracy of the inclinometer is influenced by the measuring error of the sensors.

2 Inclinometer error modeling

From Eq. (3) and Eq. (4), it can be known that inclinometer error mainly results from the measuring error of acceleration sensors. The errors of pitch θ and roll γ can be deduced by Taylor formula as

$$\Delta\theta \approx \frac{1}{g\cos\theta_0}\Delta a_x, \tag{5}$$

$$\Delta\gamma \approx -\frac{\cos\gamma_0}{g\cos\theta_0}\Delta a_y + \frac{\sin\gamma_0}{g\cos\theta_0}\Delta a_z, \tag{6}$$

where Δa_x, Δa_y and Δa_z are measuring errors of the sensor on the axes, respectively; θ_0 and γ_0 are the real values of pitch and roll; $\Delta\theta$ and $\Delta\gamma$ are measuring errors of pitch and roll.

Accelerometer error Δa obeys normal distribution with parameters μ and σ, where μ is sensor's zero bias error and σ is standard deviation of random error.

$$\Delta a \sim N(\mu, \sigma^2).$$

Suppose the errors of x axis, y axis and z axis are mutually independent, $\Delta\theta$ and $\Delta\gamma$ will obey the following statistical laws.

$$\Delta\theta \sim N\left(\frac{1}{g\cos\theta_0}\mu_x, \left(\frac{\sigma(\Delta a_x)}{g\cos\theta_0}\right)^2\right),$$

$$\Delta\gamma \sim N\left(\frac{-\cos\gamma_0}{g\cos\theta_0}\mu_y + \frac{\sin\gamma_0}{g\cos\theta_0}\mu_z,\right.$$

$$\left.\left(\frac{\cos\gamma_0}{g\cos\theta_0}\sigma(\Delta a_y)\right)^2 + \left(\frac{\sin\gamma_0}{g\cos\theta_0}\sigma(\Delta a_z)\right)^2\right),$$

where $\sigma(\Delta a_x)$, $\sigma(\Delta a_y)$ and $\sigma(\Delta a_z)$ are standard deviations of measuring errors on x axis, y axis and z axis. $\Delta\theta$ is pitch error and $\Delta\gamma$ is roll error. As the selected sensors are the same type, Δa_y and Δa_z are considered the same value approximately.

The mathematical expectations and standard deviations of $\Delta\theta$ and $\Delta\gamma$ are deduced by statistical methods as

$$E(\Delta\theta) = \frac{1}{g\cos\theta_0}\mu_x, \tag{7}$$

$$E(\Delta\gamma) = \frac{-\cos\gamma_0}{g\cos\theta_0}\mu_y + \frac{\sin\gamma_0}{g\cos\theta_0}\mu_z, \tag{8}$$

$$\sigma(\Delta\theta) = \frac{1}{g\cos\theta_0}\sigma(\Delta a_x), \tag{9}$$

$$\sigma(\Delta\gamma) = \frac{1}{g\cos\theta_0}\sigma(\Delta a_y). \tag{10}$$

From error model of the inclinometer, it can be seen that pitch error is related to the measured pitch, and roll error is related to the measured pitch and roll.

3 Error model simulation and analysis

We suppose the accelerometer's error Δa obeys the following statistical law. According to the above four statistical formulae, the measuring errors and standard deviation variation tendency of pitch and roll are simulated.

$$\Delta a \sim N(0.012\ g_n,\ 0.001\ g_n).$$

3.1 Pitch error simulation

According to Eqs. (7) and (9), the change tendencies of error and standard deviation are simulated when pitch ranges from −60° to 60°, as shown in Fig. 3 and Fig. 4, respectively.

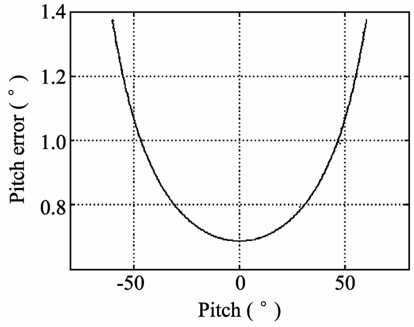

Fig. 3 Pitch error curve

By analysis of Eq. (7) and (9) and by observation of Fig. 3 and Fig. 4, it can be seen that error and standard deviation of pitch are related to the measured pitch, and they increase with the increase of absolute value of pitch.

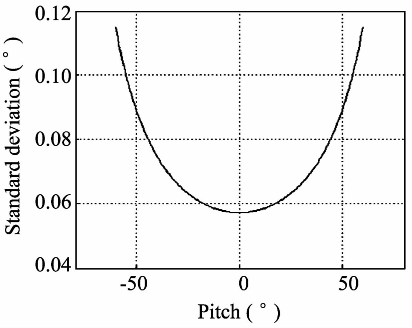

Fig. 4 Pitch standard deviation curve

3.2 Roll error simulation

Figs. 5 and 6 show the error and standard devication of roll after simulation.

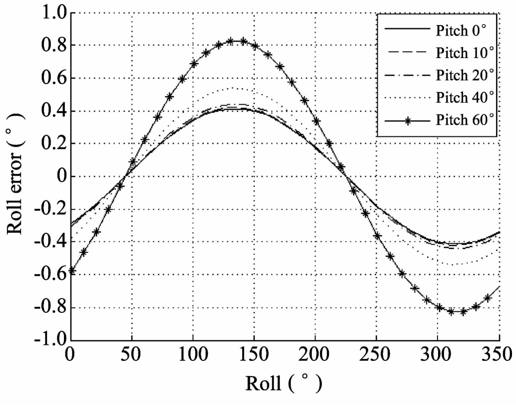

Fig. 5 Mathematical expectation of roll error under different pitches

It can be seen from Fig. 5 that mathematical expectation of roll error is related to the measured pitch and roll. Roll error changes like a sine curve when pitch is invariable. The smallest values are got at 45° and 225°, and the biggest values appear at 135° and 315°. On condition that there exists the same roll, roll error increases with the increase of pitch.

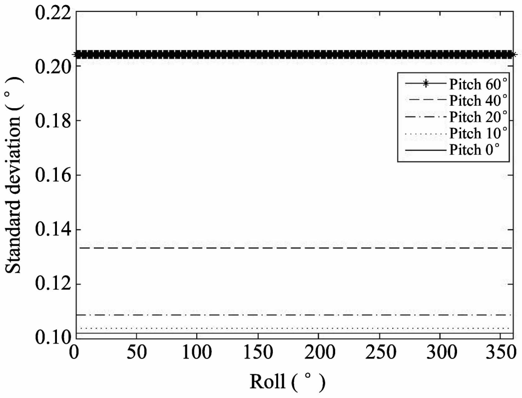

Fig. 6 Standard deviation of roll error's under different pitches

It can be seen from Eq. (10) and Fig. 6, standard deviation of roll error is independent of roll itself. When pitch is invariable, standard deviation curve may be a horizontal line, and its value has relation to pitch.

4 Experiment

Through the triaxial position turntable experiment, the established error model are verified. ADXL325 triaxial accelerometer is used as the acceleration sensors, whose range is ± 6 g_n, and the noise level of the inclinometer output is ± 250 μg_n.

4.1 Pitch experiment

Roll stays at 0° during the experiment, pitch ranges from −80° to 80°, each time increasing 10°. By comparing the pitches of turntable with the pitches solved by the inclinometer, the pitch error curve and standard deviation curve are drawn, as shown in Fig. 7.

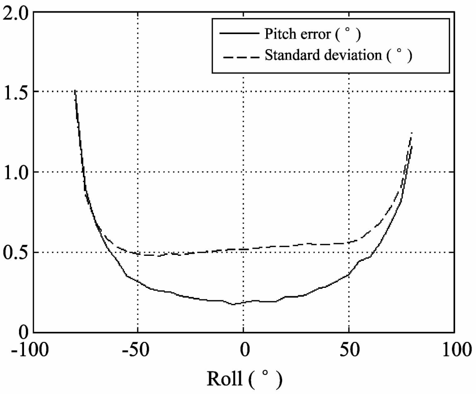

Fig. 7 Pitch error curve and standard deviation curve

From the curves we can see that error increases while the pitch increasing, the variation tendency is basically the same as the simulation result, but the error values have some bias. Standard deviation has big error while pitch ranges from −50° to 50°.

4.2 Roll experiment

The pitch are set 0°, 10°, 20°, 40° and 60°, and roll rotates from 0° to 360° for each pitch, increaing 10° one time. Comparing the solved results and simulation results, the error and standard deviation are calculated, as shown in Fig. 8 and Fig. 9.

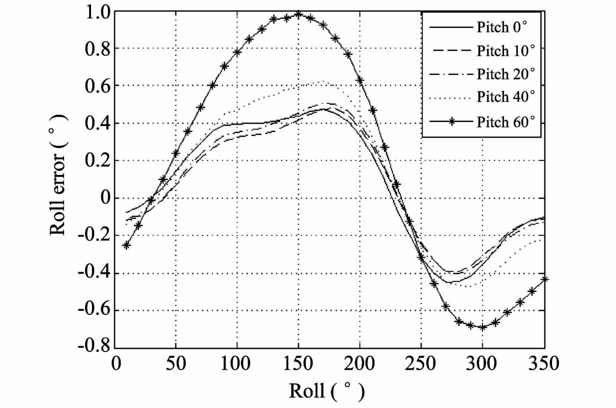

Fig. 8 Roll error curves

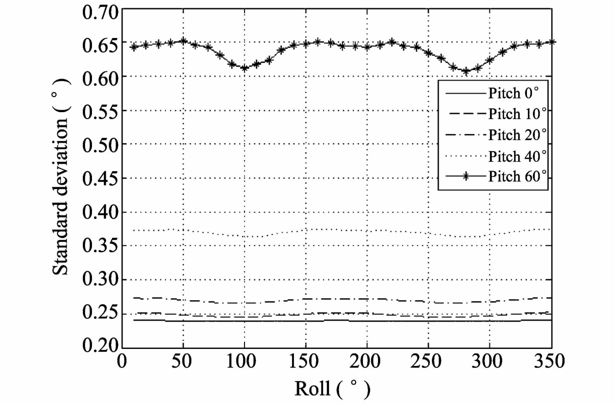

Fig. 9 Roll error standard deviation curves

According to the comparison between experiment results and simulation results, it can be seen that experiment results have the same change tendency with simulation results. But the former have bigger error values, especially when pitch is greater that 60°, roll error standard deviations are entirely 0.5° higher than those of the simulation results.

The reasons may be noisy interference in environment, turntable accuracy, sensor's error and noise level. Simulation environment is perfect, so it get the perfect results.

5 Conclusion

The principle of inclinometer based on accelerometer is introduced in this paper, and system error sources are also analyzed. Measuring error's distribution law is provided by statistical law and is analyzed. The established mathematical model in this paper can be used to not only select sensors, but also calculate the sensor' error when the system error range is given. This model can give the credible range, which provides reference while choosing inclinometer with high accuracy range.

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